The geometry of an electrical component is approximated by "a cube + a cylinder" as shown in the figure below. The entire component is made of pure copper ($\rho = 8933$ kg/m^3, $C_p = 385$ $J/kg{\cdot}K$, $k = 401$ $W/m{\cdot}$K). The temperature of the component is 150°C when it is exposed to a convective environment (h=5 W/$m^2{\cdot}K$ and T_∞=20°C). Calculate

(a) the Biot number for the problem and

(b) temperature of the component after 10 minutes

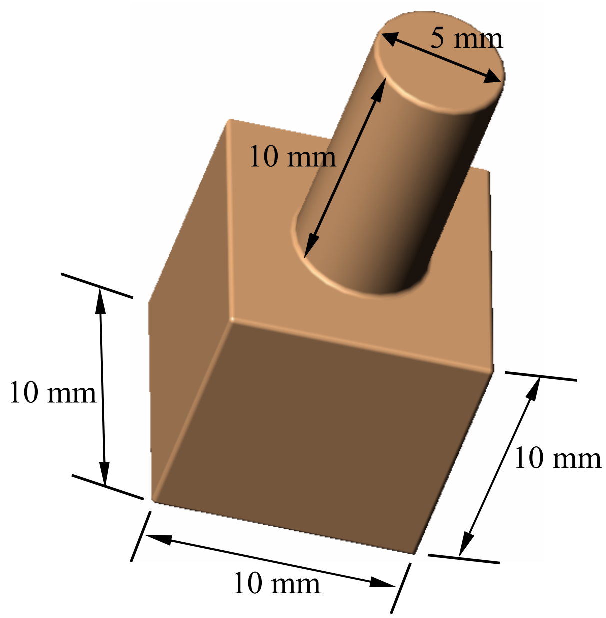


Figure 1

Solution:

Volume of the component: $\forall = \left(\dfrac{10}{1000}\right)^3 + \dfrac{\pi}{4}\left(\dfrac{5}{1000}\right)^2\left(\dfrac{10}{1000}\right) = 0.12 \times 10^{-5}\ m^3$

Surface area of the component: $A_s = 6 \times \left(\dfrac{10}{1000}\right)^2 + \pi \times \dfrac{5}{100} \times \dfrac{10}{1000} = 0.00076\ m^2$

Characteristic length: $L_c = \dfrac{\forall}{A_s} = \dfrac{0.12 \times 10^{-5}}{0.00076} = 0.0016\ m$

Biot number: $\boxed{Bi = \dfrac{hL_c}{k} = \dfrac{5 \times 0.0016}{401} = 0.00002 << 0.1}$; (Therefore, Lumped method is applicable here)

Temperature of the component after 10 minutes (=600 sec):

$$\boxed{T = T_\infty + \left(T_0 - T_\infty\right)\ \exp\left(-\frac{hA_s}{\rho C_p \forall} t\right) = 20 + \left(150 - 20\right)\ \exp\left(-\frac{5 \times 0.00076}{8933 \times 385 \times 0.12 \times 10^{-5}} \times 600\right) = 94.82\ ^\circ C}$$

An insulated iron pipe is carrying saturated steam at 250ºC. The length of the pipe is 10 m. Surrounding temperature and the convection heat transfer coefficient are 25ºC and 10 W/m^2·K, respectively. Calculate

(a) the heat loss to the surrounding if the contact between the pipe and insulation is ideal.
(b) A subsequent experiment shows that the heat loss to the surrounding is 80% of the heat loss obtained in ideal case. Determine the magnitude of the thermal contact resistance between the pipe and the insulation.

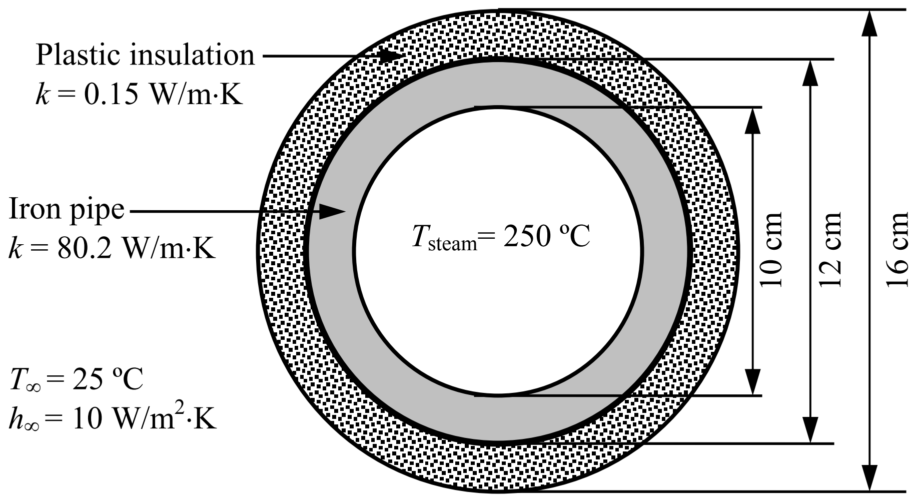
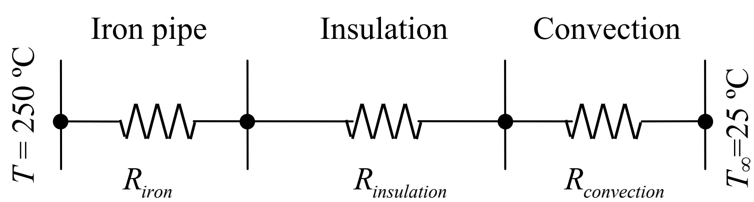


