SOLUTIONS

$$y'' - 2y' = 2 - 3x$$

RE $\quad r^2 - 2r = 0$ (1)

$r(r-2) = 0 \rightarrow r = 0, 2 \rightarrow y_h = C_1 e^{0x} + C_2 e^{2x}$

$\underline{\underline{y_h = C_1 + C_2 e^{2x}}}$

$f(x) = 2 - 3x$ $\quad$ Guess $\quad y_p = A + Bx$ (NO)

$y_p = Ax + Bx^2$ ← (1)

$y_p' = A + 2Bx$

$y_p'' = 2B$

So $y_p'' - 2y_p' = 2 - 3x$

$(2B) - 2(A + 2Bx) = 2 - 3x$

$2B - 2A - 4Bx = 2 - 3x$

$$\begin{cases} -2A + 2B = 2 \\ -4B = -3 \end{cases}$$

$\therefore B = 3/4$ (1)

$B - A = 1 \Rightarrow 3/4 - A = 1$

$\therefore A = -1/4$ (1)

$\underline{y_p} = -\frac{1}{4}x + \frac{3}{4}x^2$

Now, $y = y_h + y_p = \underbrace{C_1 + C_2 e^{2x}}_{(1)} \underbrace{- \frac{1}{4}x + \frac{3}{4}x^2}_{(1)}$

SOLUTIONS

$y'' - 4y' + 4y = 6xe^{2x}$ ①

RE $0 = r^2 - 4r + 4 = (r-2)^2 \Rightarrow r = 2, 2$

$y_h = C_1 e^{2x} + C_2 x e^{2x}$

$y_1 = e^{2x}$ $\quad y_1' = 2e^{2x}$

$y_2 = xe^{2x}$ $\quad y_2' = e^{2x} + 2xe^{2x}$

NOT REQ'D! $W(e^{2x}, xe^{2x}) = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x}+2xe^{2x} \end{vmatrix}$

$= e^{4x} + 2xe^{4x} - 2xe^{4x} = e^{4x}$

$$\begin{cases} u_1' e^{2x} + u_2' xe^{2x} = 0 \\ u_1' 2e^{2x} + u_2' (e^{2x} + 2xe^{2x}) = 6xe^{2x} \end{cases}$$ ③ for work

$$\left(\begin{array}{cc|c} e^{2x} & xe^{2x} & 0 \\ 2e^{2x} & e^{2x}+2xe^{2x} & 6xe^{2x} \end{array}\right) \sim \left(\begin{array}{cc|c} e^{2x} & xe^{2x} & 0 \\ 0 & e^{2x} & 6xe^{2x} \end{array}\right) R_2 - 2R_1$$

$$\sim \left(\begin{array}{cc|c} 1 & x & 0 \\ 0 & 1 & 6x \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -6x^2 \\ 0 & 1 & 6x \end{array}\right) R_1 - xR_2$$

$\therefore u_1 = \int -6x^2\,dx = -2x^3$ ① $\quad u_2 = \int 6x\,dx = 3x^2$ ①

$\Rightarrow y = y_h + y_p = C_1 e^{2x} + C_2 xe^{2x} - 2x^3 e^{2x} + 3x^3 e^{2x}$

$= C_1 e^{2x} + C_2 xe^{2x} + x^3 e^{2x}$

① for y_h, ① for y_p

SOLUTIONS

$4x^2y'' + 4xy' - y = x \longrightarrow y'' + \frac{1}{x}y' - \frac{1}{4x^2}y = \frac{1}{4x}$

$a=4 \quad b=4 \quad c=-1$

RE $ar^2 + (b-a)r + c = 0$

$4r^2 - 1 = (2r-1)(2r+1) = 0 \longrightarrow r = \frac{1}{2}, -\frac{1}{2}$ ①

$y_1 = x^{1/2} \quad y_1' = \frac{1}{2}x^{-1/2}$

$y_2 = x^{-1/2} \quad y_2' = -\frac{1}{2}x^{-3/2}$

$A\vec{x} = \vec{b}: \begin{pmatrix} x^{1/2} & x^{-1/2} \\ \frac{1}{2}x^{-1/2} & -\frac{1}{2}x^{-3/2} \end{pmatrix}\begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{4x} \end{pmatrix}$ ← (CAREFUL! correct $f(x)$) ①

$\det A = W(x^{1/2}, x^{-1/2}) = -\frac{1}{2}x^{-1} - \frac{1}{2}x^{-1} = -\frac{1}{x}$

$\vec{x} = A^{-1}\vec{b} = (-x)\begin{pmatrix} -\frac{1}{2}x^{-3/2} & -x^{-1/2} \\ -\frac{1}{2}x^{-1/2} & x^{1/2} \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{4x} \end{pmatrix}$

This is $\frac{1}{\det A}$

$= \begin{pmatrix} \frac{1}{2}x^{1/2} & x^{1/2} \\ \frac{1}{2}x^{1/2} & -x^{3/2} \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{4x} \end{pmatrix} = \begin{pmatrix} \frac{1}{4}x^{-1/2} \\ -\frac{1}{4}x^{1/2} \end{pmatrix}$ ③ for work

$u_1 = \frac{1}{4}\int x^{-1/2}dx = \frac{1}{2}x^{1/2}$ ① $\quad u_2 = \frac{1}{4}\int x^{1/2}dx = -\frac{1}{6}x^{3/2}$ ①

So $y_p = (\frac{1}{2}x^{1/2})(x^{1/2}) + (-\frac{1}{6}x^{3/2})(x^{-1/2}) = \frac{1}{2}x - \frac{1}{6}x = \frac{x}{3}$

Then $y = \underbrace{c_1x^{1/2} + c_2x^{-1/2}}_{①} + \underbrace{\frac{1}{3}x}_{①}$