Please Print Clearly

Last Name:__________________

First Name:__________________

Student Number:__________________________

Your Tutorial Section (CIRCLE ONE):

01 Thu. 9-11 RS211

02 Thu. 9-11 GB119

03 Tue. 10-12 SF2202

04 Tue. 10-12 SF3201

05 Tue. 13-15 GB304

06 Tue. 13-15 GB412

(YOU LOSE FOUR POINTS FOR INCORRECT TUTORIAL SECTION INFORMATION)

Your Lecture Section (CIRCLE ONE):

01 Reza Iravani **02 Reza Iravani** **03 Piero Triverio**

(YOU LOSE ONE POINT FOR INCORRECT LECTURE SECTION INFORMATION)

Mid-term – Answer All Questions **100 minutes [25 Marks]**
(Non-programmable Calculators Allowed)

Instructions:

- **Do not unstaple this exam book.**
- **Answer each question neatly and concisely.**
- **Write the final answer in the box where provided.**
- **The exam has 5 questions and 10 pages, including the cover page, the 2nd order-circuit aid-sheet and an extra sheet.**
- **Solutions steps and final answers must not be in pencil.**

Q1	**/8**
Q2	**/3**
Q3	**/6**
Q4	**/5**
Q5	**/3**
Total	**/25**

Question 1

a) Find the expressions for V_{AB} and V_O in terms of V_S in the circuit in Figure Q1.a. (The op amps are ideal). [4 points]

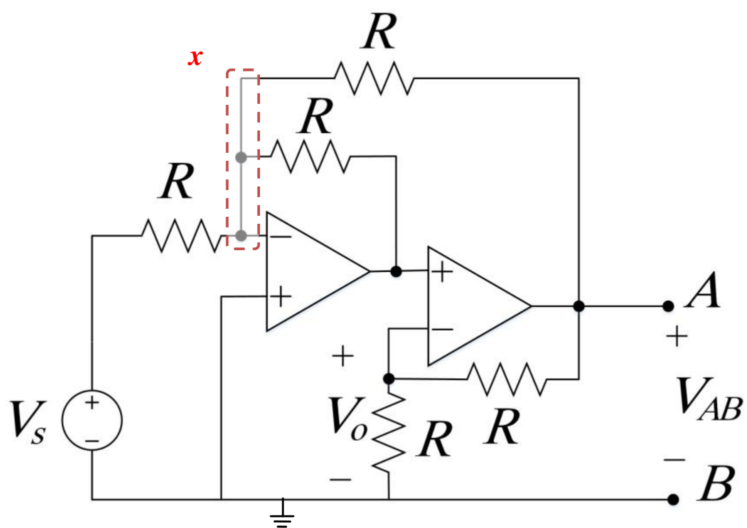


Figure Q1.a

For ideal op amps: $v^+=v^-$ and $i^+=i^-=0$

KCL at node x:

$$\frac{V_s}{R} = \frac{0 - V_o}{R} + \frac{0 - V_{AB}}{R}$$

$$V_s = -V_o - V_{AB} \quad (1)$$

Voltage division of V_{AB} and V_o:

$$V_o = V_{AB}\frac{R}{R+R} = \frac{V_{AB}}{2} \quad (2)$$

From (1) and (2),

$$V_s = -\frac{V_{AB}}{2} - V_{AB} = -\frac{3}{2}V_{AB}$$

And

$$V_s = -3V_o$$

$V_{AB} = -\frac{2}{3}V_s$

$V_O = -\frac{1}{3}V_s$

b) For the circuit in Figure Q1.b, (The op amps are ideal):

