Code:4141

DEPARTMENT OF MATHEMATICS
University of Toronto
MAT136H1S
Term Test
Winter 2012
Time allowed: 1 hour, 45 minutes

NAME OF STUDENT:
(Please PRINT full name
and UNDERLINE surname): ______________________

STUDENT NUMBER:______________________

SIGNATURE OF STUDENT:______________________

TUTORIAL CODE(e.g.,M4A, R5D, etc.):______________________

TUTORIAL TIME(e.g.,T4,R5,F3, etc.):______________________

NAME OF YOUR TA:______________________

NOTE:

1. Before you start, check that this test has 14 pages. There are NO blank pages.

2. This test has two parts:
   PART A [48 marks]: 12 multiple choice questions.
   PART B [52 marks]: 7 written questions.
   Answers to both PART A and PART B are to be given in this booklet.
   No computer cards will be used.

3. No aids allowed.
   NO CALCULATORS!

4. **DO NOT TEAR OUT ANY PAGES**

FOR	MARKERS	ONLY
PART A		/48
B1		/7
B2		/7
B3		/7
B4		/7
B5		/8
B6		/8
B7		/8
TOTAL		/100

PART A *[48 marks]*

Please read carefully:
PART A consists of 12 multiple-choice questions, each of which has exactly one correct answer. Indicate your answer to each question by **completely filling in the appropriate circle with a dark pencil.** MARKING SCHEME: 4 marks for a correct answer, 0 for no answer or a wrong answer. You are not required to justify your answers in PART A. Note that for PART A, **only your final answers (as indicated by the circles you darken) count; your computations and answers indicated elsewhere will NOT count.**
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1. If $f'(x) = 8x^3 - 4x + 5$ and $f(1) = 6$, then $f(-1) =$.

Ⓐ -6
Ⓑ 0
Ⓒ -1
Ⓓ -4
Ⓔ 5

2. If $\int_2^3 \{5f(x) + 4g(x)\}dx = 5$ and $\int_2^3 g(x)dx = 5$, then $\int_2^3 f(x)dx =$.

Ⓐ -6
Ⓑ 0
Ⓒ 5
Ⓓ -3
Ⓔ 8

3. Find the Riemann sum for $f(x) = 2x^2$ on [1,7], by partitioning [1,7] into 3 subintervals of equal length and choosing each sample point to be the left endpoint of the subinterval.

Ⓐ 140
Ⓑ 120
Ⓒ 130
Ⓓ 150
Ⓔ 160

4. If $G(x) = \int_0^x \sqrt{1+e^t}dt$, then $G'(\ln 4) =$

Ⓐ $\sqrt{5}$
Ⓑ $\sqrt{5} - 1$
Ⓒ $\ln 5$
Ⓓ $-\ln 4$
Ⓔ $\sqrt{5} - \sqrt{2}$

5. Find the area of the region enclosed between the graphs of $y = 3x$ and $y = 3x^2 - 6x$.

Ⓐ $\frac{27}{2}$
Ⓑ $\frac{24}{5}$
Ⓒ $\frac{25}{2}$
Ⓓ $\frac{26}{3}$
Ⓔ $\frac{25}{3}$

6. Let R be the region bounded by the curves $y = 4x$ and $y = 2x^2$. Find the volume of the solid generated by revolving R about the y-axis.

Ⓐ $\frac{14\pi}{3}$
Ⓑ $\frac{13\pi}{3}$
Ⓒ $\frac{16\pi}{3}$
Ⓓ $\frac{17\pi}{3}$
Ⓔ $\frac{13\pi}{2}$

7. $\int_0^{2\pi} |x - \pi| dx =$

Ⓐ $2\pi^2 - 2$
Ⓑ $2\pi^2$
Ⓒ $\pi^2 + 1$
Ⓓ π^2
Ⓔ $2\pi^2 - \pi$

8. Water flows from the bottom of a storage tank at a rate $r(t) = (200 - 4t)$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.