Figure 2

Solution:

Perfect contact: no contact resistance



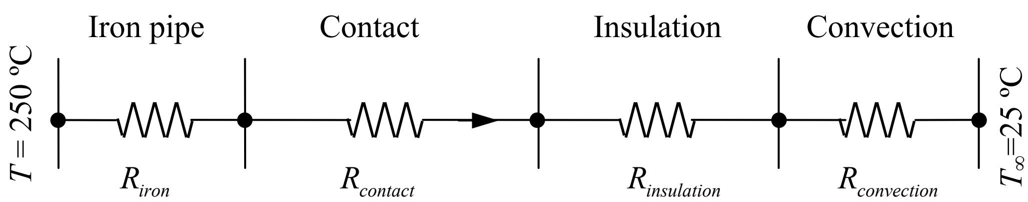
$$R_{iron} = \frac{\ln\left(\frac{r_{outer}}{r_{inner}}\right)}{2\pi\, k\, L} = \frac{\ln\left(\frac{12/2}{10/2}\right)}{2\pi \times 80.2 \times 10} = 0.00003618 \quad \frac{K}{W}$$

$$R_{insulation} = \frac{\ln\left(\frac{r_{outer}}{r_{inner}}\right)}{2\pi\, k\, L} = \frac{\ln\left(\frac{16/2}{12/2}\right)}{2\pi \times 0.15 \times 10} = 0.0305240 \quad \frac{K}{W}$$

$$R_{convection} = \frac{1}{h\,A_s} = \frac{1}{h \times \pi DL} = \frac{1}{10 \times \pi \times \frac{16}{100} \times 10} = 0.019894 \quad \frac{K}{W}$$

Therefore, heat loss: $\dot{Q}_{loss} = \frac{T_{steam} - T_\infty}{\sum R} = \frac{250 - 25}{R_{iron} + R_{insulation} + R_{convection}} \approx 4460\,W$

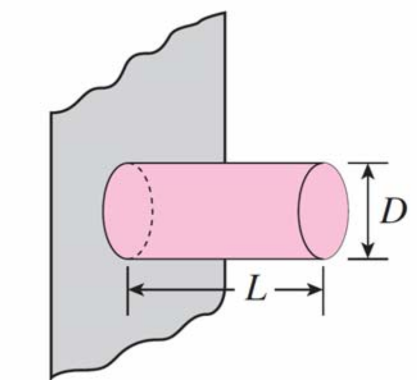
Imperfect contact with contact resistance:



$$\dot{Q}_{loss,measured} = \frac{T_{steam} - T_\infty}{\sum R} \quad \rightarrow \sum R = \frac{T_{steam} - T_\infty}{\dot{Q}_{loss,measured}} = R_{iron} + R_{contact} + R_{insulation} + R_{convection}$$

$$R_{contact} = \frac{T_{steam} - T_\infty}{\dot{Q}_{loss,measured}} - \left(R_{iron} + R_{insulation} + R_{convection}\right)$$
$$= \frac{T_{steam} - T_\infty}{0.8 \times \dot{Q}_{loss}} - \left(R_{iron} + R_{insulation} + R_{convection}\right) = 0.01261 \quad \frac{K}{W}$$

Compare the heat transfer performance of a pin fin of rectangular profile and a pin fins of triangular profile when L=20cm, D=2cm, T_b=100°C, T_∞=20°C, k=80.2 W/m^{2}K, and h=10 W/mK.



