Midterm Exam Emat 233

February, 2006

Problem 1. *Consider the surface given by the formula*

$$1 = \cos(zx)e^{z+y}$$

(a) *Find the equation of the tangent plane at the point* $(\pi, 0, 0)$.

(b) *Find normal line in symmetric or in parametric form passing through the point* $(\pi, 0, 0)$.

Problem 2 *Let* $\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + t\mathbf{k}$ *be the position vector of a moving particle.*

(a) *Find the velocity vector* $\mathbf{v}(t)$ *and the acceleration vector* $\mathbf{a}(t)$ *at any* t.

(b) *Find the tangential component* a_T *of the acceleration vector* $\mathbf{a}(t)$ *at any* t.

(c) *At what point(s) does the particle pass through the* xy*-plane?*

Problem 3. *Answer the following two questions for the function*

$$f(x, y) = e^{x^2+y^2}.$$

(a) *Find the directional derivative of the function at the point* $(1, 1)$ *in the direction of the vector* $-\mathbf{i} + \mathbf{j}$.

(b) *Find the direction along which the function* f *increases most rapidly at the point* $(1, 1)$*, and find the maximum rate.*

Problem 4.

(a) *Let* $f(x, y, z) = x^2 - 2y + zx$*; find* $\text{grad}(f)$ *and* $\text{curl}(\text{grad}(f))$. *Can you find also* $\text{curl}(f)$ *and* $\text{div}(f)$*?* **Explain.**

(b) *Let* $\mathbf{F}(x, y, z) = xz\mathbf{i} + y^2\mathbf{j} + \text{e}^z\mathbf{k}$*; find* $\text{div}(\mathbf{F})$. *Can you find* $\text{grad}(\mathbf{F})$ *?* **Explain.**