YORK UNIVERSITY

Faculty of Liberal Arts and Professional Studies

School of Administrative Studies

AP/ADMS 4503.03 DERIVATIVE SECURITIES

MIDTERM EXAMINATION

Solution

Sunday, October 14, 2012

12:00 - 2:00 p.m.

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INSTRUCTIONS

1. Allowed material: Textbook, lectures notes and a calculator.
2. This examination contains **4 questions** on **4 pages** (including this cover page) and carries a total mark of **40 points**.
3. Answer all questions in the examination booklet provided.
4. If you have to make any assumptions, state them clearly. Unrealistic assumptions, or those inconsistent with the information provided in the question, will not be accepted.
5. All interest rates are per annum and continuously compounded.
6. Keep at least 4 decimal places in your calculations and final answers, and at least 6 decimal places for interest rates.
7. Show your work, including all formulas.

Good luck!

Question 1 (10 marks)

Oats futures are traded on the CME and each contract is on 5,000 bushels. The spot price of Oats is $3.70 per bushel and the 6-month futures price is $3.76. The risk-free rate is 4% per annum continuously compounded. We denote the storage cost (paid continuously) and the convenience yield by *u* and *y* respectively.

(a) What is the theoretical 1-year futures price? (3 marks)

(b) Suppose that the 1-year market futures price is $4.10. What arbitrage would you theoretically undertake for one contract? Show all details. (4 marks)

The initial margin is $1,350 per contract while the maintenance margin is $1,000. Assume an investor takes a short 1-year futures contract at a price of $4.10. Answer parts (c) and (d) independently.

(c) What price would trigger a margin call and for what amount? (1.5 marks)

(d) What price would be an incentive to default on a margin call? (1.5 marks)

Solution

(a) First we solve for (u-y) using the 6-month futures price:

$$F_{6m} = 3.76 = 3.70 \times \exp\left((4\% + u - y)) \times 0.5\right)$$
$$\Rightarrow u - y = -0.7828\%$$

The 1-year futures price is given by:

$$F_{1y} = 3.70 \times \exp\left((4\% - 0.7828\%) \times 1\right) = \$3.8210$$

(b) There is an arbitrage since the market is overpricing the 1-year contract. By analogy with an index, (y-u) can be seen as a dividend yield q = 0.7828%. For one contract, you should:

Description	Today	In 1 year
Short one contract (on 5,000 bushels)	$0	$20,500
Borrow for 1 year at 4%	$18,355.75	-$19,104.86
Buy exp(-qT)x5,000 bushels @ $3.70	-$18,355.75	
Total	**$0**	**$1,395.14**

(c) The price that would trigger a margin call is given by:

$$(4.10 - F_1) \times 5{,}000 = 1{,}000 - 1{,}350 = -350 \Rightarrow F_1 = \$4.17$$

The margin call would be for a total amount of \$0.07 x 5,000 x 1 = \$350.

(d) There would be an incentive for default if the margin call is for an amount of \$1,350, the total initial margin balance. This would occur if the futures price increases to:

$$\$4.10 + 1{,}350 / 5{,}000 = \$4.37$$

Question 2 (10 marks)

Consider futures contracts on IBM stock. We assume that the stock pays a cash dividend in 3 months. The risk-free rate is 4% per annum continuously compounded. The 6-month futures contract is fairly priced at \$210 and the stock spot price is \$207.