Pin fins of rectangular profile

$$m = \sqrt{\frac{4h}{kD}} = 5$$

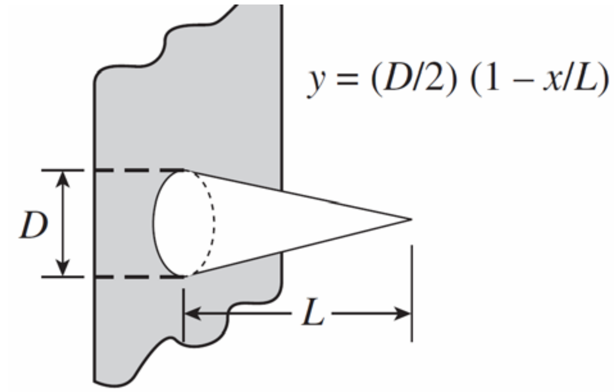
$$A_{fin} = \pi D L_c = 0.01288\ m^2$$

$$\eta_{fin} = \frac{\tanh(mL_c)}{mL_c} = 75\ \%$$

$$R_{fin} = \frac{1}{\eta_{fin} A_{fin} h} = 10.30\ \frac{^oC}{W}$$

$$\dot{Q}_{fin} = \frac{T_b - T_\infty}{R_{fin}} = 7.76\ W$$

Heat Transfer = 100%
Volume and mass = 100%



Pin fins of triangular profile

$$m = \sqrt{\frac{4h}{kD}} = 5$$

$$A_{fin} = \frac{\pi D}{2}\sqrt{L^2 + \left(\frac{D}{2}\right)^2} = 0.00629 m^2$$

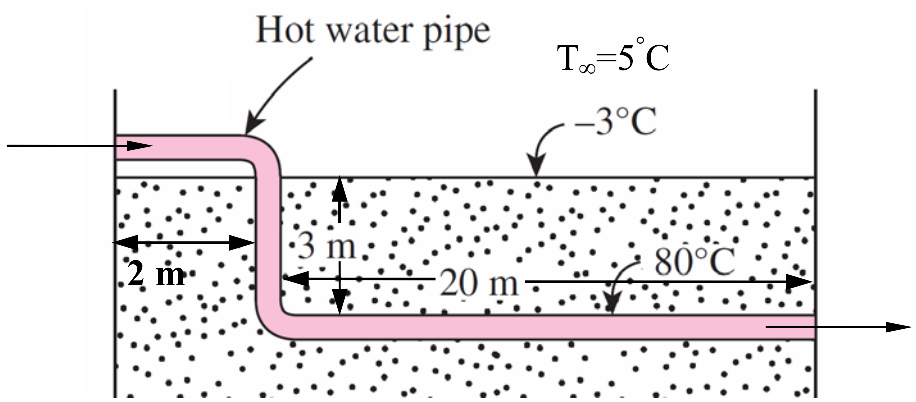
$$\eta_{fin} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} = 86\ \%$$

$$R_{fin} = \frac{1}{\eta_{fin} A_{fin} h} = 18.34\ \frac{^oC}{W}$$

$$\dot{Q}_{fin} = \frac{T_b - T_\infty}{R_{fin}} = 4.36\ W$$

Heat Transfer = 56%
Volume and mass = 50%

Hot water at an average temperature of 80°C and an average velocity of 1.5 m/s is flowing through a 25-m section of a pipe that has an outer diameter of 5 cm. The pipe extends 2 m in the ambient air above the ground, dips into the ground (k=1.5 W/m·°C) vertically for 3 m, and continues horizontally at this depth for 20 m more before it enters the next building. The first section of the pipe is exposed to the ambient air at 5°C, with a heat transfer coefficient of 22 W/m^2·°C. If the surface of the ground is covered with snow at -3°C, determine the total rate of heat loss from the hot water.



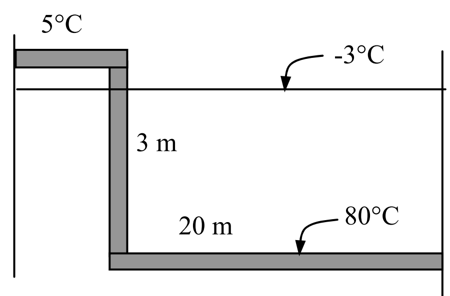
Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** The pipe is at the same temperature as the hot water.

Properties The thermal conductivity of the ground is given to be k = 1.5 W/m·°C.

Analysis (a) We assume that the surface temperature of the tube is equal to the temperature of the water. Then the heat loss from the part of the tube that is on the ground is

$$A_s = \pi DL = \pi(0.05\text{ m})(2\text{ m}) = 0.3142\text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$= (22\text{ W/m}^2.°\text{C})(0.3142\text{ m}^2)(80-5)°\text{C} = 518\text{ W}$$



Considering the shape factor, the heat loss for vertical part of the tube can be determined from

$$S = \frac{2\pi L}{\ln\left(\frac{4L}{D}\right)} = \frac{2\pi(3\text{ m})}{\ln\left[\frac{4(3\text{ m})}{(0.05\text{ m})}\right]} = 3.44\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (3.44\text{ m})(1.5\text{ W/m.°C})[80-(-3)]°\text{C} = 428\text{ W}$$

The shape factor, and the rate of heat loss on the horizontal part that is in the ground are