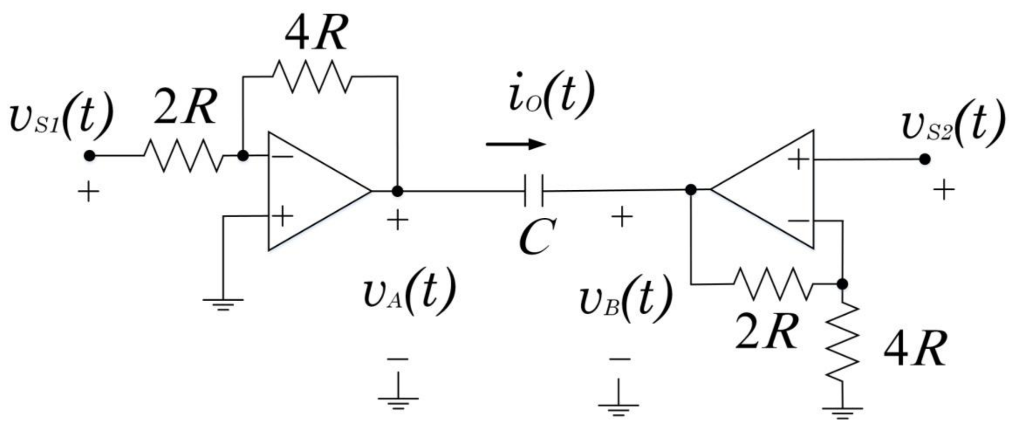


Figure Q1.b.

1) Find $v_A(t)$ in terms of $v_{s1}(t)$ and $v_{s2}(t)$. [2 points]

Using the inverting circuit equation:

$$v_A(t) = -\frac{4R}{2R}v_{S1}(t) = -2v_{S1}(t)$$

$v_A(t) = -2v_{S1}(t)$

2) Find $v_B(t)$ in terms of $v_{s1}(t)$ and $v_{s2}(t)$. [1 point]

Using the non-inverting circuit equation:

$$v_B(t) = \left(1 + \frac{2R}{4R}\right)v_{S2}(t) = \frac{3}{2}v_{S2}(t)$$

$v_B(t) = \frac{3}{2}v_{S2}(t)$

3) Find the expression for $i_o(t)$. [1 point]

Current flow through a capacitor:

$$i_o(t) = C\frac{dv_{AB}(t)}{dt} = C\frac{d\left(v_A(t) - v_B(t)\right)}{dt} = C\frac{d\left(-2v_{S1}(t) - \frac{3}{2}v_{S2}(t)\right)}{dt}$$

$i_o(t) = -2C\frac{dv_{S1}(t)}{dt} - \frac{3C}{2}\frac{dv_{S1}(t)}{dt}$

Question 2

The switch in the circuit in Figure Q2 opens at $t = 0$. (It has been closed for a long time).

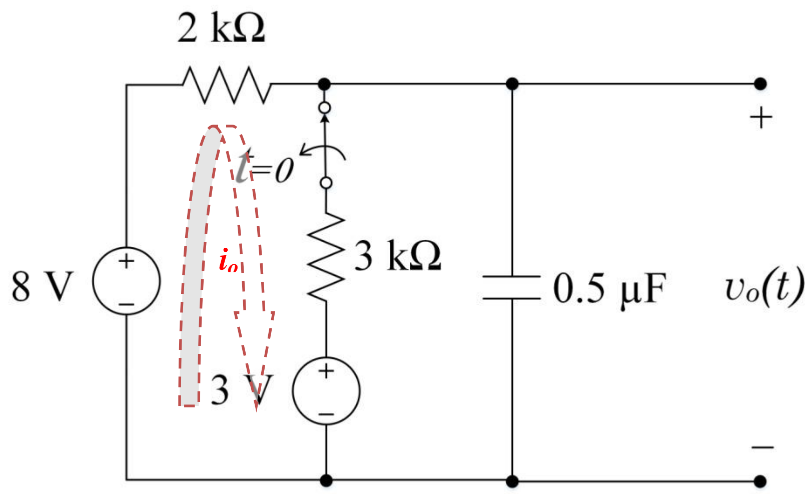


Figure Q2

1) Calculate the voltage $v_o(t)$ for $t \geq 0$. [2 points]

