

UNIVERSITY OF TORONTO
FACULTY OF ARTS AND SCIENCE

JUNE 2015 EXAMINATIONS

CHM220 EXAM

Duration: 3 hours

Exam Aid Allowed: Non-Programmable Calculator

Name: _____

Student Number: _____

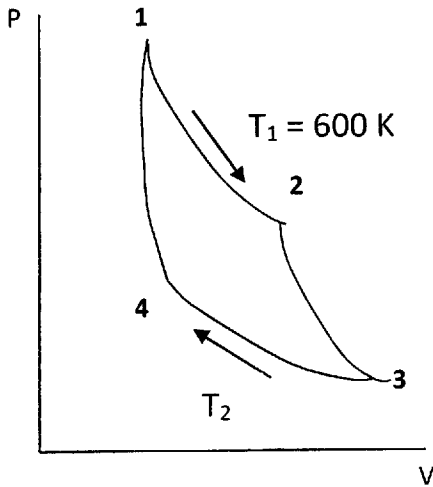
Tutorial Section: _____

No.	Maximum	Mark
1.	16	
2.	10	
3.	10	
4.	20	
5.	13	
6.	17	
7.	14	
Total	100	

Write legibly! Use the back of a page if more space is required, and indicate clearly when you have done so.

QUESTION 1 [16 pts]

Consider the Carnot cycle shown below for 0.700 moles of an ideal *monatomic* gas for which you are given:
 $T_1 = 600 \text{ K}$, $V_2 = 2V_1$ and $3V_3 = 4V_2$



(a) [3 pts] Calculate $w_{1 \rightarrow 2}$, $\Delta U_{1 \rightarrow 2}$, $q_{1 \rightarrow 2}$

(b) [2 pts] Find T_2 .

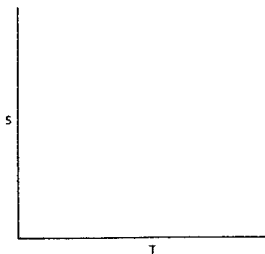
(c) [1 pts] What is the efficiency of a Carnot engine operating on this cycle?

(d) [2 pts] What is the total work around the cycle?

(e) [2 pts] How much heat is evolved at T_2 ?

(f) [2 pts] Evaluate $\Delta S_{3 \rightarrow 4}$.

(g) [2 marks] The above diagram shows a Carnot cycle in the form of a pressure-volume diagram. Sketch the corresponding entropy-temperature diagram, labeling the individual steps $A \rightarrow B$ (isothermal), $B \rightarrow C$ (adiabatic), $C \rightarrow D$ (isothermal), and $D \rightarrow A$ (adiabatic).



(h) [2 pts] What is $\Delta S_{\text{universe}}$ for the whole cycle? Show your calculation or explain your result.

QUESTION 2 [10 pts]

Consider the reaction $\text{FeO}(s) + \text{CO}(g) \rightleftharpoons \text{Fe}(s) + \text{CO}_2(g)$ for which K_p is found to have the following values:

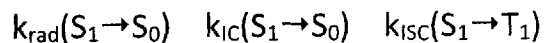
T	600°C	1000°C
K_p	0.900	0.396

- (a) [8 pts] Calculate $\Delta_r G^\circ$, $\Delta_r S^\circ$, and $\Delta_r H^\circ$ for this reaction at 600°C. Assume that $\Delta_r H^\circ$ is independent of temperature.

- (b) [2 pts] Calculate the mole fraction of $\text{CO}_2(g)$ present in the gas phase at 600°C.

QUESTION 3 (10 pts)

(a) [3 pts] Sketch a Jablonski diagram and make sure to include the S_0 , S_1 , and T_1 states of the chromophore. Label the following processes on the diagram:



(b) [3 pts] Assume that initially the chromophore is in the first excited state and $[S_0] = [T_1] = 0$. Using the rate constants provided in (a), write down expressions for $\frac{d[S_1]}{dt}$, $\frac{d[S_0]}{dt}$ and $\frac{d[T_1]}{dt}$.

Hint: This is a case of parallel reactions.

