Math 1025 Test-1 of Oct. 9th

X = your score out of 125 points.
Letter Grade ranges:

A+ if $X \geq 105$, A if, $85 \leq X < 105$,
B+ if, $75 \leq X < 85$, B if $65 \leq X < 75$
C+ if, $55 \leq X < 65$, C if, $45 \leq X < 55$
D+ if, $35 \leq X < 45$, D if, $25 \leq X < 35$.

IDCode	t1
291189	54
T2T2	48
BAAM	15
6666	13
AJAN	55
7392	15
7235	18
CJB	20
O6O6	18
AB38	35
32764	98
O7FU	35
O1255	21
2206	17
O914	5
1007	40
1114	75
1105	53
4468	35
ASD1	45
1213	46
O11d	35
J307	34
A891	22
6967	15
O52O	55
5378	17
999	48
VCDG	39
1130	30
ADAM	38
1365	39
6368	23
1100	28
1419	18
A.J	27
shi 102	35
Sevil	54
A369	74
needle	30
7236	34
23605	72
2810	68
rzjamnd	34
O711	57
I7I7	25
DJL3	53
8428	54
MBSN	37
1128	57
KNTK	10
SAK2	0
M2JSC	33
V1M1	51
olikh	35
4361	54
2597	11
7878	69
16192	85
WK#1	8
2113	20

IDCode	t1
LO62	20
7310	56
LBAH	40
O529	12
1370	0
ken1	3
3033	41
O421	32
8609	56
2913	37
golf72	30
2369	29
1305	3
2399	40
3869	13
matv	3
4290	42
7351	63
7197	20
1234	23
20502	43
JASONSAXO	42
2311	35
1237	87
1880	32
NPIZ	44
CSE9303	18
JAKE	45
91091	34
6157	28
1317	40
MI76	60
JR11	72
OO66	16
LUCK	2
1289	48
S412	84
2805	47
1264	9
3689	5
1392	50
8221`	24
O131	58
8008	8
SYTAQ	20
nt10	16
O525	87
1098	56
T0811	65
1959	80
P7003	28
5859	71
XWAN	29
O1O1	15
AKOO	93
O114	49
1310	79
730514	80
LOL1	38
6083	63

York University
Department of Mathematics and Statistics

MATH:1025 (Prof. H Joshi). Test – 1 (Ver-1) Oct 9, 2009

Qs # 0. You would not like any body to look into your term record. For this reason I suggest you choose a four character alpha numeric secret IDCode and write it (in space above: IDCode) below. Your test marks and your final grade will be posted using this IDcode and not your student number or your name. If you do not complete all entries below, you shall ***loose*** 5 points on this test.

PRINT (not scribble) ____________________ ____________________
(Last Name) (First Name)

____________________ __________
(Student Number) (IDCode)

:

Answer all questions printed on FIVE pages. Each question is worth 25 points.
You may write on both sides of a page:

1. Determine all values of parameters a, b, c for which the following linear system is consistent

$$\begin{cases} x+y+2z &= a \\ 3x+y+4z &= b \\ 2x+y+3z &= c \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 3 & 1 & 4 & b \\ 2 & 1 & 3 & c \end{array}\right] \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 0 & -2 & -2 & b-3a \\ 0 & -1 & -1 & c-2a \end{array}\right] \xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 0 & 0 & 0 & b+a-2c \\ 0 & -1 & -1 & c-2a \end{array}\right]$$ 10

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The linear system is consistent if $b+a-2c=0$

or $b = 2c - a$

or $a = 2c - b$

or $c = \frac{a+b}{2}$ 10

2. For the following nonlinear system, find complete solution set if the variables x, y, z are in the interval $[0, \pi]$. $\begin{cases} \sin x + 2\cos y + 3\tan z = 0 \\ 2\sin x + 5\cos y + 3\tan z = 0 \\ \sin x + 5\cos y - 5\tan z = 0 \end{cases}$

Let $a = \sin x$, $b = \cos y$, $c = \tan z$, then

$$\begin{cases} a + 2b + 3c = 0 \\ 2a + 5b + 3c = 0 \\ a + 5b - 5c = 0 \end{cases} \quad 3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 5 & 3 & 0 \\ 1 & 5 & -5 & 0 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 3 & -8 & 0 \end{array}\right) \quad 3$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow[R_2 \to R_2 + 3R_3]{R_1 \to R_1 - 3R_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \to 5 \quad \Rightarrow \begin{cases} a = 0 \\ b = 0 \\ c = 0 \end{cases} \to 3$$

then $\begin{cases} \sin x = 0 \\ \cos y = 0 \\ \tan z = 0 \end{cases}$ and $x, y, z \in [0, \pi]$ So $\begin{cases} x = 0, \pi \\ y = \frac{\pi}{2} \\ z = 0, \pi \end{cases} \to 6$

The Answer is $\left\{ \begin{pmatrix} 0 \\ \frac{\pi}{2} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{\pi}{2} \\ \pi \end{pmatrix} \begin{pmatrix} \pi \\ \frac{\pi}{2} \\ 0 \end{pmatrix} \begin{pmatrix} \pi \\ \frac{\pi}{2} \\ \pi \end{pmatrix} \right\} \to 5$

