

Assignment 5

Due Thursday, Oct. 9, 2014

Note: there are 5 problems

1. Consider the following statements:

P : The integer n is divisible by 2.

Q : The integer n is divisible by 3.

R : The integer n is divisible by 6.

(a) Translate the following logical expressions into good English sentences.

$$(i) \quad \neg P \vee \neg Q$$

$$(ii) \quad [(P \vee Q) \wedge \neg(P \wedge Q)] \Rightarrow \neg R$$

(b) Write down the contrapositive (in symbols) of (ii) above and simplify it so that no \neg precedes a parenthesis or bracket.

2. Consider the assertion:

If r is a non-zero rational and s is irrational then rs is irrational

(i) State the context, the premise, and the conclusion.

(ii) Prove the assertion stating the proof technique that you are using.

(iii) Write down the converse of the assertion. Is the converse true? Give reasons.

3. Consider the statement:

A sufficient condition for a real polynomial $p(x)$, of the real variable x , to have a real root is that it be of odd degree.

(i) Write down (a) the contrapositive and (b) the converse of this statement.

(ii) Is the converse true? If yes, give a proof; if not give a counter-example.

4. Show that:

(i) if p is a prime and $p|n^2$ then $p|n$. (Hint: use the Fundamental Theorem of Arithmetic.)

- (ii) Show by an example that the above is not true if p is not a prime.
5. If p is prime and $p|ab$, where $a, b \in \mathbb{Z}$, then $p|a$ or $p|b$.
- (i) State the context, the premise, and conclusion of the above statement.
- (ii) Prove the statement, stating the proof technique that you are using.