Ⓐ 1,650 liters
Ⓑ 1,700 liters
Ⓒ 1,750 liters
Ⓓ 1,600 liters
Ⓔ 1,800 liters

9. Find the value of

$$\lim_{n\to\infty}\left(\frac{1}{n+2n^2}+\frac{2}{2n+2n^2}+\frac{3}{3n+2n^2}+\ldots+\frac{n}{n^2+2n^2}\right).$$

- (A) $1-2\ln 3+2\ln 2$
- (B) $1+3\ln 3-3\ln 2$
- (C) $1+2\ln 3-2\ln 2$
- (D) $1+2\ln 3-\ln 2$
- (E) $1-2\ln 3+\ln 2$

10. Let f be a continuous function on $[-3, 6]$ such that
the average value of $f(x)$ on $[-3, -1]$ is 5,
the average value of $f(x)$ on $[-1, 2]$ is 4,
the average value of $f(x)$ on $[2, 6]$ is 2.
Let $g(x) = 2 + f(x)$. Find the average value of $g(x)$ on $[-3, 6]$.

Ⓐ $\frac{17}{4}$
Ⓑ $\frac{17}{3}$
Ⓒ $\frac{13}{3}$
Ⓓ $\frac{13}{2}$
Ⓔ $\frac{16}{3}$

11. Evaluate $\int_7^8 \sqrt{\frac{x}{x-6}} dx$.

Hint: the formula $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$ may be useful.

Ⓐ $4 - \sqrt{7} + 6\ln 3 - 3\ln(4 + \sqrt{7})$
Ⓑ $8 - \sqrt{7} + 6\ln 3 - 3\ln(4 + \sqrt{7})$
Ⓒ $8 - \sqrt{7} - 6\ln 3 - 3\ln(4 + \sqrt{7})$
Ⓓ $4 + \sqrt{7} - 6\ln 3 - 3\ln(4 + \sqrt{7})$
Ⓔ $8 + \sqrt{7} - 6\ln 3 - 3\ln(4 + \sqrt{7})$

12. $\int_1^{\sqrt{2}} \ln(x+\sqrt{x^2-1})dx =$

Ⓐ $2\ln(1+\sqrt{2})-1$
Ⓑ $\sqrt{2}\ln(1+\sqrt{2})-1$
Ⓒ $\sqrt{2}\ln(1+\sqrt{2})-\sqrt{2}$
Ⓓ $\ln(1+\sqrt{2})-1$
Ⓔ $\sqrt{2}\ln(1+\sqrt{2})$

PART B *[52 marks]*

Please read carefully:
Present your complete solutions to the following questions in the spaces provided, in a neat and logical fashion, showing all your computations and justifications. Any answer in PART B without proper justification may receive very little or no credit. Use the back of each page for rough work only. If you must continue your formal solution on the back of a page, you should indicate clearly, in LARGE letters, "SOLUTION CONTINUED ON THE BACK OF PAGE _____." In this case, you may get credit for what you write on the back of that page, but you may also be penalized for mistakes on the back of that page.
MARKS FOR EACH QUESTION ARE INDICATED BY [ ].
DO NOT TEAR OUT ANY PAGES.

1. Find $\int xe^{5x}dx$.

[7]

2. Find $\int(2x^3+\cos x)(x^4+2\sin x)^{45}dx$

Note: This is a very easy problem!

[7]

3. Find $\int \tan^{74} x \sec^4 x dx$.

[7]

4. Find $\int \frac{dx}{(25-x^2)^{3/2}}$.

[7]

5. Find $\int \frac{5x^2+3x+64}{x(x^2+16)}dx$.

[8]

6. Find $\int \frac{dx}{\sqrt{x}+x\sqrt{x}}$.

[8]

7. Note: This is not an easy question and will be marked very strictly. Very little or no credit will be given unless your solution is completely correct.

Given that $\int_0^1 (\arcsin x)^6 dx = k$, find the value of $\int_0^1 (\arcsin x)^8 dx$ in terms of k. Simplify your final answer as much as possible.