(c) [4 pts] Using rate constants provided in (a), write down expressions for:

i) The fluorescence lifetime of the chromophore

ii) The yield of S_0

iii) The yield of T_1

iv) $\frac{[S_0]}{[T_1]}$

QUESTION 4 (20 pts)

Briefly answer the following questions.

(a) [4 pts] Which of the following wave functions are eigenfunctions of the operator d^2/dx^2 ? If they are eigenfunctions, what is the eigenvalue? Show your work.

i) e^{-ikx}

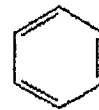
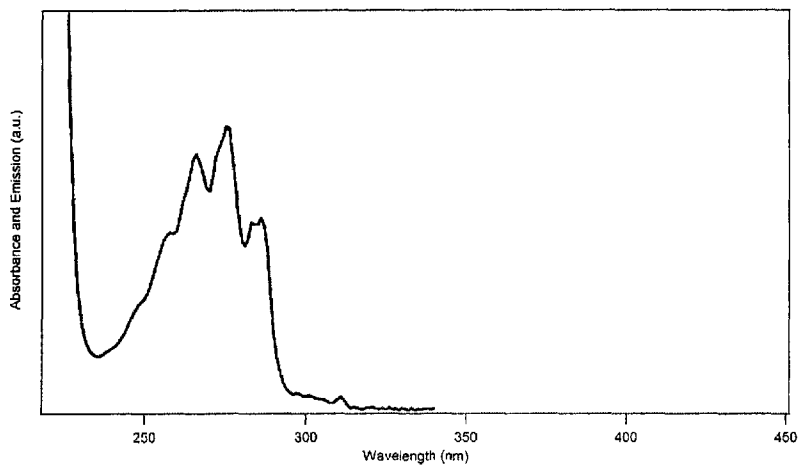
ii) $e^{-\alpha x^2}$

(b) [7 pts] Sketch the $4d_z^2$ orbital showing the axes, the relative phases, and being careful to respect the three-dimensional nature of the orbitals. Determine the number of radial and angular nodes. Also sketch the radial probability density function and the corresponding wavefunction.

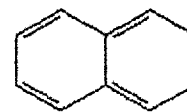
(c) [3 pts] Locate the radial nodes in the 4p orbital of an H atom. The radial wavefunction $R_{4,1}$ is proportional to:

$$(20 - 10\rho + \rho^2)\rho \quad \text{where} \quad \rho = \frac{2Zr}{na_0} \quad a_0 = 0.529\text{\AA}$$

(d) [3 pts] The spectrum below shows the absorption of naphthalene. On the same graph, use a **solid line** to sketch the absorption of benzene and a **dotted line** for the emission of naphthalene. Indicate the Stokes' shift for naphthalene



benzene



naphthalene

(e) [3 pts] Describe how resonance energy transfer can be used to determine the separation between two molecules (Spectroscopic ruler).

QUESTION 5 (13 pts)

- (a) [10 pts] Prepare a qualitative **molecular orbital energy level diagram** for BrCl. Use sketches to show clearly how the atomic orbitals interact to form MOs and clearly indicating the relative amount of electron density on each atom. Include labels for each molecular orbital giving the appropriate symmetry.

- (b) [3 pts] What is the bond order? Would you expect BrCl to have a shorter or longer bond length than BrCl?

QUESTION 6 (17 pts)

(a) [7.5 pts] If a particle is confined to a 3D cubic box with edge length $L=a=b=c$, fill in the table below by indicating the energy the first five energy levels. Also, provide (n_x, n_y, n_z) for all the states as well as the degeneracy for each of the first five levels.

