STAT 3502 Winter 2015 Assignment 1 Solutions

1. (a) $P(A \cap B \cap C) = P(A)P(B)P(C) = .85 \times .7 \times .65 = 0.38675$.

   (b) There are two ways to approach this problem

      i) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A)P(B) - P(A)P(C) - P(B)P(C) + P(A)P(B)P(C) = .85 + .7 + .65 - .85 \times .7 - .85 \times .65 - .7 \times .65 + .85 \times .7 \times .65 = 0.98425$

   OR

      ii) P(at least one flight is full) = 1 − P(no flights are full) $= 1 - P(A')P(B')P(C') = 1 - 0.15 \times 0.3 \times 0.35 = 1 - .00225 = 0.98425$

   (c) $P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C) = P(A)P(B')P(C') + P(A')P(B)P(C') + P(A')P(B')P(C) = .85 \times .3 \times .05 + .15 \times .7 \times .05 + .15 \times .3 \times .95 = 0.15525$

2. (a) $P(AAA) = P(A)P(A)P(A) = .4^3 = 0.064$

   (b) $P(AA) + P(BB) + P(ABAB) + P(OO) = P(A)P(A) + P(B)P(B) + P(AB)P(AB) + P(O)P(O) = 0.4^2 + 0.12^2 + .05^2 + .43^2 = .3618$

3. (a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .73 + .87 - .89 = .71$. This event is the probability that the lifeline usage amount is not exceeded in the two months.

   (b) There are two ways to approach this problem

      i) P(exceeded in at most one month) = P(not exceeded) + P(exceeded in exactly one of the two months) $= P(A \cap B) + P[(A' \cap B) \cup (A \cap B')] = P(A \cap B) + [P(A \cup B) - P(A \cap B)] = 0.89$

   OR

      ii) P(exceeded in at most one) = 1 - P(exceeding in both months) $= 1 - P(A' \cap B') = P(A \cup B) = 0.89$ by DeMorgan's Law

4. (a) The four possible events are (1,1), (1,2), (2,1), and (2,2), with understanding that the first component corresponds to the health plan (1 or 2), and the second component corresponds to the dental plan (1 or 2).

   (b) P(save as much as possible) = P(1,1) = 0.27

   (c) P(1,2) + P(2,2) = .14+.35 = .49

5. (a) We want the number of ways of selecting 3 individuals out of 15 individuals, order being irrelevant. This number is

$$\binom{15}{3} = \frac{15 \times 14 \times 13}{3 \times 2} = 455$$

(b) No crew should contain any of the best three machinists. That is all three-member crews have to be selected out of the worst $15 - 3 = 12$. The number of ways to do this is

$$\binom{12}{3} = \frac{12 \times 11 \times 10}{3 \times 2} = 220$$

(c) Thinking of the complement, we just need to take away crews that have all members from the best 5 machinists. The number of interest = the number of all possible crews - the number of crews formed by the worst 10 machinists =

$$\binom{15}{3} - \binom{10}{3} = 455 - \frac{10 \times 9 \times 8}{3 \times 2} = 335.$$

6. (a) Let G_1 = the customer uses regular unleaded gas, G_2= the customer uses extra unleaded gas, and G_3=the customer uses premium unleaded gas. Let A=the customer fills their tank. Then we have

$$P(G_2 \cap A) = P(A|G_2)P(G_2) = 0.6 \times 0.4 = 0.24$$

(b) Using total probability formula, we get

$$P(A) = P(A|G_1)P(G_1) + P(A|G_2)P(G_2) + P(A|G_3)P(G_3) = .75 \times .35 + .6 \times .4 + .55 \times .25 = .64$$

(c) Using Bayes rule,

$$\begin{aligned} P(G_1|A) &= \frac{P(A|G_1)P(G_1)}{P(A)} = \frac{.75 \times .35}{.64} = 0.41016 \\ P(G_2|A) &= \frac{P(A|G_2)P(G_2)}{P(A)} = \frac{.6 \times .4}{.64} = 0.375 \\ P(G_3|A) &= \frac{P(A|G_3)P(G_3)}{P(A)} = \frac{.55 \times .25}{.64} = 0.2148 \end{aligned}$$

7. P(System works) = P(Path 1 works or Path 2 works) = $P(A' \cup B') = 1 - P(A \cap B) = 1 - P(A)P(B) = 1 - 0.03 \times 0.01 = .9997$ (by DeMorgan's law)

8. (a)

$$\begin{aligned} E(X) &= 13.5 \times .2 + 15.9 \times .3 + 17.2 \times .35 + 19.1 \times .15 = 16.355 \\ E(X^2) &= 13.5^2 \times .2 + 15.9^2 \times .3 + 17.2^2 \times .35 + 19.1^2 \times .15 = 270.5585 \\ V(X) &= E(X^2) - [E(X)]^2 = 270.5585 - 16.355^2 = 3.072475 \\ SD(X) &= \sqrt{V(X)} = 1.752848 \end{aligned}$$

(b) $E(\text{price}) = E(25X - 4) = 25E(X) - 4 = 25 \times 16.355 - 4 = 404.875$

(c) $V(\text{price}) = V(25X - 4) = 25^2 V(X) = 25^2 \times 3.072475 = 1920.297$

(d) $E[h(X)] = E(X - 0.005X^2) = E(X) - 0.005E(X^2)$
$= 16.355 - 0.005 \times 270.5585 = 15.00221$

9. (a) $P(X \leq 2) = p(0) + p(1) + p(2) = .15 + .25 + .35 = .75$

(b) $P(X < 1) = p(0) = .15$

(c)

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.15 & 0 \leq x < 1 \\ 0.4 & 1 \leq x < 2 \\ 0.75 & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

(d) F(3) - F(1) = .75 - .15 = 0.6
NOTE: If the student used p(2) + p(3), only give half marks