[8]

MAT136

Answers to Term-test (February 2012)

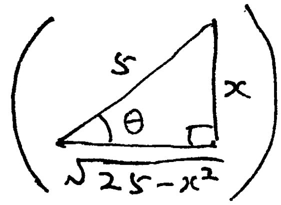
PART A: 1. -4 2. -3 3. 140 4. $\sqrt{5}$ 5. $27/2$ 6. $\frac{16\pi}{3}$ 7. π^2
8. 1800 9. $1-2\ln 3+2\ln 2$ 10. $\frac{16}{3}$ 11. $4-\sqrt{7}+6\ln 3-3\ln(4+\sqrt{7})$
12. $\sqrt{2}\ln(1+\sqrt{2})-1$ (For details, see next page.)

B1. $\int xe^{5x}\,dx = x\left(\frac{e^{5x}}{5}\right)-\int \frac{e^{5x}}{5}\,dx = \frac{xe^{5x}}{5}-\frac{e^{5x}}{25}+C.$

B2. Let $u=x^4+2\sin x$. Then $du=(4x^3+2\cos x)\,dx=2(2x^3+\cos x)\,dx$.
Hence $\int (2x^3+\cos x)(x^4+2\sin x)^{45}\,dx=\frac{1}{2}\int u^{45}\,du=\frac{u^{46}}{2(46)}+C$
$=\frac{(x^4+2\sin x)^{46}}{92}+C.$

B3. $\int \tan^{74}x\sec^4 x\,dx=\int(\tan^{74}x\sec^2 x)\sec^2 x\,dx$
$=\int(\tan^{74}x)(1+\tan^2 x)\sec^2 x\,dx=\int u^{74}(1+u^2)\,du$ (by letting $u=\tan x$)
$=\int(u^{74}+u^{76})\,du=\frac{u^{75}}{75}+\frac{u^{77}}{77}+C=\frac{\tan^{75}x}{75}+\frac{\tan^{77}x}{77}+C.$

B4. Let $x=5\sin\theta$. Then $dx=5\cos\theta\,d\theta$.
Also, $(25-x^2)^{3/2}=(25-25\sin^2\theta)^{3/2}=\{25(1-\sin^2\theta)\}^{3/2}$
$=(25\cos^2\theta)^{3/2}=5^3\cos^3\theta.$
Hence $\int\frac{dx}{(25-x^2)^{3/2}}=\int\frac{5\cos\theta\,d\theta}{5^3\cos^3\theta}=\frac{1}{25}\int\sec^2\theta\,d\theta$
$=\frac{1}{25}\tan\theta+C=\frac{x}{25\sqrt{25-x^2}}+C.$



B5. By Partial Fractions,
$\int\frac{5x^2+3x+64}{x(x^2+16)}\,dx=\int\left(\frac{4}{x}+\frac{x+3}{x^2+16}\right)dx$
$=4\ln|x|+\frac{1}{2}\ln|x^2+16|+\frac{3}{4}\arctan\left(\frac{x}{4}\right)+C.$

B6. Let $u=\sqrt{x}$. Then $u^2=x$ and $2u\,du=dx$.
Hence $\int\frac{dx}{\sqrt{x}+x\sqrt{x}}=\int\frac{2u\,du}{u+u^3}=2\int\frac{du}{1+u^2}=2\arctan u+C$
$=2\arctan(\sqrt{x})+C.$

NOTE: In the Final Exam, for an indefinite integral, if you forget the "+C" in your answer, marks will be deducted!