Given: $\psi_{n_x, n_y, n_z} = \sqrt{\frac{8}{abc}} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{b}\right) \sin\left(\frac{n_z \pi z}{c}\right)$

$$E = \frac{h^2}{8m} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right)$$

$$\int A \sin^2(bx) dx = A \left[\frac{x}{2} - \frac{\sin 2bx}{4b} + c \right]$$

$$n_x = 1, 2, 3, \dots \quad n_y = 1, 2, 3, \dots \quad n_z = 1, 2, 3, \dots$$

Energy Level	Energy	(n_x, n_y, n_z)	Degeneracy
5			
4			
3			
2			
1			

(b) [4 pts] Consider an electron in a cubic box of 1 nm on a side. Calculate the ratio of the probabilities (N_2/N_1) of finding the electron in the first two energy levels at

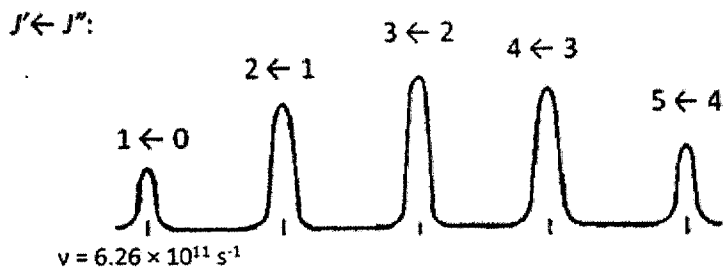
- i) 10 K ii) 10'000 K

(c) [4.5 pts] With what probability will the particle described by the wavefunction in (a) be found in the volume $0 < x, y, z < L/4$ in the ground state?

(d) [1 pts] What would be the probability of finding the particle in the region described in (d) if the box was classical?

QUESTION 7 (14 pts)

- (a) [6 pts] Below is a rotational spectrum of H^{35}Cl . Use the frequency for the $J=0$ to $J=1$ transition to determine the equilibrium internuclear distance R_e .



- (b) [1 pts] Calculate the moment of inertia for H^{35}Cl .
- (c) [4 pts] Calculate the difference in rotational energy between $J = 1$ and $J = 2$ levels and determine the wavelength of radiation that will be absorbed in promoting the molecule from $J = 1$ to $J = 2$. Show your work.
- (d) [1 pts] What is the degeneracy of the rotational level with $J = 5$?
- (e) [1 pts] What is the number of vibrational modes for HCl ?
- (f) [1 pts] Which of the following has the highest fundamental frequency of vibration? Circle the correct answer.

H^{35}Cl

D^{35}Cl

Physical and Chemical Constants

Atomic mass unit	1 a.m.u. = $1.6605402 \times 10^{-27}$ kg
Mass of an electron	$m_e = 9.109 \times 10^{-31}$ kg
Mass of a neutron	$m_n = 1.674 \times 10^{-27}$ kg
Mass of a proton	$m_p = 1.672 \times 10^{-27}$ kg
Avogadro's number	$N_A = 6.0221367 \times 10^{23}$ mole ⁻¹
Boltzmann's constant	$k = 1.380658 \times 10^{-23}$ J K ⁻¹
Faraday's constant	$F = 9.6485309 \times 10^4$ C mole ⁻¹
Fundamental unit charge	$e = 1.60217733 \times 10^{-19}$ C
Gas constant	$R = 8.314510$ J mole ⁻¹ K ⁻¹ $= 0.08315$ bar dm ³ mole ⁻¹ K ⁻¹ $= 0.082058$ L atm mole ⁻¹ K ⁻¹
Heat capacity of water	$C = 4.184$ J g ⁻¹ K ⁻¹ $= 75.4$ J mole ⁻¹ K ⁻¹
Planck's constant	$h = 6.6260755 \times 10^{-34}$ J s
Rydberg's constant	$R_H = 2.1798 \times 10^{-18}$ J $= 1.097 \times 10^7$ nm ⁻¹
Speed of light	$c = 2.99792458 \times 10^8$ m s ⁻¹
Zero point	0°C = 273.15 K
K_w of H ₂ O at 25°C	$K_w = 1.00 \times 10^{-14}$
Pi	$\pi = 3.1415927$