3. Answer the following:
   a. Show that if a square matrix A satisfies $A^2 - 3A + I = \theta$, then $A^{-1} = 3I - A$.
   b. Prove: If $A^T A = A$, then matrix A is symmetric and $A = A^2$.
   c. Find all 2 by 2 diagonal matrices A that satisfy the equation $A^2 - 3A - 4I = \theta$.

a) since $(3I-A)A = 3A - A^2$, but $A^2 - 3A + I = \theta$ 2

$\therefore 3A - A^2 = I \quad \therefore (3I-A)A = I$ 3

$\therefore A^{-1} = 3I - A$ 2

b) since $A = A^TA$, so $A^T = (A^TA)^T = A^T(A^T)^T = A^TA = A$ 4 2

$\therefore$ A is symmetric

since A is symmetric, $A^T = A$, and $A = A^TA \Rightarrow A^TA = A^2$ 2 2

$\therefore A^2 = A$

c) suppose 2x2 diagonal matrix A is given by $A = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$ 2

Hence, $A^2 - 3A - 4I = 0 \Rightarrow \begin{bmatrix} x^2 & 0 \\ 0 & y^2 \end{bmatrix} - 3\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$ 3

$\Rightarrow \begin{cases} x^2 - 3x - 4 = 0 \\ y^2 - 3y - 4 = 0 \end{cases} \Rightarrow \begin{matrix} x = 4, -1 \\ y = 4, -1 \end{matrix}$ 2

$\therefore$ The Answer is $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

3

4. Consider the matrix $A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}$

   a. Find a sequence of elementary matrices $E_1, E_2, \dots, E_k$ such that $E_k E_{k-1} \dots E_2 E_1 A = I$. You must use notations $E(R_i \leftrightarrow R_j)$, $E(R_j \to kR_j)$, and $E(R_i \to R_i + kR_j)$ for elementary matrices.

   b. Write A^{-1} as a product of elementary matrices, and hence compute A^{-1}. No credit will be given if you compute A^{-1} by any other method.

   c. Write the matrix A as a product of elementary matrices.

(a)

$$A=\begin{pmatrix}2 & 3\\ 5 & 7\end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix}2 & 3\\ 1 & 1\end{pmatrix} \xrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}$$

$$\xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \longrightarrow 4$$

$\therefore E(R_1 \leftrightarrow R_2)\, E(R_2 \to R_2 - R_1)\, E(R_1 \to R_1 - 2R_2)\, E(R_2 \to R_2 - 2R_1)\, A = I \to 4$

Here denote $E_1 = E(R_2 \to R_2 - 2R_1)$ $E_2 = E(R_1 \to R_1 - 2R_2)$ $E_3 = E(R_2 \to R_2 - R_1)$
$E_4 = E(R_1 \leftrightarrow R_2)$ then $E_4 E_3 E_2 E_1 A = I$

(b) since $E_4 E_3 E_2 E_1 A = I$ $A^{-1} = E_4 E_3 E_2 E_1 \longrightarrow 2$

$$A^{-1} = E_4 E_3 E_2 E_1 = E_4 E_3 E_2 \begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix} = E_4 E_3 \begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix} \quad 2$$

$$= E_4 E_3 \begin{pmatrix}5 & -2\\ -2 & 1\end{pmatrix} \; 2 \; = E_4 \begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & -2\\ -2 & 1\end{pmatrix} = E_4 \begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix} \; 2$$

$$= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix} = \begin{pmatrix}-7 & 3\\ 5 & -2\end{pmatrix} \; 2$$

(c) $(A^{-1})^{-1} = A = (E_4 E_3 E_2 E_1)^{-1} = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} \longrightarrow 3$

$= E(R_2 \to R_2 + 2R_1)\, E(R_1 \to R_1 + 2R_2)\, E(R_2 \to R_2 + R_1)\, E(R_1 \leftrightarrow R_2) \to 4$

5. If it exists, find the multiplicative inverse, using any method, of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ -1 & -5 & 5 \end{bmatrix}$.

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ -1 & -5 & 5 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1}$$

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$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -3 & 8 & 1 & 0 & 1 \end{array}\right] \quad R_3 \to R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 3 & 1 \end{array}\right] \xrightarrow[R_2 \to R_2 - 3R_3]{R_1 \to R_1 + 3R_3}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 9 & 3 \\ 0 & 1 & 0 & 13 & -8 & -3 \\ 0 & 0 & -1 & -5 & 3 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to -R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 25 & 9 \\ 0 & 1 & 0 & 13 & -8 & -3 \\ 0 & 0 & 1 & 5 & -3 & -1 \end{array}\right] \qquad \therefore A^{-1} = \begin{pmatrix} -40 & 25 & 9 \\ 13 & -8 & -3 \\ 5 & -3 & -1 \end{pmatrix}$$

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