In the given network: $i_o(0) = \frac{(8-3)V}{(2+3)k\Omega} = 1mA$, $v_o(0) = 1mA \times 3k\Omega + 3V = 6V$ and $v_o(\infty) = 8\ V$

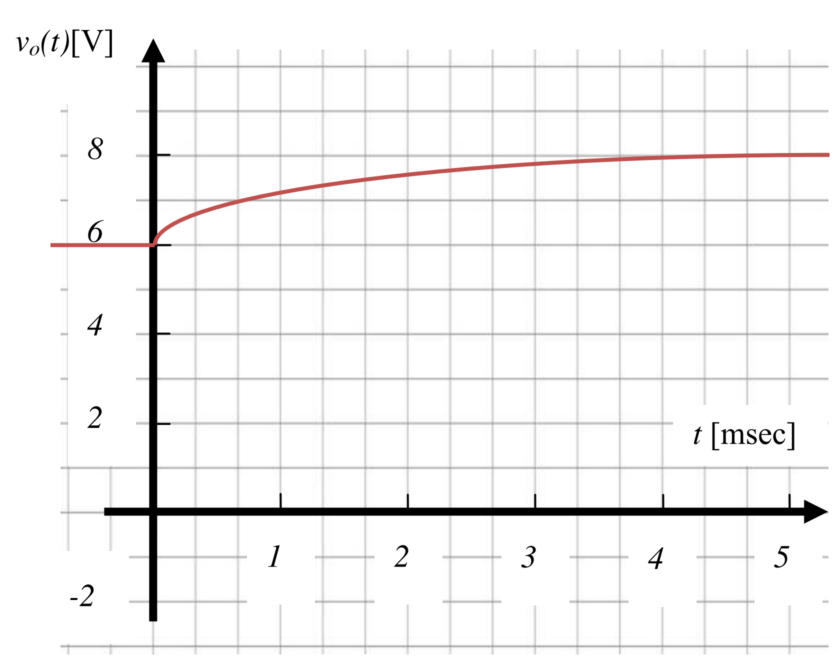
Circuit time constant, $T_C = RC = 2k\Omega \times 0.5\mu F = 1msec = \frac{1}{1000} sec$

Using 1st order circuit equation:

For $t \geq 0,\ v_o(t) = [v_o(0) - v_o(\infty)]e^{-t/T_C} + v_o(\infty) = [6-8]e^{-1000t} + 8 = 8 - 2e^{-1000t}\ V$

$v_o(t) = 8 - 2e^{-1000t}\ V$ for $t \geq 0$ or $v_o(t) = (8 - 2e^{-1000t})u(t)\ V$

2) Sketch $v_o(t)$ waveform for $t \geq 0$. [1 point]



Question 3

The switch in Figure Q3 has been in position 1 for a long time and is moved to position 2 at $t = 0$.
For $L = 4$ mH, $C = 10$ µF and $I_s = 2$ A.

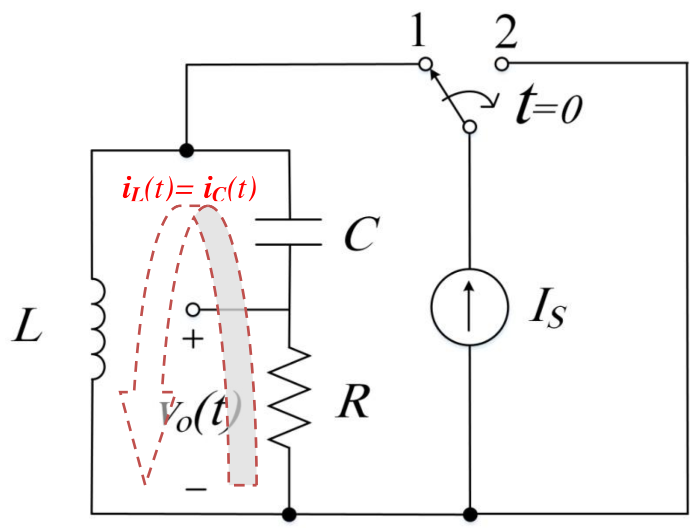


Figure Q3

a) Determine the values of R that make the circuit response (i)-overdamped, (ii)-critically damped and (iii)-underdamped. [3 points]

