FACULTY OF ENGINEERING

PROBABILITY AND STATISTICS FOR ENGINEERS
ENGG 319

Final Examination

December 19, 2007 — Time: 3 hrs.

THE EXAM IS OPEN BOOK

CALCULATORS: SANCTIONED SCHULICH SCHOOL OF ENGINEERING CALCULATORS ARE ALLOWED

There are 30 questions.
Answer all 30 questions by indicating the letter of the correct answer on the scoring sheet.

Each question answered correctly is awarded 1 mark.
Each question answered incorrectly is awarded 0 marks.

Total possible for entire exam is 30 marks.

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1. Calculate the sample standard deviation from the 11 sample data below:

   25 30 35 40 45 50 55 60 65 70 75

   a) 50　　b) 16.58　　c) 250　　d) 275
   e) None of the above

2. A student is testing an electric motor with a 100 RPM at a constant speed. He has randomly sampled the RPM of the motor 25 times and the sample mean was computed at 100RPM. Assume that the RPM follows a normal distribution with a true standard deviation of 10, what is the 95% CI for the mean (μ) RPM of the 100 RPM motor?

   a) $98.10 < \mu < 101.90$　　b) $90.00 < \mu < 110.00$
   c) $92.43 < \mu < 107.57$　　d) $96.08 < \mu < 103.92$
   e) None of the above

3. A laser printer normally print 150 page/min. 20 test printers printed mean of 125 pages per minute with a variance of 80. What is the 99% confidence interval for the page/min? Assume a normal distribution.
   a) 125.0 ± 6.338　　b) 62.5 ± 3.920　　c) 150.0 ± 3.920
   d) 125.0 ± 56.69　　e) None of the above

4. **A.** What hypothesis is a juror testing if he/she commits a Type **I** error by finding a defendant "*guilty*".
   B. What hypothesis is a juror testing if he/she commits a Type **II** error by finding a defendant "*guilty*".

   a) **A.** The defendant is guilty; **B.** The defendant is guilty.
   b) **A.** The defendant is NOT guilty; **B.** The defendant is NOT guilty.
   c) **A.** The defendant is guilty; **B.** The defendant is NOT guilty.
   d) **A.** The defendant is NOT guilty; **B.** The defendant is guilty.
   e) None of the above (can NOT decide)

5. A beverage company is testing the performance of its bottling system. One measure of performance is the average number of bottle filled in a minute. To estimate this value, an employee recorded the number from random attempts of 121. The following sample statistics were computed: $\overline{X}$ of 120 bottles with an S^2 of 49. Estimate the true mean (μ) number of bottle filled in a minute with a 90% CI.

   a) $120.0 - 2.365 \times 4.455 < \mu < 120.0 + 2.365 \times 4.455$
   b) $120.0 - 1.645 \times 0.636 < \mu < 120.0 + 1.645 \times 0.636$
   c) $60.0 - 1.943 \times 0.636 < \mu < 60.0 + 1.943 \times 0.636$
   d) $120.0 - 1.645 \times 4.455 < \mu < 120.0 + 1.645 \times 4.455$
   e) None of the above

For Questions **6** and **7**, use the information provided in the question below.

In a study to determine how the skill in doing a complex assembly job is influenced by the amount of training, 20 new recruits were given varying amounts of training ranging between 5 and 20 hours. After training, their times to perform the job were recorded. Denoting x = duration in training (hours) and y = time to do the job (in minutes), the following summary statistics were obtained.

$$\sum x_i = 100.0 \quad \sum y_i = 700.0 \quad S_{xx} = 24.0 \quad S_{yy} = 150.0 \quad S_{xy} = -48.0$$

6. Compute the least-squares estimates of the intercept "***a***" and slope "***b***".
   a) a = 53.5, b = -0.5 b) a = 45.0, b = -2.0
   c) a = 48.0, b = 0.35 d) a = 48.0, b = -0.35
   e) None of the above

7. What proportion of the time is explained by the linear relationship to the number of hours in training in Question 6?
   a) 34% b) 44% c) 54% d) 64%
   e) None of the above

For Questions **8** and **9**, use the information provided in the question below.

Population mean value of junior athletes' 10K run record is 45 minute with a population standard deviation of 6 min.