B7. $\displaystyle\int_0^1 \underbrace{(\arcsin x)^8}_{u}\underbrace{dx}_{dv} = \underbrace{(\arcsin x)^8}_{u}\underbrace{x}_{v}\Big]_0^1 - \int_0^1 \underbrace{x}_{v}\underbrace{\frac{8(\arcsin x)^7}{\sqrt{1-x^2}}dx}_{du}$

$\displaystyle = \left(\frac{\pi}{2}\right)^8 - 8\int_0^1 \underbrace{(\arcsin x)^7}_{u}\underbrace{\left(\frac{x}{\sqrt{1-x^2}}\right)dx}_{dv}$

$\displaystyle = \left(\frac{\pi}{2}\right)^8 - 8\left\{\underbrace{(\arcsin x)^7}_{u}\underbrace{(-\sqrt{1-x^2})}_{v}\Big]_0^1 - \int_0^1 \underbrace{(-\sqrt{1-x^2})}_{v}\underbrace{\frac{7(\arcsin x)^6 dx}{\sqrt{1-x^2}}}_{du}\right\}$

$\displaystyle = \left(\frac{\pi}{2}\right)^8 - 8\left\{0 + \int_0^1 7(\arcsin x)^6 dx\right\} = \underline{\underline{\left(\frac{\pi}{2}\right)^8 - 56k}}.$

NOTE: This question is marked very strictly. Very little credit is given unless your solution is completely correct.

A8. Answer $\displaystyle = \int_0^{10} r(t)\,dt = \int_0^{10}(200-4t)\,dt = 200t - 2t^2\Big]_0^{10} = 1{,}800.$

A9. Limit $\displaystyle = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{in+2n^2} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{i}{i+2n} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{i/n}{i/n+2}$

$\displaystyle = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} f\left(\frac{i}{n}\right)$ (by letting $f(x) = \dfrac{x}{x+2}$)

$\displaystyle = \int_0^1 f(x)\,dx = \int_0^1 \frac{x}{x+2}dx = \int_0^1 \frac{(x+2)-2}{x+2}dx = \int_0^1\left(1-\frac{2}{x+2}\right)dx = 1-2\ln 3+2\ln$

A11. $\displaystyle\int_7^8 \sqrt{\frac{x}{x-6}}\,dx = \int_7^8 \sqrt{\frac{x}{x-6}}\left(\frac{\sqrt{x}}{\sqrt{x}}\right)dx = \int_7^8 \frac{x\,dx}{\sqrt{x^2-6x}} = \int_7^8 \frac{x\,dx}{\sqrt{(x-3)^2-9}}$

$\displaystyle = \int_4^5 \frac{(u+3)\,du}{\sqrt{u^2-9}}$ (by letting $u = x-3$)

$\displaystyle = \int_4^5 \frac{u}{\sqrt{u^2-9}}du + 3\int_4^5 \frac{du}{\sqrt{u^2-9}} = \sqrt{u^2-9}\Big]_4^5 + 3\ln\left|u+\sqrt{u^2-9}\right|\Big]_4^5$

$= 4-\sqrt{7}+6\ln 3-3\ln(4+\sqrt{7}).$

A12. $\displaystyle\int \underbrace{\ln(x+\sqrt{x^2-1})}_{u}\underbrace{dx}_{dv} = \underbrace{\ln(x+\sqrt{x^2-1})}_{u}\underbrace{x}_{v} - \int \underbrace{x}_{v}\underbrace{\left(\frac{1}{x+\sqrt{x^2-1}}\right)\left(1+\frac{x}{\sqrt{x^2-1}}\right)dx}_{du}$

$\displaystyle = x\ln(x+\sqrt{x^2-1}) - \int\left(\frac{x}{x+\sqrt{x^2-1}}\right)\left(\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right)dx$

$\displaystyle = x\ln(x+\sqrt{x^2-1}) - \int\frac{x}{\sqrt{x^2-1}}dx = x\ln(x+\sqrt{x^2-1}) - \sqrt{x^2-1} + C.$

Hence $\displaystyle\int_1^{\sqrt{2}} \ln(x+\sqrt{x^2-1})\,dx = \sqrt{2}\ln(\sqrt{2}+1) - 1.$

A10. average value of f on $[-3,6] = \dfrac{(5)(2)+(4)(3)+(2)(4)}{9} = \dfrac{10}{3}$

So average value of $2+f$ on $[-3,6] = 2+\dfrac{10}{3} = \dfrac{16}{3}.$
