Formula Sheet Midterm MAT3320

$$r^2 + (p_0 - 1)r + q_0 = 0$$

$$y_1 = \sum_{n=0}^{+\infty} a_n x^{n+r_1}, \quad y_2 = \sum_{n=0}^{+\infty} b_n x^{n+r_2}$$

$$y_1 = \sum_{n=0}^{+\infty} a_n x^{n+r_1}, \quad y_2 = y_1 \ln x + \sum_{n=1}^{+\infty} b_n x^{n+r_2}$$

$$y_1 = \sum_{n=0}^{+\infty} a_n x^{n+r_1}, \quad y_2 = Ay_1 \ln x + \sum_{n=0}^{+\infty} b_n x^{n+r_2}$$

$$ax^2y'' + bxy' + cy = 0, \qquad ar^2 + (b-a)r + c = 0$$

$$y_1 = x^{r_1}, \quad y_2 = x^{r_2}$$

$$y_1 = x^{r_1}, \quad y_2 = x^{r_1} \ln x$$

$$y_1 = x^{\lambda} \cos(\mu \ln x), \quad y_2 = x^{\lambda} \sin(\mu \ln x)$$

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$$

$$-\frac{\alpha(\alpha+1)}{2!}c_0, \quad \frac{(\alpha-2)\alpha(\alpha+1)(\alpha+3)}{4!}c_0, \ \ldots$$

$$-\frac{(\alpha-1)(\alpha+2)}{3!}c_1, \ \frac{(\alpha-3)(\alpha-1)(\alpha+2)(\alpha+4)}{5!}c_1, \ \ldots$$

For $-1 \le x \le 1, \quad P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2 - 1)^n]$

$$\int_{-1}^{1} P_m(x) P_n(x) dx = \begin{cases} 0, & \text{if } m \neq n, \\ \frac{2}{2n+1}, & \text{if } m = n. \end{cases}$$

$$a_n = \frac{2n+1}{2} \int_{-1}^{1} f(x) P_n(x) dx$$

If $p > 0$, $\quad \Gamma(p) = \displaystyle\int_0^{+\infty} x^{p-1} e^x dx, \qquad \Gamma(p+1) = p\ \Gamma(p), \qquad \Gamma(1/2) = \sqrt{\pi}$

If $p < 0$ and $p \notin \mathbb{Z} \quad \Gamma(p) = \dfrac{\Gamma(p+1)}{p}$

$$x^2 y'' + xy' + (\lambda^2 x^2 - \nu^2) y = 0, \qquad \sum_{m=0}^{+\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu}\ m!\ \Gamma(\nu+m+1)}$$

$$\frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos\left(\frac{n\pi}{L}\right) + b_n \sin\left(\frac{n\pi}{L}\right)$$

$$\frac{1}{L}\int_{-L}^{L} f(x) \cos\left(\frac{n\pi}{L}\right), \frac{1}{L}\int_{-L}^{L} f(x) \sin\left(\frac{n\pi}{L}\right), \ \frac{2}{L}\int_{0}^{L} f(x) \cos\left(\frac{n\pi}{L}\right), \ \frac{2}{L}\int_{0}^{L} f(x) \sin\left(\frac{n\pi}{L}\right)$$