

Assignment 4

Due Thursday, Oct. 2, 2014

Note: there are 5 problems

1. Consider the statement,
“ $a \mid b$ whenever b is a multiple of a ”.
 - (a) Write this statement as a “sufficient condition” (as discussed in the class).
 - (b) Write it as a “necessary condition”.
 - (c) Is the converse of the statement true? Give reasons.
2. Consider the statement: $q_1 \in \mathbb{Q}$ and $q_2 \in \mathbb{Q} \Rightarrow q_1 q_2 \in \mathbb{Q}$.
 - (a) Identify the context, premise and conclusion.
 - (b) Give a direct proof.
 - (c) Write down the converse of this statement. Is the converse true? If yes, give a proof, if not give a counter-example.
3. (a) Show that if x is a non-negative real number then $x^2 + 1 \geq 2x$. (Hint: Use backward-forward method).

(b) Would the statement still be true if x were not assumed to be non-negative? Explain your answer.
4. Show that if x and y are real numbers then $2xy \leq x^2 + y^2$.
5. Show that if $a < 1$, $b < 1$ then $ab + 1 > a + b$.