Final Exam – Solutions
ECON 3240 A: Labour Economics

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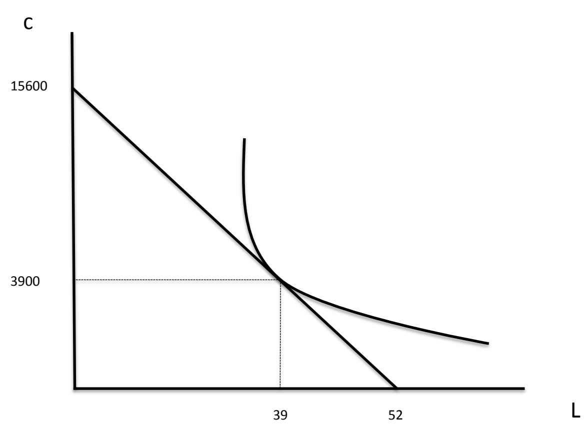
1. Gil works in real estate. He has 52 weeks per year, which he can split between work and leisure. Gil enjoys consuming the goods that he buys with his labor income (c) and he also enjoys taking time off for leisure (L). Gil's utility function is $U = c^{1/4}L^{3/4}$. Gil can earn W=$300 per week at his job, and he has no non-labour income. **[For this utility function,** $MRS = \frac{3c}{L}$**]**

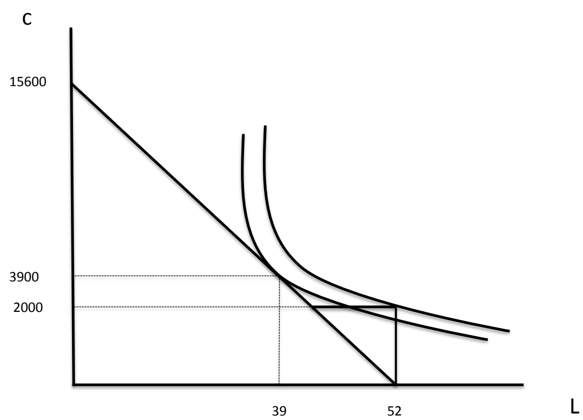
   (a) **(5 pts)** How much leisure will Gil choose to take? What will his level of consumption be? What will his level of utility be? Illustrate your answer with a carefully labelled graph.

   To figure out Gil's optimal level of consumption and leisure, set $MRS = W$, *or* $\frac{3c}{L} = 300$, *and* $c + wL = wT + Y_N$, *or* $c + 300L = 15600$. *The solution to these two equations is* $L^* = 39$ *and* $c^* = 3900$. *Gil's utility will be* $U = (3900)^{1/4}(39)^{3/4} = 123.3$. *See graph below.*



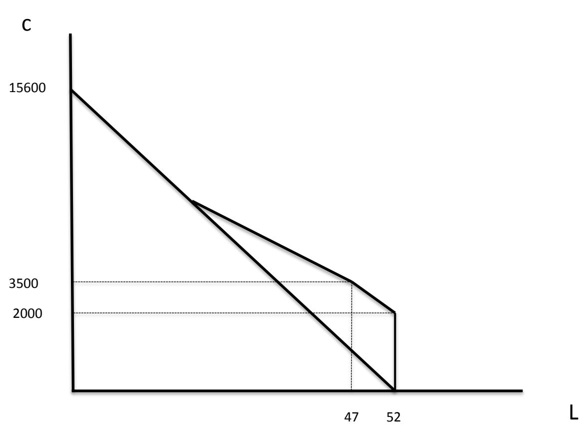
   (b) **(5 pts)** Suppose that the government introduces a traditional welfare program, in which workers can receive $2,000 per year if they have no labour income. However, this benefit is reduced dollar for dollar with a worker's labour income. After this program is introduced, how much will Gil choose to work? Illustrate your answer with a carefully labelled graph.