Using the 2nd order equation [a series RLC circuit] **page 9**:

$$\frac{d^2 v_C(t)}{dt^2} + \frac{R}{L}\frac{dv_C(t)}{dt} + \frac{1}{LC} v_C(t) = 0$$

Laplace Transform of this equation:

$$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$$

Compare it with the 2nd order standard form:

$$S^2 + 2\zeta\omega_o S + {\omega_o}^2 = 0$$

Then, $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4mH \times 10\ \mu F}} = 5000\ rad/sec$

& $2\zeta\omega_o = \frac{R}{L}$

$2\zeta \times 5000 = \frac{R}{4mH}$

$\zeta = \frac{R}{40}$

For the overdamped response $\zeta > 1 \quad \therefore R > 40\ \Omega$

For the critically damped response $\zeta = 1 \quad \therefore R = 40\ \Omega$

For the underdamped response $\zeta < 1 \quad \therefore R < 40\ \Omega$

Overdamped response
$R > 40\ \Omega$

Critically damped response
$R = 40\ \Omega$

Underdamped response
$R < 40\ \Omega$

b) If $R = 100\ \Omega$, find $v_o(t)$ for $t \geq 0$. [2 points]

$R = 100\ \Omega > 40\ \Omega$ ∴ Overdamped response

$$S^2 + 25000S + 5000^2 = 0$$

$$S_{1\&2} = -1043.56 \quad \& \quad -23956.44$$

$$v_C(t) = K_1 e^{-1043.56\,t} + K_2 e^{-23956.44t}\ \text{V} \qquad \text{for } t \geq 0$$

From the given circuit: $v_C(0) = 0$ V, $i_L(0) = I_S = 2A$, $v_o(0^-) = 0$ V & $v_o(0^+) = -200$ V

Using the initial conditions, we can calculate K_1 & K_2 as follows:

$$v_C(0) = K_1 + K_2 = 0 \qquad (1)$$

$$i_L(t) = i_C(t) = C\frac{dv_C(t)}{dt} = -1043.56CK_1e^{-1043.56t} - 23956.44CK_2e^{-23956.44t}\text{A} \quad \text{for } t \geq 0$$

$$i_L(0) = 2A = -1043.56CK_1 - 23956.44CK_2 \qquad (2)$$

Solving (1) & (2)