8. What is the probability that 2 groups of randomly selected 36 junior athletes (n_1=n_2=36) will differ in their mean record by more than $\sqrt{2}$ minutes?

   a) 0.217 b) 0.317 c) 0.417 d) 0.500
   e) None of the above

9. What is the probability that 2 groups of randomly selected 36 junior athletes (n_1=n_2=36) will differ in their mean record by an amount between $\sqrt{2}$ and $2\sqrt{2}$ minutes?

   a) 0.8413 - 0.5427 b) $2 \times (0.8413 - 0.5427)$
   c) 0.9772- 0.8413 d) $2 \times (0.9772 - 0.8413)$
   e) None of the above

10. Environment Protection Agency wants to test a chemical manufacturer's claim that the average amount of toxic material in a can of paint is 20 mg. If it is known that σ = 2.4 mg, from a sample of 36 cans of paints, determine the probability of a Type **I** error if the criteria for rejecting the paint manufacturer's claim is observing $\overline{\boldsymbol{X}} > 20.5$ mg.

a) 0.055 b) 0.89 c) 0.11 d) 0.945
e) None of the above

11. Given the information in Question 10, determine the probability of computing a Type II error if $\mu = 21$ mg.

a) 0.11 b) 0.89 c) 0.055 d) 0.945
e) None of the above

12. Determine the sample correlation coefficient from the given numbers: a drug trial in which $\boldsymbol{x}$ is the dosage in mg and $\boldsymbol{y}$ is the number of days of relief from allergies, the following information was obtained:

intercept a = -0.1 slope b = 0.7 $S_{xx} = 10$ $S_{yy} = 6$ SSE = 1.10 n = 5

a) 60.4% b) 70.4% c) 80.4% d) 90.4% e) None of the above

13. Estimate the difference ($\mu_1 - \mu_2$) between engineering (group #1) and social science (group #2) college junior tuitions at 95% CI, when a random sample of 50 engineering college juniors produced a sample mean of $25,000 with a normal population variance of $3,000, while 50 social science college juniors produced a sample mean of $24,000 with a normal population variance of $2,700.

a) $979.07 < \mu_1 - \mu_2 < 1020.93$ b) $987.62 < \mu_1 - \mu_2 < 1012.38$
c) $-979.07 < \mu_1 - \mu_2 < -1020.93$ d) $-987.62 < \mu_1 - \mu_2 < -1012.38$
e) None of the above

14. A Calgary ski equipment store sells 12 similar goggles, 7 of which have special prices. Find the 95% CI for the difference of the means ($\mu_x - \mu_y$). Assume both populations are normal with equal variances.

Regular Price, *x* ($): 40 50 40 60 50 ($\Sigma\,(x_i - \bar{x}) = 280$)

Special Price, *y* ($): 30 40 27 35 43 40 30 ($\Sigma\,(y_i - \bar{y}) = 228$)

a) $52 - (2.069 \times 7.127 \times 0.343) < \mu_x - \mu_y < 52 + (2.069 \times 7.127 \times 0.343)$
b) $52 - (2.228 \times 6.353 \times 0.586) < \mu_x - \mu_y < 52 + (2.228 \times 6.353 \times 0.586)$
c) $13 - (2.069 \times 7.127 \times 0.586) < \mu_x - \mu_y < 13 + (2.069 \times 7.127 \times 0.586)$
d) $13 - (2.228 \times 7.127 \times 0.586) < \mu_x - \mu_y < 13 + (2.228 \times 7.127 \times 0.586)$
e) None of the above

15. The maintenance yard of a mine has 4 tool cribs. The Chief Maintenance Engineer wants to analyze the level of utilization of these tool cribs. The following data have been collected on the number of cribs using during a day:

Number of Tool Cribs Used	0	1	2	3	4
Frequency	15	23	18	20	14

These data are used to conduct the hypothesis test that the number of tool cribs used in a day follows a uniform distribution. Using a 5% significance level, what is the conclusion of the hypothesis test?

a) $0.17 < \chi^2_{cri} = 9.488$, therefore accept H_o
b) $0.80 < \chi^2_{cri} = 9.488$, therefore accept H_o
c) $3.00 < \chi^2_{cri} = 9.488$, therefore accept H_o
d) $3.03 < \chi^2_{cri} = 9.488$, therefore reject H_o
e) None of the above

16. A city health department wishes to determine if the mean bacteria counts per unit volume of water at a lake beach is below the safety level of 200. A researcher collected 10 water samples of unit volume and found the bacteria counts to be:

208 210 191 196 184 175 193 215 180 198

What is the value of the appropriate statistic, and do the data indicate that there is no cause for concern at the 0.05 significance level? *(Assume that the normally requirement is met.)*

a) -1.197; accept H_o b) -1.197; reject H_o c) 1.197; reject H_o
d) -2.821; accept H_o e) None of the above

17. An engineer of a factory concluded that the entire manufacturing process is normal what the true standard deviation σ of the weight per flour bag is less than 0.3 lb. In fact, the factory produces 30 lb bag of flour. He randomly selected and measured the weight of 12 bags to test the sample standard deviation s. When $\sigma = 0.1$, find the probability that s^2 exceeds 0.02.

a) 0.05 to 0.10 b) 0.025 to 0.05 c) 0.02 to 0.025 d) 0.01 to 0.02
e) None of the above

18. A chicken farm claims that the weight of the egg will have an average of 200 grams. To maintain this average, the farmer weighs 25 eggs every day. If the computed t-value falls between " $-t_{0.01}$ and $t_{0.01}$", the farmer will be satisfied with his claim. What conclusion should the farmer draw from a sample that has a mean of 202 gm and a variance of 9 gm? Assume the distribution of the egg's weight to be approximately normal. Is the farmer's claim valid?

a) The farmer's claim is valid. b) The farmer's claim is *NOT* valid
c) We can *NOT* decide (we need more information).

19. An owner of a local Steak House is going to buy a box steak knife made by either X or Y manufacturer. To estimate the difference in the two different knives, the owner conducted a simple test with 10 of each knife. The knives were repeatedly used to cut meat until they need sharpening. As a result, mean of X manufacturer's knives are 40 with a standard deviation of 10, while mean of Y manufacturer's knives are 35 with a standard deviation of 5. Compute the 90% Confidence Interval for ($\mu_x - \mu_y$) assuming the populations to be approximately normal. Assume that the variances are not equal.

a) 5 ± 7.428 b) 5 ± 2.101 c) 5 ± 7.637 d) 5 ± 2.106
e) None of the above

20. After a simple lab, pressure temperature change was obtained 1 hour and 10 hours after the initial heating of the specimens. What is the 95% CI for the difference in temperature change?

Observation	1 hour	10 hours	Diff.
1	1000	980	20
2	1020	1000	20
3	1010	970	40

Observation	1 hour	10 hours	Diff.
4	1040	950	90
5	1030	990	40
6	1050	960	90

a) $50 \pm 2.571 \times 11.461$ b) $50 \pm 2.571 \times 13.166$
c) $25 \pm 2.776 \times 13.166$ d) $50 \pm 2.776 \times 11.461$
e) None of the above

21. Plastic sheets produced by a machine are periodically monitored for possible fluctuations in thickness. Uncontrollable heterogeneity in the viscosity of the liquid mold makes some variation in thickness measurements unavoidable. However, if the true standard deviation of thickness exceeds 1.5 millimetres, there is a cause to be concerned about the product quality. Thickness measurements (in millimetres) of 10 specimens produced on a particular shift with a standard deviation as 2.079. The engineer undertaking the test wants to decide if the data substantiate the suspicion that the process variability exceeded the stated level on this particular shift at the 0.05 significance level. What is the value of the appropriate statistic and associated conclusion for the test?

a) 12.48; accept H_o b) 17.289; reject H_o c) -17.306; accept H_o
d) 25.959; reject H_o e) None of the above

22. A manufacturer of nickel-hydrogen batteries randomly selects 100 nickel plates for test cells, cycles them 2400 times, and determines that 14 of the plates have blistered. The manufacturer wants to devise a hypothesis test to establish if more than 10% of the plates blister under these conditions. What hypothesis test should be undertaken? *(Select the most appropriate answer.)*

a) H_o: $p = 0.10$, H_1: $p > 0.10$
b) H_o: $p = 0.14$, H_1: $p < 0.14$

c) H_o: p = 0.10, H_1: p ≠ 0.10
d) H_o: p = 0.14, H_1: p ≠0.14
e) None of the above

For Questions **23** and **24**, use the information provided in the question below.

A contract engineer studied the rate at which a spilled liquid spread across a surface. The engineer used derived empirical formulas to calculate the mass of the spill after a period of time ranging from 0 to 16 min (n=17) and the following summary statistics were obtained.