(a) What is the dividend amount implied by the 6-month contract? (3 marks)

(b) What is the theoretical 1-year futures price? (2 marks)

(c) Actually, the 1-year futures market price is \$216. Assuming that you are not allowed to trade the stock (neither sell nor buy) in the spot market, how would you undertake this arbitrage? Show all details (5 marks)

Solution

(a) The dividend amount implied by the 6-month contract is:

$$F_{6m} = 210 = \left(207 - Div \times \exp(-4\% \times 0.25)\right) \times \exp\left(4\% \times 0.5\right)$$
$$\Rightarrow Div = \$1.1699$$

(b) The 1-year futures price is:

$$F_{1y} = \left(207 - 1.1699 \times \exp(-4\% \times 0.25)\right) \times \exp\left(4\% \times 1\right) = \$214.24$$

(c) Since the market is overpricing the 1-year futures contract, and without using the spot market, one can undertake the following arbitrage:

Today		In 6 months	In 1 year
Long a 6-month futures		-\$210.00	
Short a 1-year futures			\$216
		Borrow \$210.00 for 6 months	-\$214.24
Total	**\$0**	**\$0**	**\$1.76**

Note that a traditional arbitrage would lead to the same profit:

Description	Today	In 1 year
Long one share	-$207.00	
Short one 1-year futures		$216
Borrow for 1 year	$207.00	-$215.45
Receive Div compounded		$1.2055
Total	**$0**	**$1.76**

Question 3 (10 marks)

MNL stock bid/ask prices are $99.90 and $100.10 respectively, and the risk-free rate bid/ask values are 3.90%–4.10%. There are transaction costs of $1 for the purchase or sale of the stock on the spot market. We assume that the stock pays no dividend and we consider a 1-year futures contract.

(a) What price should you quote for a short contract? Show all details. (3 marks)

(b) What price should you quote for a long contract? Show all details. (3 marks)

(c) For simplicity, we assume that there are no transaction costs for going either long or short on the futures contract. Is there an arbitrage and what is the profit if the 1-year futures market price is $104? Same question if the market price is $107? (4 marks)

Solution

(a) We need to create a synthetic short position as follows:

- Short the stock at $99.9
- Pay the transaction fee of $1
- Invest the net proceeds of $98.9 at 3.9%
- The future value of the investment is therefore 98.9 x exp(3.9% x 1) = $102.83, that is the 1-year futures bid price.

(b) We need to create a synthetic long position as follows:

- Borrow $101.1 at 4.1%
- Long the stock at $100.1
- Pay the fee $1
- The future value of the loan is therefore 101.1 x exp(4.1% x 1) = $105.33, that is the 1-year futures ask price.

(c) The market price of $104 falls within the no-arbitrage bounds, so there is no arbitrage. For the market price of $107 on the other hand, there is an arbitrage because the market is overpricing the contract. The profit is 107 - 105.33 = $1.67.

Question 4 (10 marks)

It is early August and farmer Robinson is making final estimates of this year's wheat crop. His production is turning out to be much better than expected. This causes concern because if his production is better than expected, other farmers must be experiencing the same situation. The current spot price is $8.75 per bushel, and the September wheat futures (5,000 bushels per contract) price is $8.35 per bushel. Farmer Robinson anticipates a production of 60,000 bushels. His wheat will not be ready to harvest until September and his break-even point is $300,000.

(a) What can farmer Robinson do to hedge his profit? Show all details? (4 marks)

(b) At harvest time in September (that is also the contract's maturity); our farmer's concerns are realized and the spot price has dropped to $7.75 per bushel. Compute his total revenue if his final yield was actually 80,000 bushels. What is the effective price per bushel if he sells the entire production? (3 marks)

(c) What is the net profit if his production turned out to be only 50,000 bushels? (3 marks)

Solution

(a) Farmer Robinson can sell his anticipated wheat production in the futures market, which will guarantee him a selling price of $8.35. This is a short hedge because he is selling his production forward. He needs to take 12 short September wheat futures contracts.

(b) At maturity, farmer Robinson must deliver 60,000 bushels at $8.35 for a total of $501,000 and the remaining 20,000 bushels will be sold spot at $7.75 for a total of $155,000. So the total revenue is $656,000, and the effective price is 656,000/80,000 = $8.20.

(c) In this scenario, framer Robinson must buy 10,000 bushels in the spot market for a total of $77,500, and deliver 60,000 bushels as per the contract for a total of $501,000. His net profit is therefore 501,000 – 77,500 – 300,000 = $123,500.