extra problems - chapter 11
(one-price monopoly)

(#1) Given the following demand

$$P = 10 - Q$$

and costs

$$C(Q) = cQ \quad \text{where } c > 0$$

a) calculate the equilibrium price and quantity
b) how will an increase in costs, i.e "c" affect the answer in a)

(#2) Given the following demand

$$P = a - bQ \quad \text{where } a, b > 0$$

and costs $C(Q) = cQ$ where $c > 0$

a) calculate the equilibrium price and quantity
b) how would an increase in costs, i.e. "c" affect the answer in a)

#3 At the equilibrium quantity, the monopolist charges a price that is clearly above the extra cost of making the last unit. As such, would it not be profitable to sell an extra unit at a slightly lower price thereby increasing profits even further

#1 - Answer

Max_Q Profits

$\text{Max}_Q P(Q)Q - C(Q)$

$\text{Max}_Q (10-Q)Q - cQ$

$\text{Max}_Q 10Q - Q^2 - cQ$

$10 - 2Q - c = 0$

$$Q = \frac{10-c}{2}$$

$P = 10 - Q$

$= 10 - \left(\frac{10-c}{2}\right)$

$$P = \frac{10+c}{2}$$

b) $Q = \frac{10 - c}{2}$

$= \frac{10}{2} - \frac{c}{2}$

$Q = 5 - \frac{1}{2}c$

$\frac{dQ}{dc} = -\frac{1}{2} < 0$ the firm produces less when costs increase

$P = \frac{10 + c}{2}$

$= \frac{10}{2} + \frac{c}{2}$

$P = 5 + \frac{1}{2}c$

$\frac{dP}{dc} = \frac{1}{2} > 0$ the firm charges a higher price if cost increase

#2 - Answers

$\max_Q$ profits

$\max_Q$ Revenues - Costs

$\max_Q P(Q)Q - c(Q)$

$\max_Q (a - bQ)Q - cQ$

$\max_Q aQ - bQ^2 - cQ$

$a - 2bQ - c = 0$

$$Q = \frac{a-c}{2b}$$

$P = a - bQ$

$= a - b\left(\frac{a-c}{2b}\right)$

$= a - \left(\frac{a-c}{2}\right)$

$$P = \frac{a+c}{2}$$

b) $P = \frac{a}{2} + \frac{c}{2} = \frac{a}{2} + \left(\frac{1}{2}\right)c$

$\frac{dP}{dc} = \frac{1}{2} > 0$ an increase in costs leads to a higher price

$Q = \frac{a}{2b} - \frac{c}{2b} = \frac{a}{2b} - \left(\frac{1}{2b}\right)c$

$\frac{dQ}{dc} = -\frac{1}{2b} < 0$ an increase in costs leads to a lower equilibrium quantity

#3 - Answer

No it would not. If it wants to sell an extra unit it must charge a lower price for the extra unit and all previous units which is not profit maximizing