$\bar{x}$ **= 25.0** $\bar{y}$ **= 45.0** S_{xx} **= 100.0** S_{yy} **= 80.0** S_{xy} **= 50.0**

23. Find a **90% Confidence Interval** for mean mass of all spills with an elapsed time of 10 min.

a) 25.0 ± 1.721 × 1.414 × 4.750
b) 25.0 ± 1.753 × 1.414 × 4.750
c) 37.5 ± 1.753 × 1.414 × 1.519
d) 37.5 ± 1.753 × 1.915 × 1.519
e) None of the above

24. Find a **95% Prediction Interval** for the mass of a spill when the elapsed time is 10 min.

a) 37.5 ± 2.131 × 1.753 × 1.819
b) 37.5 ± 2.131 × 1.915 × 1.819
c) 25.0 ± 1.753 × 1.414 × 1.819
d) 37.5 ± 1.721 × 2.000 × 1.819
e) None of the above

25. An engineer wishes to determine the 98% confidence interval on the ratio of two variances. The first sample of data has n_1 = 14 data points and the second sample has n_2 = 13 data points. Determine $\dfrac{1}{f_{1-\alpha/2}(df_1, df_2)}$.

a) 3.96
b) 3.67
c) 27.23
d) 2.70
e) None of the above

26. Exactly 50% of UofC students do not agreed to pay higher tuition to improve their education environment when 200 students were asked at the campus survey. Find a 95% CI for the population proportion of the UofC students said "yes" on the survey.

a) 0.431 < p < 0.569
b) 0.236 < p < 0.825
c) 0.465 < p < 0.535
d) 0.495 < p < 0.505
e) None of the above

27. A factory worker knows that the exact amount each water bottle contains may vary. The mean fill per bottle is important, but equally important is the variation, S^2, of the amount of fill. To estimate the variation of fill, he randomly selects 10 bottles and weighs the content of each. If the sample standard deviation of the 10 bottle measurements is 0.4, construct a 99% CI for the true variance, σ^2.

a) $0.791 < \sigma^2 < 1.756$
b) $0.012 < \sigma^2 < 0.102$
c) $0.012 < \sigma^2 < 0.088$
d) $0.061 < \sigma^2 < 0.830$
e) None of the above

28. A manufacturer of a specific pesticide useful in the control of household bugs claims that his product retains most of its potency after six months and the drop in potency will exhibit at most with variance as 4. To test the manufacturer's claim, a consumer group obtained a random sample of 21 containers of pesticide from the manufacturer. Each can was tested for potency after being stored for six months and the sample potency was measured and the sample variance of the drops in potencies was computed to be 5.89. What is the value of the test statistic, and does it suggest that there is sufficient evidence to indicate that the population of potency drops has more variability than that claimed by the manufacturer? Use an $\alpha = 0.05$.

a) 30.14; accept Ho
b) 30.14; reject Ho
c) 29.45; accept Ho
d) 58.9; reject Ho
e) None of the above

29. The following data have been collected to test the null hypothesis $\sigma_1 = \sigma_2$. The alternative hypothesis for this test has been defined as $\sigma_1 \neq \sigma_2$.

$$\bar{X}_1 = 14.2 \quad \bar{X}_2 = 16.1$$
$$S_1 = 8.9 \quad S_2 = 5.2$$
$$n_1 = 8 \quad n_2 = 10$$

For $\alpha = 0.10$, what conclusion can be reached with the available data?

a) $0.27 < 2.93 < 3.29$, therefore accept H_o
b) $0.27 < 1.46 < 3.29$, therefore accept H_o
c) $0.27 < 2.78 < 3.29$, therefore accept H_o
d) $11.72 > 3.29$, therefore reject H_o
e) None of the above

30. A factory is testing with 2 different sets of its product line. It has been determined that both sets yield the same average number of items per day. To obtain a set that produces a better QC, it is suggested that the set with the smaller variance in the number of item produced per day be accepted. The 2 independent random samples yield the following data: Based on the 90% CI for the ratio (σ_1^2/σ_2^2), which product line will be rejected?

Line #1: $n_1 = 25$, $\bar{\boldsymbol{X}}_1 = 499.36$, $S_1^2 = 3729.41$
Line #2: $n_2 = 21$, $\bar{\boldsymbol{X}}_2 = 491.095$, $S_2^2 = 1407.89$

a) Line #1
b) Line #2
c) does *NOT* make any difference ($\sigma_1^2/\sigma_2^2 = 1.0$)
d) can *NOT* decide (need more information)

********End of Exam*******JW/K & X/W********