York University, Department of Mathematics and Statistics
Math 1013 "Applied Calculus I", Fall 2014
Test 1 (Sections A and D)

2014-10-03

Last name ....

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Instructor ....

1. (10 pts) Consider a function $f(x) = \ln(x^2 - 4)$.

- (5 pts) Find its domain and range.
- (5 pts) Prove that $f(x)$ is not one-to-one on its domain. Restrict the domain and find the inverse function on the restricted domain (Hint: recall the construction of the inverse sin and cos functions).

(1a) domain: $x^2-4>0$; $x^2>4$;
$x \in (-\infty; -2) \cup (2; \infty)$ +3

range: $\boxed{y \in (-\infty; \infty)}$ (see the computations of the inverse f-n in (1b)) +2

(1b) Not one-to-one, since $\boxed{f(3) = f(-3)}$ +2

restrict $x \in (2; \infty)$. Then

$\ln(x^2-4) = y$

$x^2 - 4 = e^y$

$x = \sqrt{4+e^y}$

+3

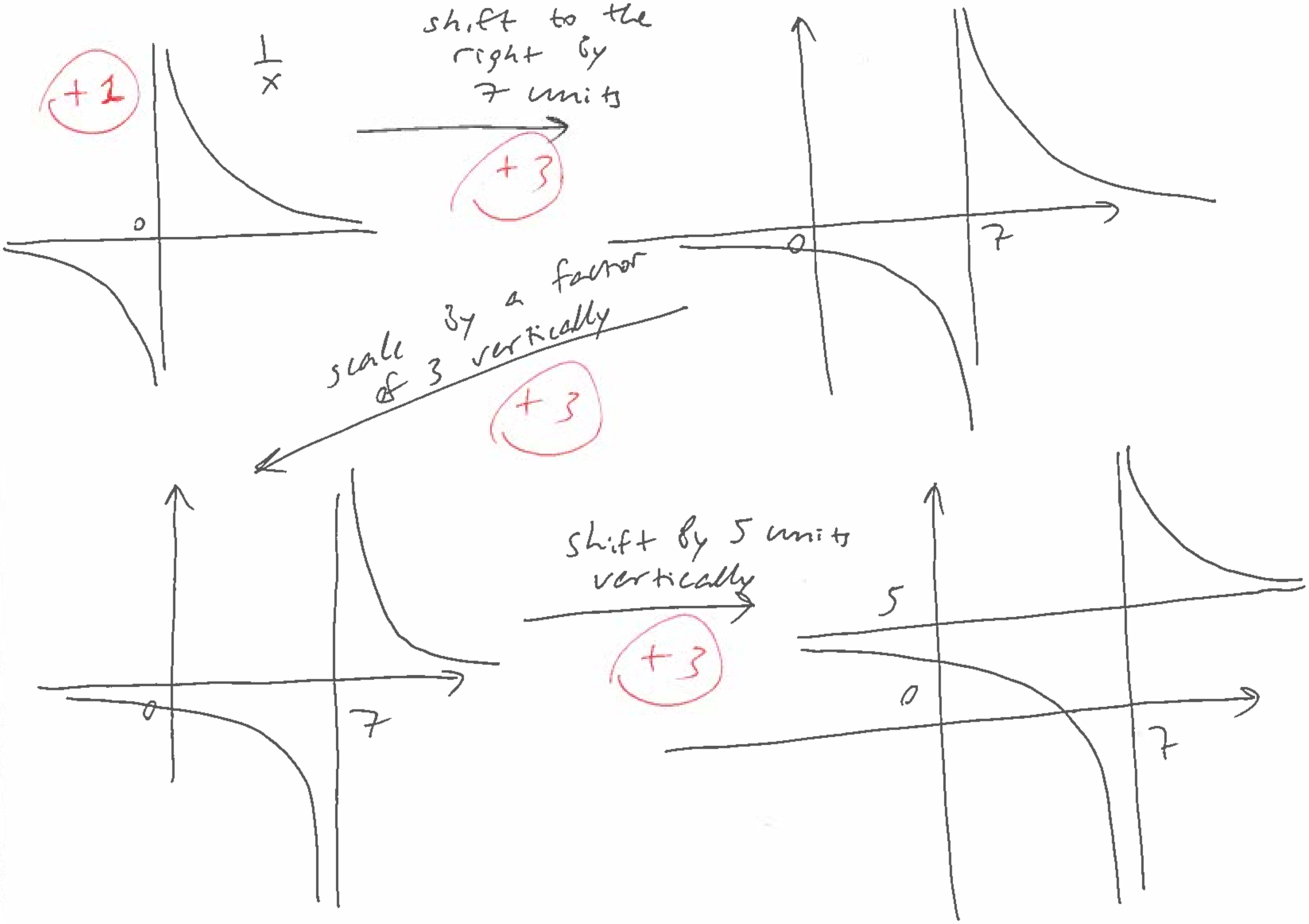
so the inverse function is

$f^{-1}(x) = \sqrt{4+e^x}$

its domain is $(-\infty;\infty)$, so the range of $f(x)$ is $(-\infty;\infty)$

Extra space for question 1

2. (10 pts) Sketch the graph of $y = f(x) = \frac{3}{x-7} + 5$ by starting with the graph of $y = 1/x$ and applying the shifting/scaling transformations. Make sure to indicate the vertical and horizontal asymptotes of $y = f(x)$ on the graph.



3/10 if only the final graph is show

Extra space for question 2

3. (15 pts) Evaluate the following limits:

- (5 pts)

$$\lim_{t\to 1^-} \frac{\cos(\pi t)}{1-t}$$

- (5 pts)

$$\lim_{x\to -2} \frac{\sqrt{x+11}-3}{2+x}$$

- (5 pts)

$$\lim_{y\to 0} y^2 \sin(\pi/y)$$

(3a) $\cos(\pi) = -1$ +1

$1-t > 0$ as $t \to 1^-$ +1 $\Big\}$ $\lim_{t\to 1^-} \frac{\cos(\pi t)}{1-t} = -\infty$ +3