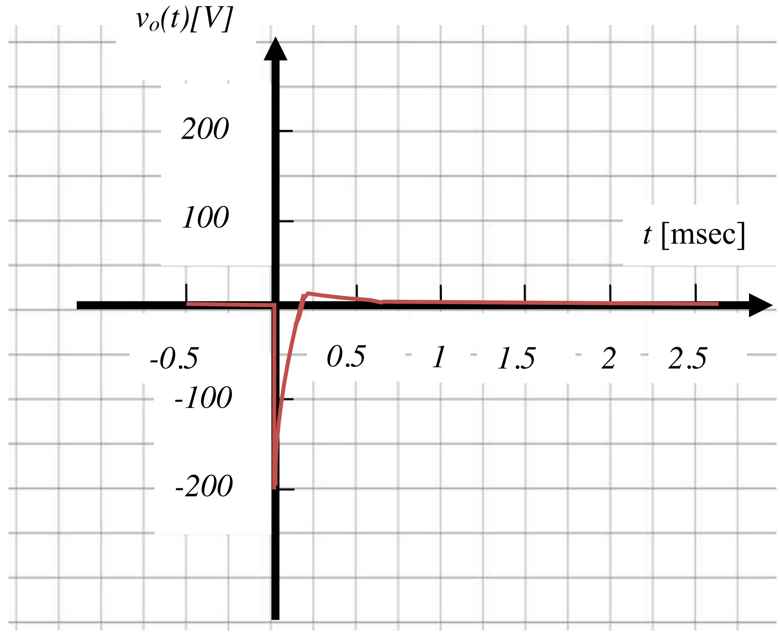
$K_1 = 8.7287$ & $K_2 = -8.7287$

$$i_L(t) = -0.0911e^{-1043.56t} + 2.0911e^{-23956.44t}\ \text{A} \quad \text{for } t \geq 0$$

$$v_o(t) = -i_L(t)R = 9.11e^{-1043.56t} - 209.11e^{-23956.44t}\text{V} \quad \text{for } t \geq 0$$

$v_o(t)$=$9.11e^{-1043.56t} - 209.11e^{-23956.44t}$V for $t \geq 0$

c) Plot $v_o(t)$ (from part b above) for $t \geq -0.5$ msec. [1 point]



Question 4

For the network given in Figure Q4:

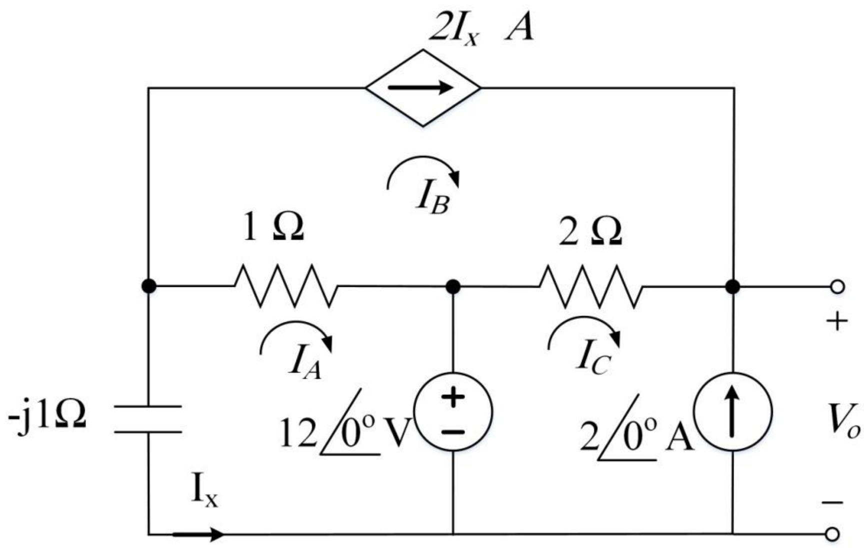


Figure Q4

1) Write the mesh-current equations using the three loop currents identified on the figure. [3 points]

KVL at loop "A":

$$-j1I_A + 1(I_A - I_B) + 12\angle 0^o = 0$$

$$(1 - j1)I_A - I_B = 12\angle 180^o \quad (1)$$

Using the dependent current source equation (loop "B"), we have:

$$I_B = 2I_x$$

and $I_x = -\,I_A$ from (loop "A")

$$\therefore I_B = -2I_A$$

$$2I_A + I_B = 0 \quad (2)$$

From loop "C":

$$I_c = -2\angle 0^o \quad (3)$$

The mesh-current equations

$$(1 - j1)I_A - I_B = 12\angle 180^o$$

$$2I_A + I_B = 0$$

$$I_c = -2\angle 0^o$$

2) Use the mesh-current analysis to find the phasor voltage V_o. [2 points]

KVL at Loop "C":

$$V_O = 2(I_B - I_c) + 12\angle 0^o$$

Solving equations (1) & (2) for I_B,

$$I_B = \frac{12\sqrt{10}}{5}\angle 18.435^o A$$

$$V_O = 2\left(\frac{12\sqrt{10}}{5}\angle 18.435^o + 2\angle 0^o\right) + 12\angle 0^o = \frac{8\sqrt{370}}{5}\angle 8.973^o V$$