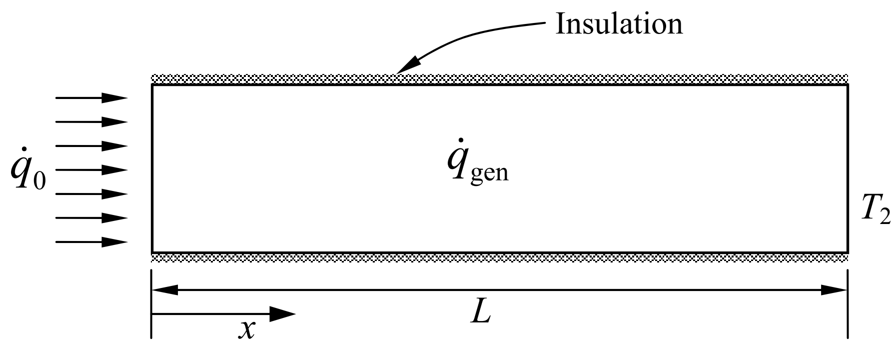
$$S = \frac{2\pi L}{\ln\left(\frac{4z}{D}\right)} = \frac{2\pi(20\text{ m})}{\ln\left[\frac{4(3\text{ m})}{(0.05\text{ m})}\right]} = 22.9\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (22.9\text{ m})(1.5\text{ W/m.°C})[80-(-3)]°\text{C} = 2851\text{ W}$$

and the total rate of heat loss from the hot water becomes

$$\dot{Q}_{\text{total}} = 518 + 428 + 2851 = \mathbf{3797\text{ W}}$$

In a nuclear power plant, a metal rod of constant cross-sectional area is generating $\dot{q}_{\text{gen}}$ (W/m^3) amount of heat internally. The left wall of the rod is exposed to a constant heat flux ($\dot{q}_0$, W/m^2) condition. While the right wall of the rod is exposed to an environment having constant temperature (T_2, $^\circ$C). The remaining surfaces of the rod are properly insulated so that you can consider a one dimensional heat transfer. Determine an expression of the temperature distribution inside the rod. Assume that the thermal conductivity of the rod is constant. Also verify your solution.



$$\frac{1}{A_c}\frac{d}{dx}\left(kA_c\frac{dT}{dx}\right)+\dot{q}_{gen}=\rho C\frac{dT}{dt}$$

$$\Rightarrow \frac{kA_c}{A_c}\frac{d}{dx}\left(\frac{dT}{dx}\right)+\dot{q}_{gen}=0$$

$$\Rightarrow \frac{d^2T}{dx^2}=-\frac{\dot{q}_{gen}}{k}$$

$$\Rightarrow \frac{dT}{dx}=-\frac{\dot{q}_{gen}}{k}x+C_1 \qquad (1)$$

$$\Rightarrow T=-\frac{\dot{q}_{gen}}{k}\frac{x^2}{2}+C_1x+C_2 \qquad (2)$$

First boundary condition: constant heat flux

$$x=0, -k\frac{dT}{dx}=\dot{q}_0 \quad \rightarrow \frac{dT}{dx}=-\frac{\dot{q}_0}{k}$$

$$\Rightarrow \frac{dT}{dx}=-\frac{\dot{q}_0}{k}=-\frac{\dot{q}_{gen}}{k}x+C_1=-\frac{\dot{q}_{gen}}{k}0+C_1=C_1$$

$$\Rightarrow C_1=-\frac{\dot{q}_0}{k}$$

Second boundary condition: constant temperature

$$x = L, T = T_2$$

$$T_2 = -\frac{\dot{q}_{gen}}{k}\frac{L^2}{2} + C_1 L + C_2 = -\frac{\dot{q}_{gen}}{k}\frac{L^2}{2} - \frac{\dot{q}_0}{k}L + C_2$$

$$\Rightarrow C_2 = T_2 + \frac{\dot{q}_{gen}}{k}\frac{L^2}{2} + \frac{\dot{q}_0}{k}L$$

$$T = -\frac{\dot{q}_{gen}}{k}\frac{x^2}{2} + C_1 x + C_2$$

$$\Rightarrow T = -\frac{\dot{q}_{gen}}{k}\frac{x^2}{2} - \frac{\dot{q}_0}{k}x + T_2 + \frac{\dot{q}_{gen}}{k}\frac{L^2}{2} + \frac{\dot{q}_0}{k}L$$

$$\Rightarrow T = \frac{\dot{q}_{gen}}{2k}\left(L^2 - x^2\right) + \frac{\dot{q}_0}{k}\left(L - x\right) + T_2$$

Boundary condition check

$$x = L, T = T_2$$

$$T = \frac{\dot{q}_{gen}}{2k}\left(L^2 - L^2\right) + \frac{\dot{q}_0}{k}\left(L - L\right) + T_2 = T_2$$