Under a traditional welfare scheme, Gil will either choose to work or not. If he chooses to work, he will choose $L = 39$ *and* $c = 3900$*, as this is his optimal point conditional on working. So to solve this problem, you must simply check if Gil's utility is higher than before if he chooses to work zero hours and collect his full welfare benefit. If he does this, his level of leisure will be 52 and his level of consumption will be 2000, which will give him utility of* $U = 2000^{1/4}52^{3/4} = 129.5$*. This is greater than his utility from part (a), so he will choose to work zero hours. See graph below.*



(c) **(5 pts)** The government decides to alter its welfare program in the following way. Workers still receive \$2,000 per year if they have no labour income. However, they may keep the entire welfare benefit if they earn \$1,500 or less in labour income. Once they have earned \$1,500 in labour income, they must start paying back there welfare benefit at a rate of \$0.50 for every additional dollar earned. **Draw Gil's budget constraint under this new welfare scheme.**

See graph below.



(d) **(5 pts)** Discuss **in words** the effect that the program in part (c) will have on Gil's labour supply, relative to parts (a) and (b), making reference to income and substitution effects.

Relative to part (a), this new program should reduce labour supply. The program increases Gil's "wealth", so this income effect will encourage him to work less. And, in the range of the budget line that Gil was operating under when before the policy was enacted, his wage has declined, since he loses fifty cents of his welfare subsidy with every dollar earned. So, the substitution effect encourages him to work less as well. Relative to part (b), this program should increase Gil's labour supply. Under the traditional welfare scheme, Gil chose not to work at all. The new welfare program removes the extreme disincentive to work at all by allowing him to keep some or all of his welfare subsidy even if he works a little.

2. Arty owns a surveillance firm, which operates in a competitive market for labour and surveillance services. In the operation of his business, Arty employs labor (N) at a wage of $w = 15$ per hour, and he employs computers (K), which cost him $r = 100$ each to maintain for one day. With N hours of labour and K computers, Arty can produce $Q = 10K^{1/2}N^{1/2}$ units of surveillance output in one day, which he sells for \$10 each. Arty currently has 9 computers. **[For this production function, $MP_N = 5K^{1/2}N^{-1/2}$, and $MRTS = \frac{K}{N}$.]**

   (a) **(5 pts)** In the **short run**, how much labour will Arty hire per day? How many units of surveillance will he produce?

   In the short run, Arty will choose the quantity of labour at which $MRP_N = W$:

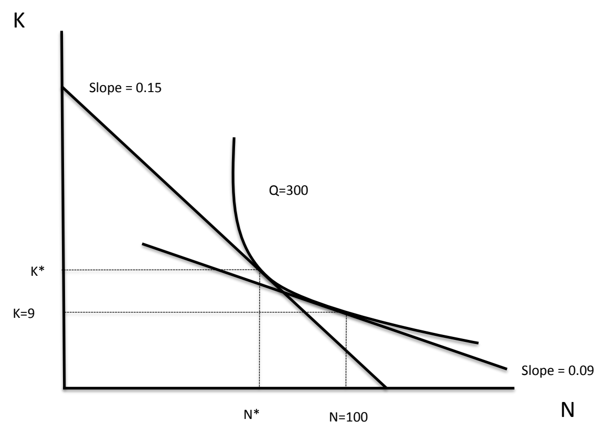
$$
\begin{aligned}
P \times MP_N &= W \\
&\Rightarrow 10 \times 5K^{1/2}N^{-1/2} = 15 \\
&\Rightarrow 50(9)^{1/2}N^{-1/2} = 15 \\
&\Rightarrow N^{-1/2} = \frac{15}{150} = \frac{1}{10} \\
&\Rightarrow N^{1/2} = 10 \\
&\Rightarrow N = 10^2 = 100
\end{aligned}
$$

   So, he will hire 100 units of labour. This will produce $Q = 10 \times 9^{1/2} \times 100^{1/2} = 300$ units of surveillance.

