Solutions

YORK UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 2015 (M) 3.0 , WINTER 2015

APPLIED MULTIVARIATE AND VECTOR CALCULUS

TEST #1

FEBRUARY 2, 2015

Instructor : Igor Poliakov

Name (print) ____,
(Family) (Given)

Student I.D. ____

Instructions:

1. This is a closed-book Test. Calculators are permitted.
2. Show all significant steps. No marks will be given for the answer alone.
3. Solve the problems in the spaces provided. If you need more space, use the back of the page (indicate this fact on the original page).
4. **YOU ARE NOT ALLOWED TO USE AN AID SHEET.**
5. **USE PEN TO WRITE THE FINAL ANSWER.**
6. **DRAW A BOX AROUND YOUR FINAL ANSWERS.**

Test #1 contains 6 questions on 7 pages. Read the instructions carefully and sign below:

____
(Signature)

QUESTION	1	2	3	4	5	6	Total
MARKS	5	5	5	5	5	5	30
SCORE							

QUESTION 1 [5]

Determine the set of points at which the function is continuous:

$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2} & if\ (x,y) \neq (0,0) \\ 1 & if\ f(x,y) = (0,0) \end{cases}$$

1) $x^2 \le 2x^2 + y^2 \Rightarrow |x^2y^3/(2x^2+y^2)| \le |y^3|$;

$\lim_{(x,y)\to(0,0)} |y^3| = 0$ by the Squeeze Th.

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \dfrac{x^2y^3}{2x^2+y^2} = 0$;

But $f(0,0) = 1$, so f is discontinuous at $(0,0)$.

Therefore, f is continuous on the set

$\{(x,y) \mid (x,y) \neq (0,0)\}$;

QUESTION 2 [5]

Use the definition of partial derivatives as limit to find $f_x(x,y)$:

$$f(x,y) = xy^2 - x^3y.$$

$$f_x(x,y) \overset{(1)}{=} \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h} =$$

$$\overset{(1)}{=} \lim_{h\to 0} \frac{(x+h)y^2 - (x+h)^3y - (xy^2 - x^3y)}{h} =$$

$$\overset{(1)}{=} \lim_{h\to 0} \frac{h(y^2 - 3x^2y - 3xyh - yh^2)}{h} =$$

$$\overset{(1)}{=} \lim_{h\to 0} (y^2 - 3x^2y - 3xyh - yh^2) \overset{(1)}{=} \boxed{y^2 - 3x^2y}$$

5

QUESTION 3 [5]

Given that f is a differentiable function with

$f(2,5) = 6,\; f_x(2,5) = 1,\; and\; f_y(2,5) = -1,$

use a linear approximation to estimate $f(2.2, 4.9)$.

$$f(x,y) \simeq f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5) =$$

$$= 6 + 1(x-2) + (-1)(y-5) = x - y + 9;$$

Therefore: $f(2.2, 4.9) \simeq 2.2 - 4.9 + 9 = \boxed{6.3}$

$\boxed{5}$

QUESTION 4 [5]

Given: $e^z = xyz$.
Find $\partial z/\partial x$.
(do not use implicit differentiation).

$$F(x,y,z) \overset{1}{=} e^{x} - xyz = 0;$$

$$\frac{\partial z}{\partial x} \overset{2}{=} (-)\frac{F_x}{F_z} \overset{1}{=} (-)\frac{(-)yz}{e^z - xy} \overset{1}{=} \boxed{\frac{yz}{e^z - xy}}$$

$\boxed{5}$

QUESTION 5 [5]

At what point on the paraboloid $y = x^2 + z^2$ is the tangent plane parallel to the plane
$x + 2y + 3z = 1$?

$f(x,y,z) = x^2 + z^2 - y$;

① $\nabla f(x,y,z) = \langle 2x, -1, 2z \rangle$ is a normal vector to the surface at (x,y,z) and it is a normal vector for the tangent plane.

The tangent plane is parallel to the plane $x+2y+3z=1$ when the normal vectors of the planes are parallel:

$\langle 2x_0, -1, 2z_0 \rangle \overset{①}{=} k\langle 1,2,3 \rangle \overset{①}{\Rightarrow} k = -\frac{1}{2}$

$\Rightarrow \langle 2x_0, -1, 2z_0 \rangle \overset{①}{=} \langle -\frac{1}{2}, -1, -\frac{3}{2} \rangle$

$\Rightarrow \boxed{(x_0, y_0, z_0) \overset{①}{=} \left(-\frac{1}{4}, \frac{5}{8}, -\frac{3}{4}\right)}$

5

QUESTION 6 [5]

Find the limit, if it exists, or prove that the limit does not exist:

$$\lim_{(x,y\to(0,0)} \frac{x^2 y e^y}{x^4+4y^2};$$

$f(x,y) = \dfrac{x^2 y e^y}{x^4+4y^2}$;

① $f(x,0) = 0$ for $x \neq 0$,
so $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along the x-axis.

② Along the y-axis or $y = x$ gives a limit of 0. But

$$f(x,x^2) = \frac{x^2 x^2 e^{x^2}}{x^4 + 4(x^2)^2} = \frac{x^4 e^{x^2}}{5x^4} = \frac{e^{x^2}}{5} \quad \text{if } x \neq 0.$$

So $f(x,y) \to \frac{e^0}{5} = \frac{1}{5}$ as $(x,y) \to (0,0)$ along $y = x^2$.

Conclusion: $\boxed{\text{the limit DNE}}$

Total marks: 30

THE END