MATH 2030 3.0
Midterm test, February 8th, 2016

NAME **STUDENT NUMBER**

Question 1. 5*points* You roll two fair dice. What is the probability that the sum of the numbers showing is equal to 7?

Solution $S = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$ so that $P(S) = \frac{6}{36}$.

Question 2 10*points* Events F, G ad H are such that

$$P(F) = .7,\ P(G) = .6,\ P(H) = .5,$$
$$P(F \cap G) = .4,\ P(F \cap H) = .3,\ P(G \cap H) = .2,\ P(F \cap G \cap H) = .1.$$

Find $P(F \cup G)$, $P(F^c \cap G)$ and $P(F^c \cap G^c)$.

Solution

$$P(F \cup G) = P(F) + P(G) - P(F \cap G) = .7 + .6 - .4$$
$$P((F \cap G) \cup (F^c \cap G)) = P(G) = .6 = .4 + P((F^c \cap G)),\ \ P((F^c \cap G)) = .2$$
$$P(F^c \cap G^c) = P((F \cup G)^c) = 1 - P(F \cup G) = 1 - .9 = .1$$

Question 3 You have a game of 52 cards. You draw four cards in sequence without replacement.

(a) 5*points* What is the probability of drawing in that order an ace of diamonds, a king of heart, a jack of spade and a 10 of clubs?

Solution $\frac{1}{52}\frac{1}{51}\frac{1}{50}\frac{1}{49}$

(b) 5*points* What is the probability of drawing the same hand as in (a) but in any order?

Solution $4!\frac{1}{52}\frac{1}{51}\frac{1}{50}\frac{1}{49}$

(c) 5*points* What is the probability of drawing four cards of the same suit?

Solution $4 \times \frac{13}{52}\frac{12}{51}\frac{11}{50}\frac{10}{49}$

Question 4 A manufacturing process produces integrated circuit chips. Over the long run, the fraction of bad chips produced by the process is around 20%. Thoroughly testing a chip to determine whether it is good or bad is rather expensive, so a cheap test is tried. All good chips will pass the cheap test but so will 10% of the bad chips.

(a) 5*points* Given a chip passes the cheap test, what is the probability that it is a good chip?

Solution Let G be the event "being good" and B the event being "bad". Let $Pass$ be the event "passes the test. The question is to compute $P(G|pass)$. By Bayes formula, we have

$$p(G|pass) = \frac{P(pass|G)P(G)}{P(pass|G)P(G) + P(pass|B)P(B)} = \frac{.8}{.8 + .02} = \frac{8}{8.2}$$

(b) 5*points* If a company using this manufacturing process sells all chips which pass the cheap test, over the long run, what percentage of chips sold will be bad?

Solution The question is what percentage of chips that pass the test are actually bad, i.e.

$$P(B|pass) = 1 - P(G|pass) = \frac{.2}{8.2}.$$

Question 5 Suppose that in 4-child families, each child is equally likely to be a boy or a girl, independently of the others.

(a) 5*points* What is the probability of a family with two boys and two girls?

Solution Each child is considered as an independent trial. Let "girl" be "success" and let X be the total number of successes in 4 independent trials. The question is then "what is $P(X = 2)$? We have

$$P(X = 2) = \binom{4}{2}\frac{1}{2^4} = \frac{6}{2^4}.$$

(b) 5*points* What is the probability of a family with four boys?

Solution $P(X = 0) = \binom{4}{0}\frac{1}{2^4} = \frac{1}{2^4}$.

Question 6 Let X be the number of successes in 1000 independent trials with two possible outcomes "success" or "failure". Let $p = \frac{1}{2}$ be the probability of success on each trial.

(a) $5 points$ Approximate the probability that $200 \leq X \leq 300$.

Solution $np = 500, npq = 250, \sqrt{npq} = 5\sqrt{10}$

$$P(200 \leq X \leq 300) = P(\frac{200 - .5 - 500}{5\sqrt{10}} \leq Z \leq \frac{300 + .5 - 500}{5\sqrt{10}})$$

$$= P(-\frac{60.1}{\sqrt{10}} \leq Z \leq -\frac{39.9}{\sqrt{10}}) = 0$$

(b) $5 points$ Approximate the probability that $X = 500$.

Solution

$$P(X = 500) \approx P(500 - .5 \leq X \leq 500.5) = P(\frac{500 - .5 - 500}{5\sqrt{10}} \leq Z \leq \frac{500 + .5 - 500}{5\sqrt{10}})$$

$$= P(\frac{-.5}{5\sqrt{10}} \leq Z \leq \frac{.5}{5\sqrt{10}}) = P(-\frac{1}{10\sqrt{10}} \leq Z \leq \frac{1}{10\sqrt{10}})$$

$$= P(-0.03162278 \leq Z \leq 0.03162278) = 2(.0120) = .0240$$