   (b) **(5 pts)** Suppose that, in the long run, Arty chooses to produce the same quantity as he did in part (a). However, because it is the long run, he is able to alter his input of computers. Given w and r, do you expect Arty to employ the same amount of capital and labour as he did in part (a)? If not, do you expect him to employ more labour and less capital, or more capital and less labour? **[Hint: what is Arty's**

MRTS at his input of K and N from part (a)? You may want to draw a graph]

In part (a), Arty is producing 300 units of output with 100 units of labour and 9 units of capital. His MRTS at this point is: $MRTS = \frac{K}{N} = \frac{9}{100} = 0.09$*. At Arty's optimal long-run choice of capital and labour, MRTS will be equal to w/r:* $MRTS = 15/100 = 0.15 > 0.09$*. So, in the short run, Arty's MRTS is lower than what is optimal. Then, in the long run, Arty will move towards a combination of capital and labour with a larger MRTS, or a larger* $\frac{K}{N}$*. This means that he will use more capital and less labour. See graph below for an illustration.*



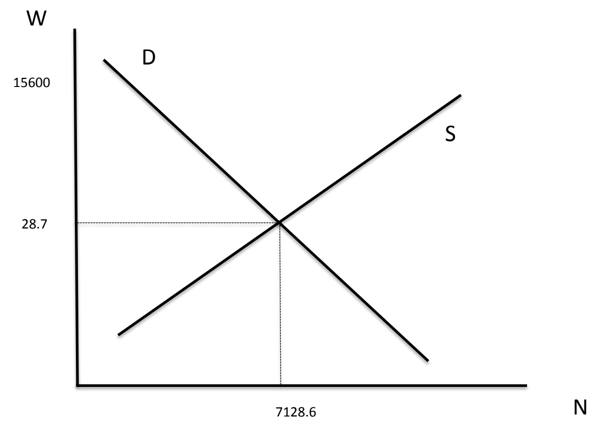
(c) **(5 pts)** Suppose that Arty is employing his long-run equilibrium quantity of K and N. However, due to an influx of qualified workers into the labour market, the wage he must pay his workers falls to $w = 10$. In the long run, will this cause Arty to hire more or less labour? Will it cause him to hire more or less capital? Explain your answer with reference to scale and substitution effects.

If the wage has fallen and the price of capital has not, this means that labour has become relatively cheaper and capital has become relatively more expensive. So, the substitution effect will encourage Arty to hire more labour and less capital. Overall production costs have fallen, so Arty will want to produce more output. So, the scale effect encourages Arty to hire more capital and more labour. Then, Arty will certainly hire more labour; however, the overall effect on capital is ambiguous.

3. The market for nuclear physicists is competitive. The market demand for these workers is: $N^D = 10,000 - 100W$, and the market supply is $N^S = -50 + 250W$, where W is the hourly wage paid to nuclear physicists.

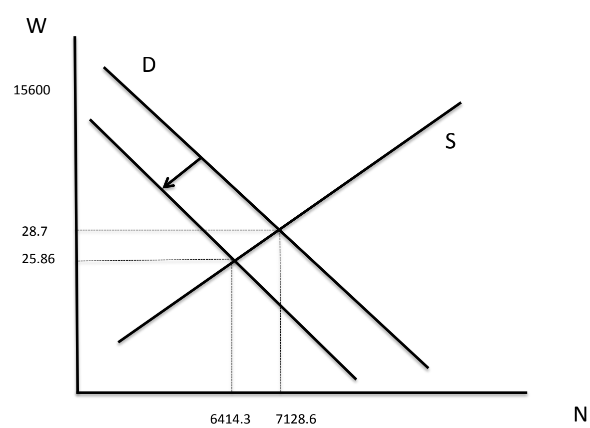
(a) **(5 pts)** Derive the equilibrium wage and level of employment for nuclear physicists in this market. Illustrate your answer with a carefully labelled graph.

To find equilibrium wages, set $N^D = N^S$, *or* $10000 - 100W = -50 + 250W$, *and solve for* W. *This gives you* $W^* = 28.71$, *which means that* $N^* = -50 + 250(28.7) = 7128.6$ *(don't take off points for rounding errors). See graph below.*



(b) **(5 pts)** The government introduces a payroll tax of \$10 on nuclear power plants. So, instead of paying W for each unit of labour, power plants must pay $W + 10$. Derive the new equilibrium wage and level of employment. Illustrate your answer with a carefully labelled graph.

