MAT337: Homework 1

These problems are to be handed in by the beginning of class on Friday 30 September.

1. Evaluate $\lim_{n\to\infty}(1 - \frac{n}{n^2-1})$ in two different ways: (a) directly from the definition and (b) using the arithmetic of limits.

2. Define a real sequence (x_n) by $x_1 = 0$ and $x_{n+1} = \frac{1}{4(1-x_n)}$ for $n \geq 1$. Show that (x_n) is convergent, and find the limit.

3. Let (x_n) be a sequence of real numbers that converges to x, and let $a, b \in \mathbb{R}$.
(a) Show that, if $x_n \leq b$ for every $n \in \mathbb{N}$, then $x \leq b$. What can you conclude if $x_n < b$ for every $n \in \mathbb{N}$?
(b) Deduce from (a) that, if $x_n \geq a$ for every $n \in \mathbb{N}$, then $x \geq a$.

4. Let S and T be nonempty subsets of $\mathbb{R}$ that are both bounded above. Prove that the set $S + T = \{s + t \mid s \in S, t \in T\}$ is nonempty and bounded above, and that $\sup(S + T) = \sup S + \sup T$.

5. Prove that, if S is a nonempty subset of $\mathbb{R}$ that is bounded below, then there is a monotone decreasing sequence of elements of S that converges to $\inf S$.