

## Assignment 9

*Due Thursday, Nov. 13, 2014*

**Note: there are 5 problems**

1. Conjecture a formula about the sum of the first  $k$  even numbers and then try proving your conjecture by induction.

2. Prove that for every integer  $p > 1$

$$\left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{p^2}\right) = \frac{p+1}{2p}.$$

3. Write, in proper mathematical terminology, what it means for a function  $f : A \longrightarrow B$  **not** to be injective, and **not** to be surjective.

4. Let  $\alpha$  and  $\beta \in \mathbb{R}$  with  $\alpha \neq 0$ . Show that the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  defined by  $f(x) = \alpha x + \beta$  is injective. Is the map surjective? In this case construct the inverse map.

5. Show that the two sets  $(a, b)$ ,  $a \neq b$  and  $(c, d)$ ,  $c \neq d$  have the same cardinality.