(3b) $\lim_{x\to -2} \frac{\sqrt{x+11}-3}{x+2} = \lim_{x\to -2} \frac{\sqrt{x+11}-3}{x+2} \cdot \frac{\sqrt{x+11}+3}{\sqrt{x+11}+3} =$ +2

$= \lim_{x\to -2} \frac{\cancel{x+11-9}}{\cancel{x+2}} \cdot \frac{1}{\sqrt{x+11}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$ +3

cancell

(3c) let $f(y) = y^2 \sin(\frac{\pi}{y})$

for $y \neq 0$ we have $-y^2 \le f(y) \le y^2$ +3

Apply Squeeze thm $\Rightarrow$ $\lim_{y\to 0} f(y) = 0$

+2

Extra space for question 3

4. (10 pts) Prove that the equation $s^5 - 3s + 1 = 0$ has a solution on the interval $(1, 2)$.

let $f(s) = s^5 - 3s + 1$

+2 (i) $f(s)$ is a continuous function

+2 (ii) $f(1) = -1$

+2 (iii) $f(2) = 27$

$\Rightarrow$

$\Rightarrow$ +4 Apply Intermediate Value theorem $\Rightarrow$

$\Rightarrow$ solution to $f(s) = 0$ exists on $(1, 2)$

5. (10 pts) Find all vertical and horizontal asymptotes for $f(x) = \frac{\sqrt{x^2+1}}{x+1}$.

(i) vertical ~~hor.~~ asymptote : $x = -1$

since $\lim_{x \to -1^+} f(x) = +\infty$ +3

(ii) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\sqrt{x^2} \cdot \sqrt{1+\frac{1}{x^2}}}{x(1+\frac{1}{x})} =$

$= \lim_{x \to +\infty} \frac{x}{x} \cdot \frac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{1}{x}} = 1$ +4

(iii) $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\sqrt{x^2} \cdot \sqrt{1+\frac{1}{x^2}}}{x(1+\frac{1}{x})} =$

$= \lim_{x \to -\infty} \frac{(-x)\sqrt{1+\frac{1}{x^2}}}{x(1+\frac{1}{x})} = -1$ +3

Answer: Asymptotes at

$x = -1 \Leftarrow$ ~~hor.~~ vertical

$\left.\begin{array}{l} y = 1 \\ y = -1 \end{array}\right\} \Leftarrow$ horizontal

Extra space for question 5