CONVERSION FACTORS

1 atmosphere (atm)	= 1.01325×10^5 Pa (N m ⁻²) = 760.0 mm Hg (torr) = 1.01325 bar
1 calorie (cal)	= 4.184 joules (J)
1 debye (D)	= 3.335617×10^{-30} C m
1 eV/particle	= 96.485 kJ mole ⁻¹ = 23.061 kcal mole ⁻¹
1 eV	= 1.602×10^{-19} J = 8067 cm ⁻¹
1 kcal mole ⁻¹	= 4.184 kJ mole ⁻¹ = 349.73 cm ⁻¹
1 kJ mole ⁻¹	= 0.23901 kcal mole ⁻¹ = 83.591 cm ⁻¹
1 L atm	= 101.325 J = 24.217 cal
ln x	= 2.3026 log x

EQUATION SHEET

You may tear this sheet off

$PV = nRT$ $P = \frac{nRT}{(V - nb)} - \frac{an^2}{V^2}$	$\Delta U = q + w$ $dU = nC_{v,m}dT$ $H = U + PV$ $dH = nC_{p,m}dT$
$dw = -P_{ext}dV$ $W_{rev} = -\int_{V_1}^{V_2} \frac{nRT}{V} dV = -nRT \ln \frac{V_2}{V_1} = -nRT \ln \frac{P_1}{P_2}$	$\Delta H^\circ_{rxn} = \sum u_i \Delta H^\circ_{f,i} + \Delta C_p (T - 25^\circ C)$ $\Delta C_p = \sum u_i C_{p,i}$ For ideal gases, $C_{p,m} = C_{v,m} + R$
$P_1 V_1^\gamma = P_2 V_2^\gamma$ $T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$ $\left(\frac{\bar{C}_p}{\bar{C}_v}\right) = \gamma$ $T_1 P_1^{((1-\gamma)/\gamma)} = T_2 P_2^{((1-\gamma)/\gamma)}$	Efficiency = $ w /q_{in}$ $\epsilon = 1 - T_C/T_H$
$dS \equiv \frac{dQ_{reversible}}{T}$ $\Delta S^\circ(T_2) = \Delta S^\circ(T_1) + \int_{T_1}^{T_2} \Delta C_p \frac{dT}{T}$ $\Delta S^\circ = \sum coeff_p S^\circ(pdts) - \sum coeff_r S^\circ(rcts)$ $\Delta S_{univ} = \Delta S_{sys} + \Delta S_{surr}$ $S = k \ln W$	$\Delta S = nR \ln \frac{V_f}{V_i}$ $\Delta S = nC_v \ln \frac{T_f}{T_i}$ $\Delta S = nC_p \ln \frac{T_f}{T_i}$
$\Delta P_1 = P_1^* x_2$ $P_2 = K'm$ $\pi = MRT$ $S_{gas} = K_H P_{gas}$	$P_1 = a_1 P_1^*$ $\Delta P_1 = a_2 P_1^*$ $a_1 = \gamma x_1$ $a_2 = \gamma x_2$
$\ln\left(\frac{K(T_1)}{K(T_2)}\right) = -\frac{\Delta H^\circ_{reaction}}{R} \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$ $\frac{dP}{dT} = \frac{\Delta \bar{H}}{T \Delta \bar{V}}$ $\frac{dP}{dT} = \frac{P \Delta_{vap} \bar{H}}{RT^2}$	$E = hv$ $\lambda = c/v$ $\lambda = h/mv$ $E_{photon} = hv_0 + E_k$ $E_n = -R_H Z^2/n^2$
First order reaction: $\ln\{[A]_0/[A]\} = kt$ Second order reaction: $1/[A] - 1/[A]_0 = kt$ Zero order reaction: $[A] = -kt + [A]_0$ Arrhenius equation: $k = Ae^{-E_a/RT}$	Particle in a 1D box $E_n = \frac{n^2 h^2}{8mL^2}$ $\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$
$-\log \frac{I}{I_0} = -\log T = A$ $A = \epsilon bc$	$E_{rot} = \frac{J(J+1)h^2}{8\pi^2 I}$ $\mu = \frac{m_1 m_2}{m_1 + m_2}$ $I = \mu r^2$ $E_{vib} = \left(v + \frac{1}{2}\right) h \nu$ $\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-(E_i - E_j)/k_B T}$