With a payroll tax of \$10, the labour demand curve will shift to $N^D = 10000 - 100(W+10)$, *as firms will have to pay* $W+10$ *per unit of labour instead of just* W. *So, the new equilibrium wage will be the solution to* $10000 - 100W - 1000 = -50 + 250W$, *which gives you* $W^* = 25.86$ *and* $N^* = -50 + 250(25.86) = 6414.3$. *See graph below.*



(c) **(5 pts)** Suppose that there is a minimum wage of $W = 27.50$ for nuclear physicists. Discuss **in words** the effect this would have on the equilibrium wage and level of employment you derived in parts (a) and (b).

If there is a minimum wage of 27.50, this will have no effect on your answer to part *(a), since this minimum wage would not be binding – it is below the equilibrium wage in part (a). However, it would be binding in part (b). So, you would expect it to increase the wage and lower employment relative to your answer to part (b).*

4. Allison has recently graduated with a BA in economics. She is considering investing in a one year Master's program, which costs 8,000 in tuition and related expenses. Allison cannot work while she is participating in this program. However, if she completes the Master's program, she will earn 75,000 dollars per year, forever. If she does not invest in any further education, she will earn 65,000 dollars per year, forever. Allison can borrow and lend at an interest rate of 6% per year.

   (a) **(5 pts)** Will Allison participate in the program or not? Justify your answer by showing **one** of the following: the present value of lifetime income of Allison's earnings with and without the Master's program, or the internal rate of return on the Master's program. [Hint: recall that, for any amount X and interest rate r, $\sum_{t=1}^{\infty} \frac{X}{(1+r)^t} = \frac{X}{r}$.]

   Using notation from class, $\Delta Y = 75000 - 65000 = 10000$*;* $Y = 65000$*;* $D = 8000$*;* $r = 0.06$*. You could answer this one of two ways: first, you could calculate the IRR on the program:* $IRR = \frac{\Delta Y}{Y+D} = \frac{10000}{65000+8000} = 0.137$*, which is greater than 0.06, so Allison should do the program. Alternatively, you could calculate the marginal benefit of the year of schooling –* $MB = \frac{\Delta Y}{r} = 166,666,67$ *– and the marginal cost of schooling –* $MC = Y + D = 65,000 + 8,000 = 73,000$*. Since* $MB > MC$*, she should do the program. Finally, you could calculate the present value of lifetime income without schooling –* $PV_1 = Y + \frac{Y}{r} = 65,000 + \frac{65,000}{0.06} = 1,148,333.3$ *– and the present value of lifetime income with schooling –* $PV_2 = 0 + \frac{Y+\Delta Y}{r} - D = \frac{75,000}{0.06} - 8,000 = 1,242,000$*. Since* $PV_2 > PV_1$*, she should do the program.*

   (b) **(5 pts)** Why does Allison's annual income increase after completing a Master's program? Give **two** explanations based on theoretical models studied in class.

   Income might increase after a master's program if the master's program makes people more productive. This would be the "human capital" theory of the return to schooling. Alternatively, income might increase after a master's program if firms use master's degrees as a signal of a person's innate productivity level. Specifically, if firms believe that only high productivity people get MA degrees, then they will pay people with MA degrees more money in a competitive market. This is the "signalling" model.

   (c) **(5 pts)** Suppose the government is considering offering a subsidy to students obtaining a Master's degree. Do you think this is a socially beneficial policy? Does your answer depend on which theoretical model from part (b) you believe? **Explain your answer.**

   Your answer should depend on which model you believe from part (b). If the MA program actually makes people more productive, then it would be socially beneficial *to subsidize it. However, if the MA program merely serves to signal to firms that a person is a high productivity person, then there is no social gain to subsidizing it.*

5. You obtain the following immigration statistics from the 1990 census of Springfield. Among native-born workers, average earnings are $42,500; average earnings for workers who immigrated from Shelbyville to Springfield between 1985 and 1990 are $31,000; and average earnings for workers who immigrated to Springfield between 1980 and 1985 are $33,500.

   (a) **(5 pts)** Calculate the entry effect on earnings. Express your answer both in dollars, and as a percentage of native born earnings.