$V_O = 30.7766\angle 8.973^o V$

Question 5

In the circuit shown in Figure Q4, determine the value of the inductance such that the current is in phase with the source voltage, i.e., they have the same phase angle. [3 points]

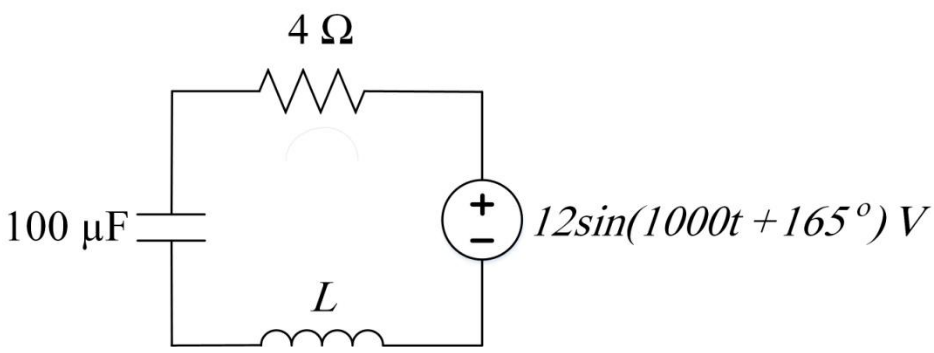


Figure Q4

The voltage source in time domain:

$$12\sin(1000t + 165^o)\,V = 12\cos(1000t + 165^o - 90^o)V = 12\cos(1000t + 75^o)V$$

The voltage source in phasor domain: $12\angle 75^o V$

It is required to have the current in phase with the voltage source; this means both, the voltage source and the current, have the same phase angle (75^o).

Or in other words, $$j\omega L - j\frac{1}{\omega C} = 0$$

In the given circuit:

$$\omega = 1000\ rad/sec$$

$$C = 100\ \mu F$$

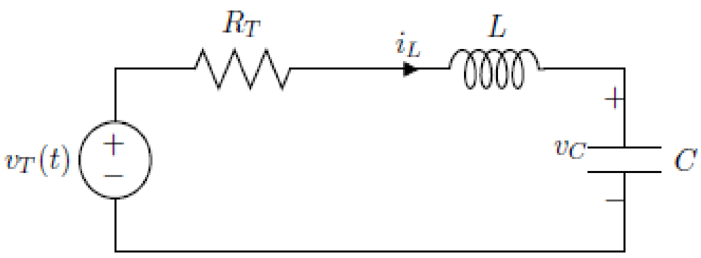
Solving for L

$$\therefore L = 0.01\ H = 10\ mH$$

L=10 mH

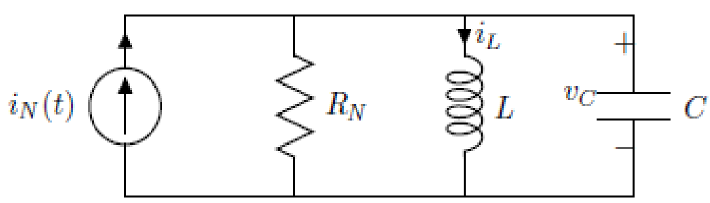
ECE212 Circuit Analysis - Aid Sheet
Fall 2014

Second order circuits



$$\frac{d^2v_C}{dt^2} + \frac{R_T}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{v_T}{LC}$$

$$\frac{d^2i_L}{dt^2} + \frac{R_T}{L}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{1}{L}\frac{dv_T}{dt}$$



$$\frac{d^2i_L}{dt^2} + \frac{1}{R_NC}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{i_N}{LC}$$

$$\frac{d^2v_C}{dt^2} + \frac{1}{R_NC}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{1}{C}\frac{di_N}{dt}$$

Standard form:

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2x(t) = f(t)$$

Extra sheet