McGill University

Department of Mathematics and Statistics

MATH 314 ASSIGNMENT 1

Due : January 28, 2016

In examples 1, 2, and 3 find the volume of the solid of revolution obtained by rotating the area bounded by the curves about the line indicated. Problems to submit 1, 6, 10, 13, 14, 15, 17, 19 and 22.

1. $y = x^2 - 2$, $y = 0$ about $y = -1$. Consider only that part above $y = -1$.

2. $y = |x^2 - 1|$, $x = -2$, $x = 2$, $y = -1$, about $y = -2$

3. $x = \sqrt{40 + 12y^2}$, $x - 20y = 24$, $y = 0$, about $y = 0$.

   Note: Ex. 2 and 3 you may leave the answer in each case as sums of single Riemann integrals.

4. Find the area common to the two circles $x^2 + y^2 = 4$, $x^2 + y^2 = 6x$. Do this problem in both cartesian and polar coordinates.

5. Evaluate

   a) $\int_0^{\pi\backslash 2} \sin^6 \theta \, d\theta$

   b) $\int_0^{\pi\backslash 2} \sin^4 \theta \cos^5 \theta \, d\theta$

   c) $\int_0^{\pi} \cos^4 \theta \, d\theta$

6. Obtain the centroid of the disk segment $x^2 + y^2 \leq 2$, $x \geq 1$.

7. Find the volume of the region lying inside the circular cylinder $x^2 + y^2 = 2y$ and inside the parabolic cylinder $z^2 = y$.

8. A volcano is represented by the surface $z = he^{-\frac{(x^2+y^2)^{1/2}}{4h}}$, $z > 0$. After an eruption in which a volume V of lava adheres to the mountain it has a similar shape. Find the percentage change in the height of the mountain, i.e., express $\left(\frac{h_2 - h_1}{h_1}\right)100$ in terms of V and h_1. Hint: Transform to polar coordinates with $0 \leq r < \infty$.

9. Calculate the volume between the surfaces $x^2 + y^2 + z^2 = 2a^2$ and $z = \frac{x^2 + y^2}{\alpha}$, α a positive constant

10. Find the moment of inertia I_z (about the z axis) of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$ if the density is proportional to the distance from the z axis.

11. Find I_z for the region bounded by the cylinders $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ and the planes $z = 1$, $z = 0$, $x = 0$ and $x = y$. Assume constant density. Take $y \geq 0$.

12. Find the volume and the center of the mass of the region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z = x^2 + y^2$. Assume constant density.

13. Find the center of mass of the uniform solid bounded by the surfaces $x^2 + y^2 = 2x$, $z = \sqrt{x^2 + y^2}$, $z = 0$. Assume constant density.

14. Find the volume generated by revolving the region in the first quadrant bounded by the parabolas $y^2 = x, y^2 = 8x, x^2 = y, x^2 = 8y$ about the x axis by transforming coordinates.

15. Evaluate $\int_0^1 \int_0^{1-y} \cos\left(\frac{x-y}{x+y}\right) dx\ dy$ by transforming coordinates .

16. Evaluate

$$\iint_R \sin\left[\left(\frac{x+y}{2}\right)\right] \cos\left[\left(\frac{x-y}{2}\right)\right] dA$$

where R is the region bounded by the lines $x - y = 0$, $x + y = 2$, and $y = 0$

17. Consider the region in the first quadrant of the xy plane bounded by the hyperbolas $xy = 1$, $xy = 9$ and the lines $y = x, y = 4x$. Evaluate

$$\iint_R \left[\sqrt{\frac{y}{x}} + \sqrt{xy}\right] dxdy.$$

18. Evaluate $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+2y^2+3z^2)} dxdydz$

19. Evaluate $\iiint_D \frac{dV}{(x+y+z)^3}$ over the region bounded by the six planes $z = 1$, $z = 2$, $y = 0$, $y = z, x = 0$, and $x = y + z$.

20. Find the volume of the region bounded by the paraboloid $x = y^2 + 2z^2$ and the parabolic cylinder $x = 2 - y^2$.

21. (a) Obtain 14 methods to find the volume of a sphere of radius a and evaluate in each case.
(b) Same as (a) for a flat circular cone of height H and radius a, and evaluate 11 of them.

22. Evaluate the following integrals

(a) $\int_0^{2\pi} \sin^8 \theta\, d\theta$

(b) $\int_0^{\infty} \sqrt{y}\, e^{-y^3} dy$

(c) $\int_0^{\infty} 3^{-4z^2} dz$

(d) $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$

(e) $\int_0^{\infty} x^m e^{-ax^n} dx$ where a is a positive constant, and as special cases

- $\int_0^{\infty} x^3 e^{-2x^2} dx$
- $\int_0^{\infty} x^5 e^{-3x^4} dx$