Fact: If f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $\vec{u} = (a, b)$ and. (22-10)

$$D_u f(x,y) = f_x(x,y) \cdot a + f_y(x,y) b.$$

Ex. If the unit vector $\vec{u}$ makes an angle θ with the positive x-axis, then.

$\vec{u} = (\cos\theta, \sin\theta)$, so that

$$D_u f(x,y) = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$$

Find the directional derivative (22-2)

$D_u f(x,y)$ if $f(x,y) = x^3 - 3xy + 4y^2$

$\vec{u}$ is given by angle $\theta = \pi/6$.

$$D_u f(x,y) = (3x^2 - 3y)\frac{\sqrt{3}}{2} + (-3x + 8y)\frac{1}{2} =$$

Find $D_u f(1,2)$

$$D_u f(1,2) = \frac{13 - 3\sqrt{3}}{2}.$$

The Gradient Vector

$$D_u f(x,y) = \underbrace{(f_x(x,y),\ f_y(x,y))}_{\nabla f \text{ or } \operatorname{grad} f} \cdot \vec{u}$$

Ex. $f(x,y) = \sin x + e^{xy}$

$$\nabla f(x,y) = (f_x, f_y) = (\cos x + y e^{xy},\ x e^{xy})$$

$f(x,y,z)$.

$\vec{u} = (a,b,c)$ $\quad |\vec{u}| = 1$.

$$D_u f(x_0, y_0, z_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb, z_0 + hc) - f(x_0, y_0, z_0)}{h}$$

$$D_u f(x,y,z) = \nabla f(x,y,z) \cdot \vec{u}, \quad \text{where}$$

$\nabla f = (f_x, f_y, f_z)$ is the gradient of f.

Ex. If $f(x,y,z) = x \sin yz$

Find the gradient of f and

Find the directional derivative of f

at $(1,3,0)$ in the direction $\vec{v} = i + 2j - k$.

22-5

$$\nabla f(x,y,z) = (f_x, f_y, f_z) =$$
$$= (\sin yz,\ xz\cos yz,\ xy\cos yz)$$

At $(1,3,0)$ $\quad \nabla f(1,3,0) = (0,0,3)$

Unit vector in the direction $\vec{i}+2\vec{j}-\vec{k}$ is

$$\vec{u} = \frac{1}{\sqrt{6}}\vec{i} + \frac{2}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k}$$

$$D_u f(1,3,0) = \nabla f(1,3,0)\cdot\vec{u} = 3\vec{k}\cdot\left(\frac{1}{\sqrt{6}}\vec{i} + \frac{2}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k}\right)$$
$$= 3\left(-\frac{1}{\sqrt{6}}\right) = -\sqrt{\frac{3}{2}}$$