   The entry effect is the difference between recent immigrants' earnings (IM8590) and native born earnings. This is $42,500 - 31,000 = 11,500$*, or* $\frac{11500}{42500} = 27\%$ *of native born earnings.*

   (b) **(5 pts)** Using the 1990 census cross section, estimate the growth rate of new immigrants' earnings over a five year period. Express your answer in dollars, and as a percentage of new immigrants' earnings in the year 1990.

   The estimated growth rate over five based on a cross-section like this would be the difference in earnings between recent immigrants (IM8590) and immigrants from the cohort five years earlier (IM8085). This is $33,500 - 31,000 = 2,500$*, or* $\frac{2500}{31000} = 8\%$ *of the new immigrants' starting earnings.*

   (c) **(5 pts)** Suppose that, contrary to your estimates from part (b), workers who immigrated from Shelbyville to Springfield between 1985 and 1990 experienced earnings growth of 5 percent between 1990 and 1995. Provide a plausible explanation for why this might have happened. Be specific about any potential differences between this immigration cohort and previous immigration cohorts.

   You were expecting new immigrants' earnings to grow by 8%; instead, they grew by 5%. So, they did worse than you were expecting. This may be because everyone's earnings declined between 1990 and 1995. Or it could be because the most recent cohort had a larger entry effect on earnings than the previous cohort. Or it could be because the recent cohort's earnings grew at a slower rate over five years than the previous cohort's earnings.

   (d) **(5 pts)** You discover that, around 1990, the wage distribution in Springfield became very unequal relative to the wage distribution in Shelbyville. According to the theory of migrant selection, can this fact explain your findings from part (c)? **Explain your answer**

In part (c), you found that new immigrants' earnings grew at a slower rate than expected. This may be because this cohort was less skilled than the previous immigration cohort. The question is: can the change in the Springfield wage distribution explain why less skilled migrants would have chosen to migrate there? The answer is no: when the earnings distribution in a destination becomes more unequal than the earnings distribution in the sending area, this tends to cause migrants to be more ***positively selected****. A more unequal earnings distribution indicates a higher return to skill; so, it should attract more skilled workers, not less skilled workers. So, the change in the earnings distribution in Springfield would predict the opposite of what you found in part (c).*

6. There are 200 women and 200 men in the town of Bronson. For both men and women, the marginal product of labour is: $MP_N = 150 - 15N$. However, employers in Bronson dislike hiring women; every unit of female labour they hire costs them \$15 in utility. There are 30 identical firms in Bronson. [NOTE = Assume $p = 1$, so $MRP_N = MP_N$.]

   (a) **(5 pts)** Derive an individual firm's labour demand function for men and women. Illustrate your answer with a carefully labelled graph.

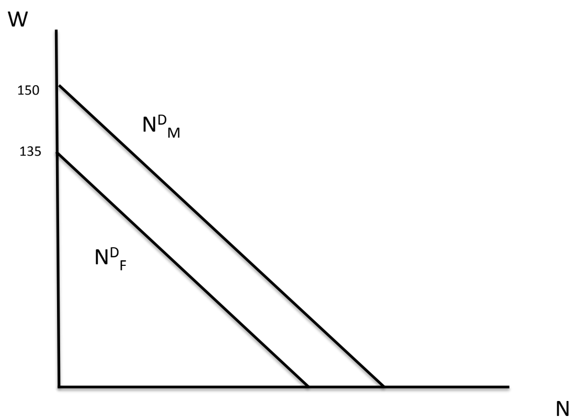
   An individual firm will set $MRP_N = MC_N$. So, for men:

$$\begin{aligned} MRP_N &= W_M \\ &\Rightarrow 150 - 15N = W_M \\ &\Rightarrow N_M^D = 10 - \frac{W_M}{15} \end{aligned}$$

   For women, $MC_N = W_F + 15$, so:

$$\begin{aligned} MRP_N &= W_F + 15 \\ &\Rightarrow 135 - 15N = W_F \\ &\Rightarrow N_F^D = 9 - \frac{W_F}{15} \end{aligned}$$

   See graph below.



