CONCORDIA UNIVERSITY

Faculty of Engineering and Computer Science

Department of Engineering

FINAL EXAMINATION ENGR 213, 13. 4. 2007

Sections: F (Dr. A. B. Keviczky), G (Dr. S. T. Ali), J (Dr. A. Stancu)

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Answer each question! Calculators Permitted!

(10)#1...Solve the following two ordinary differential equations:

$$\text{(A)}\ \frac{dy}{dx} = \frac{y^2 e^x}{1+e^{2x}}, \qquad \text{(B)}\ \frac{dy}{dx} - \frac{2}{x}y = x^2 \cos x \text{ with } y(\pi) = 4.$$

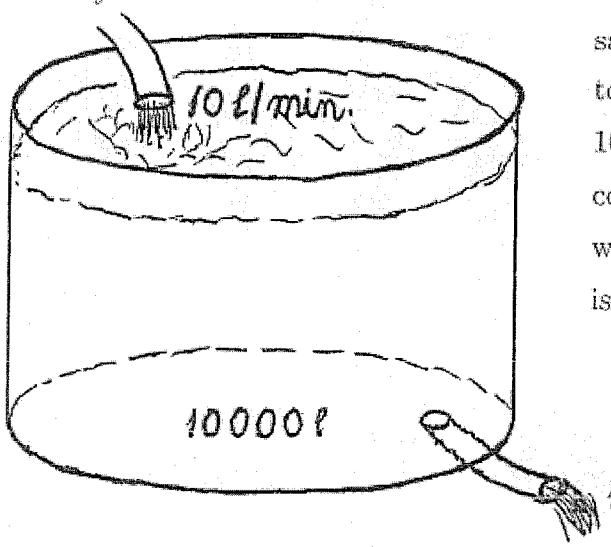
(10)#2...Solve the following ordinary differential equations by means of a suitable integrating factor $\mu = \mu(x)$ or $\mu = \mu(y)$, which shall make it exact:

$6xy\,dx + (4y + 9x^2)dy = 0$. Note: $\dfrac{\mu'(x)}{\mu(x)} = \dfrac{M_y - N_x}{N}$ or $\dfrac{\mu'(y)}{\mu(y)} = \dfrac{N_x - M_y}{M}$.

(10)#3...Solve the following two ordinary differential equations by means of a suitable substitution:

$$\text{(A)}\ \frac{dy}{dx} = \frac{x+3y}{3x+y}, \qquad \text{(B)}\ \frac{dy}{dx} + y = 8xy^4.$$

(10)#4...A cylindrical tank is filled with 10000 liters of brine containing 500 kg of salt initially. Entering from the top of the tank, at a rate of 10 l/min, is a saline solution containing 0.3 kg/l. From the well-stirred tank 10 l/min is discharged at the bottom.



(A) How much salt is contained in the tank at time t (in minutes)?

(B) What does this tell you about about the amount of salt in the tank as $t \to \infty$?

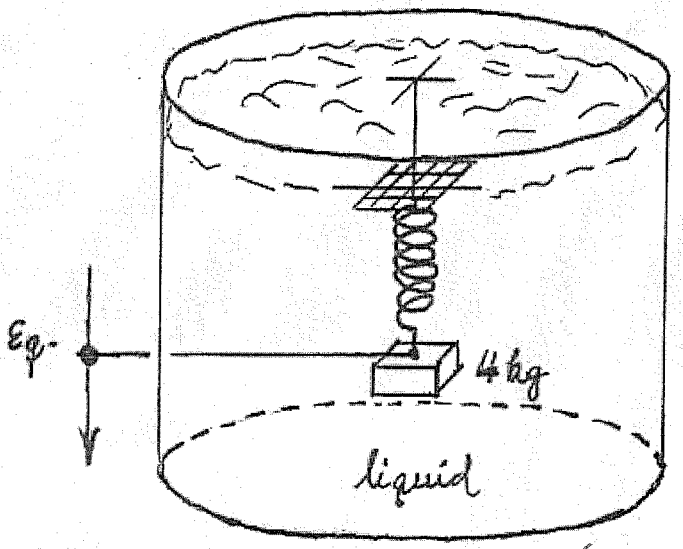
(10)#5... Solve the following linear homogeneous second order ordinary differential equations with the given initial conditions:

$$x^2y'' - 6xy' - 18y = 0 \quad \text{satisfying: } y(1) = 2 \text{ and } y'(1) = 16.$$

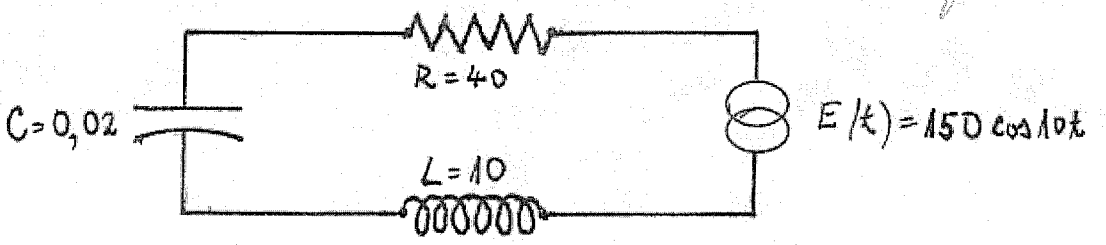
(10)#6...Find the general solutions of the following linear inhomogeneous second order ordinary differential equation by means of variation of parameter:

$$y'' - 4y = x^{-1}e^{2x}.$$

(10)#7...A mass of 4 kg is attached to a spring with spring constant 36 N/m and the entire system is immersed in a liquid whose damping is 24 times the velocity. Find the equation of motion of this spring system if it is released 2 m below the equilibrium position with an upward velocity of 12 m/sec., where m stands for the metric unit ~~of meter~~. system.



(10)#8...Given the L-R-C circuit (inductor, resistor, capacitor)



with charge $Q = Q(t)$ satisfying the linear second order ordinary differential equation

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t) \text{ where: } R = 40 \text{ ohms}, L = 10 \text{ henries}, C = 0.02 \text{ farads}$$

and an electromotive force $E(t) = 150\cos 10t$ volts. Charge Q further satisfies the initial conditions $Q(0) = 1500$ volts and $Q'(0) = 0$.

(A) Solve for the charge $Q(t)$.

(B) Indicate in $Q(t)$ the transient and steady state terms.

(10)#9...Find the general solution of the linear system of ordinary differential equations

$$\begin{array}{rcrcl} (d/dt+1)x & + & (d/dt-1)y & = & 2 \\ 3x & + & (d/dt+2)y & = & -1 \end{array},$$

where d/dt is the derivative with respect to time t.

(10)#10...Find two linearly independent power series solutions $\phi_1(x)$ and $\phi_2(x)$ of the second order ordinary differential equation

$$y'' + 2xy' + y = 0,$$

assuming that the solution has the power series form $y = y(x) = \sum_{n=0}^{\infty} A_n x^n$.

Calculate only the first three non-zero terms of each of the functions $\phi_1(x)$ and $\phi_2(x)$.