

## Tutorial 1 - Remarks

### Chain Rule

The example I did on the board for the chain rule was pretty bad, so hopefully this example will better demonstrate the applications of the chain rule.

Let  $f$  be a function of  $g$  and let  $g$  be a function of  $x$ . Recall that the chain rule states that the derivative of  $f$  with respect to  $x$  is the following:

$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f}{\partial g} \frac{\partial g}{\partial x} = f'(g(x))g'(x)$$

For this example, let  $f = 2y^6$ , and let  $y = x^5 + 3x^2 + 9$ . One way to find  $\partial f/\partial x$  is to substitute  $y$  into  $f$ , expand  $2(x^5 + 3x^2 + 9)^6$ , and then take the derivative. However, expanding  $2(x^5 + 3x^2 + 9)^6$  is very difficult. An easier way to calculate  $\partial f/\partial x$  is to use the chain rule:

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial f}{\partial y} \frac{\partial y}{\partial x} \\ \frac{\partial f}{\partial y} &= 12y^5 \\ \frac{\partial y}{\partial x} &= 5x^4 + 6x\end{aligned}$$

Using the above:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} \frac{\partial y}{\partial x} = (12y^5)(5x^4 + 6x)$$

Substituting  $y = x^5 + 3x^2 + 9$ :

$$\frac{\partial f}{\partial x} = 12(x^5 + 3x^2 + 9)^5(5x^4 + 6x)$$

## Log Rules

For this course, you are only responsible for logs with base  $e$  (i.e.  $\ln$  or natural logarithms). The following, in no particular order, are some (but not necessarily all) of the most common log rules:

$$\begin{aligned}\log_e x &= \ln x \\ \frac{\partial \ln x}{\partial x} &= \frac{1}{x} \\ \ln x^a &= a \ln x \\ \ln xy &= \ln x + \ln y \\ \ln \frac{x}{y} &= \ln x - \ln y \\ \ln e^a &= a \text{ since } \ln e = 1\end{aligned}$$

The following is not required for this course, but is a useful rule nonetheless. For any log with base  $a$ , we can change the base to  $b$  as follows:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

In economics, the most common use of this is to change the base to  $e$ . This is extremely useful when we want to take derivatives of logs with a base other than  $e$ :

$$\frac{\partial \log_a x}{\partial x} = \frac{\partial \log_e x}{\partial x \log_e a} = \frac{\partial \ln x}{\partial x \ln a} = \frac{1}{x \ln a}$$