(b) **(5 pts)** Derive the market demand function for men and women. Use this to derive the equilibrium wage and level of employment for men and women. Illustrate your answer with a carefully labelled graph.

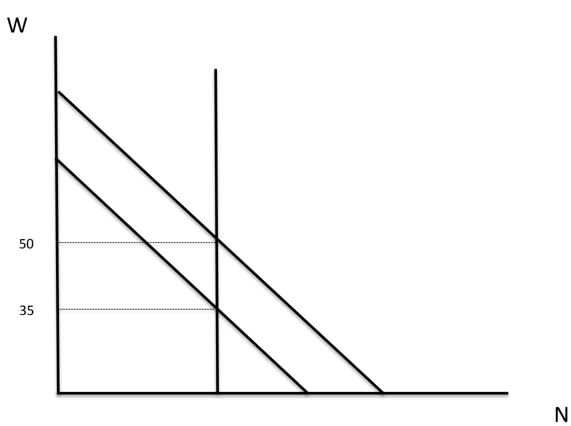
The market demand function for men and women will just be 30 × the individual firm's demand function for men and women:

$$N_M^D = 30\left(10 - \frac{W_M}{15}\right) = 300 - 2W_M$$
$$N_F^D = 30\left(9 - \frac{W_M}{15}\right) = 270 - 2W_M$$

So, the equilibrium wage for men and women will be:

$$N_M^D = N_M^S \Rightarrow 300 - 2W_M = 200 \Rightarrow W_M^* = 50$$
$$N_F^D = N_F^S \Rightarrow 270 - 2W_F = 200 \Rightarrow W_F^* = 35$$

See graph below



(c) **(5 pts)** The mayor of Bronson disagrees with the assertion that Bronson employers discriminate against women. Instead, he argues that women earn less than men because, on average, women are less educated than men. Specifically, average years of schooling for men is 15, while average years of schooling for women is only 12. To demonstrate that schooling is positively correlated with wages, the mayor estimates regressions of wages on schooling separately for men and women. The results from these regressions imply the following average wages for men and women, respectively.

$$\overline{W}_M = \hat{\alpha}_M + 2.1\overline{S}_M \quad (1)$$

$$\overline{W}_F = 17 + \hat{\beta}_F\overline{S}_F \quad (2)$$

The W's represent average wages, and the S's represent average years of schooling for men and women, respectively. Which estimates of β^F and α^M would generate the average wages for men and women that you found in part (b)?

For men, you know that $\overline{W}_M = 50$ *(from the previous problem), and that* $\overline{S}_M = 15$*. So, plug these numbers into equation (1) to solve for the* α^M *that is consistent this level of wages and average schooling:*

$$50 = \hat{\alpha}_M + 2.1(15) \Rightarrow \hat{\alpha}_M = 18.5$$

Similarly, $\overline{W}_F = 35$ *and* $\overline{S}_F = 12$*, which implies that* β^F *must be:*

$$35 = 17 + \hat{\beta}_F(12) \Rightarrow \hat{\beta}_F = 1.5$$

(d) **(5 pts)** Using a Oaxaca decomposition, decompose the raw gender wage gap into a portion attributable to differences in educational attainment and a portion that is unexplained, using the α's and β's you derived in part (c). What fraction of the gender wage gap is explained by education differences? What fraction is due to discrimination? Do you agree with the mayor of Bronson's claim that the gender wage gap is entirely caused by differences in education?

Solve for $\tilde{W}$*, where* $\tilde{W}$ *is the wage that a person compensated as a man would earn if he had the same average schooling as a woman:*

$$\tilde{W} = \hat{\alpha}_M + \hat{\beta}_M\overline{S}_F = 18.5 + 2.1(12) = 43.7$$

Then the portion attributed to differences in schooling is $W_M - \tilde{W} = 50 - 43.7 = 6.3$*, and the portion not explained by differences in schooling is* $\tilde{W} - W_F = 43.7 - 35 = 8.7$*. Or, 42% of the total $15 gap is explained, and 58% of the gap is not explained.*