York University

Faculty of Science and Engineering

Math 1190 A

Class Test 1

NAME (print): _____
(Family) (Given)

SIGNATURE: _____

STUDENT NUMBER: _____

SOLUTIONS

Instructions:

1. Time allowed: 50 minutes.
2. **NO CALCULATORS OR OTHER AIDS PERMITTED**
3. Show your work. Your work must justify any answers you give. Use page backs for any scrap work.
4. Use pen to fill in cover. If you use pencil for your solutions, you may not submit your paper for regrading.
5. There are 6 questions on 5 pages.

Question	Points	Marks
1	20	
2	20	
3	16	
4	24	
5	10	
6	10	
Total	100	

1. (20 points) Establish whether or not each of the following is a tautology. Use truth tables to justify your answer.

(a) $(p \to q) \wedge \neg p \to \neg q$.

p	q	$(p \to q)$	$\wedge$	$\neg p$	$\to$	$\neg q$
T	T	T	F	F	T	F
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

NOT a tautology.

(b) $(p \to q) \vee (\neg p \to q) \to q$.

p	q	$(p \to q)$	$\vee$	$(\neg p \to q)$	$\to$	q
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	F	T	F	F	F	F

NOT a tautology.

2. (20 points) Establish whether each of these pairs of statements are logically equivalent. Use truth tables to justify your answer.

(a) $(p \to q) \to r$ and $p \to (q \to r)$.

p	q	r	$(p \to q) \to r$	$p \to (q \to r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	T

Values do not match.
NOT logically equivalent

(b) $p \to q \vee r$ and $p \wedge \neg q \to r$.

p	q	r	$p \to q \vee r$	$p \wedge \neg q \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	F

Values match. Statements logically equivalent.

3. (16 points) Let a be an integer. Consider the statement,

If a is even then $2a$ is even.

(a) Is the statement true? Justify your answer.

YES.
Reason: 2a always divisible by 2.

(b) State its converse. Is its converse true? Justify your answer.

If 2a is even then a is even.
NOT TRUE.
Reason: (For example) 2(3) is even by 3 is not even.

(c) State its contrapositive. Is its contrapositive true? Justify your answer.

If 2a is not even (i.e, odd)
then a is not even (i.e., odd)
The statement is true as
"2a is not even" is false.

(d) State its negation. Is its negation true? Justify your answer.

a is even and 2a is not even.
This is false as it's the
negation of a true statement.

4. (24 points) Let $P(x, y)$ be the statement, "$x + y = x - y$". If the universe of discourse consists of all integers (negative, zero and positive whole numbers), what are the truth values of each of the following. Briefly justify your answers.

(a) $P(2, 2)$.

F as $2+2 \neq 2-2$.

(b) $\forall y P(1, y)$.

F as $1+y = 1-y$ only for $y = 0$.

(c) $\exists y P(1, y)$.

T Take $y = 0$.

(d) $\exists x\, P(x, 2)$.

F $x+2 = x-2$ is never true.

(e) $\exists x \exists y\, P(x, y)$.

T For example, take $x=0$ and $y=0$.

(f) $\forall x \exists y\, P(x, y)$.

T Take $y = 0$. Then $x+0 = x-0$ for all x.

(g) $\exists x \forall y\, P(x, y)$.

F $x+y = x-y$ only when $y = 0$.

(h) $\exists y \forall x\, P(x, y)$.

T Take $y = 0$. Then $x+0 = x-0$ for any x.

5. (10 points) Express each of the following so that all negation symbols immediately precede predicates.

(a) $\neg(\ \exists y \forall x P(x,y) \wedge \forall y Q(y)\ )$.

$\forall y \exists x \neg P(x,y) \vee \exists y \neg Q(y)$

(b) $\neg(\ \forall x\ (R(x) \rightarrow \neg Q(x))\ )$.

$\exists x (R(x) \wedge Q(x))$

6. (10 points) Consider the statement $(P(x) \rightarrow Q(1)) \rightarrow (\ \forall x P(x) \rightarrow \forall x Q(x)\ )$ with universe of discourse the set of integers. Show that it is **not** a tautology by choosing P and Q and a value for the free occurrence of x to make the statement false.

Let $P(x)$ be T for all x.
Let $Q(x)$ be $x=1$ (so Q is true for $x=1$, false otherwise)
Take x to be 0.

$P(0) \rightarrow Q(1)$ is T.
$\forall x P(x)$ is T.
$\forall x Q(x)$ is F.
$\forall x P(x) \rightarrow \forall x Q(x)$ is F

$(P(0) \rightarrow Q(1)) \rightarrow (\forall x P(x) \rightarrow \forall x Q(x))$
is F as required.

The end