ADM2304: One Proportion
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- **1. Hypothesis Testing:**

  All Scientific methodologies depend on first using a hypothesis or or plausible explanation and then proving or disproving this hypothesis.

  In the Science of Statistics the Hypothesis Testing is a highly specialized methodology which is used for Accepting or Rejecting the Hypotheses for the **Parameters of a Population.**

  Remember, Hypothesis Testing is always done for Population Parameters, and different Statistic-s are used for Accepting or Rejecting the Hypotheses.

So far, we have learnt about the Central Limit Theorem, CLT, for Means & Proportions, Confidence Intervals, CI, for Means & Proportions and Sample Size, n, when dealing with Proportions. It is now an appropriate time to consider Hypothesis Testing of Proportions.

- **2. Hypothesis Tests for One Proportion:**

Hypothesis Tests for **all parameters** come in three forms.

Test #1: Two Tailed Tests

Structure of this Test for One Proportion is as follows:

H_0: $p = p_0$ ➔ Null (Notice '0' in H_0) Hypothesis.

H_A: $p \neq p_0$ ➔ Alternate (Notice 'A' in H_A) Hypothesis.

Here the assumed value is p_0 and it is a value which we want to disprove. The test is called "Two Tailed" because it is possible that $p > p_0$ or $p < p_0$ or $p \neq p_0$.

If the Null Hypothesis, H_0 cannot be disproved then we say, Do Not Reject H_0 . If we can disprove the Null Hypothesis, then we say Reject H_0 . We rarely say Accept Alternate Hypothesis, H_A . This is the typical statistical usage of expressions which can be called Statistical "jargon". We also establish a formal methodology for performing Hypothesis Tests. An Example would demonstrate it clearly.

- **3. Example for Two Tailed Test for One Proportion:**

Remember, in the "Voting Fraud" example we did,

n = 100, and $\bar{p} = 0.90$

Test the hypothesis that that the population proportion, p is equal to 0.80.

- **S1** (Step 1): State the Hypothesis for Parameters:

$H_0 : p = 0.80$

$H_A : p \neq 0.80$

S2 (Step 2): Calculate the Test Statistic, 'Z' (Based on CLT)

$$SD(\bar{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.8(0.2)}{100}} = .04$$

$$Z_{Calc} = Z_{Test} = \frac{(\bar{p} - p)}{SD(\bar{p})} = \frac{(0.90 - 0.80)}{0.04} = 2.5$$

S3 (Step 3): Establish the Z* or $Z_{Crit.}$

If Level of Significance, LS = α = 0.05, then

$Z_{Crit.} = Z_{\alpha/2} = Z_{0.025} = 1.96$

S4 (Step 4): Reach an appropriate Conclusion.

Since $\{|Z_{Calc}| = 2.5\} > \{Z_{Crit} = 1.96\}$ ➔ Reject H_0

What the conclusion means is that at an LS of 5%, we must Reject the Null Hypothesis, H_0, that is $p = 0.80$. In simple English, this means that when we assert that $p \neq 0.80$, we could be making an error with a probability of 0.05 (5%)!

S3 and S4(Variation):

S3: If $LS = \alpha = 0.01$ (1%), then $Z_{\alpha/2} = 2.576$

S4: Since $\{|Z_{Calc}| = 2.5\} < \{Z_{Crit} = 2.576\}$ ➔Do Not Reject H_0.

In other words, we cannot reject H_0 that is ‘p’ = 0.80, if we want to make an error in our conclusion of only 0.01 (1%).

Since the α, the LS, is smaller, or more stringent, we have to be more careful in the conclusion we reach. Take note that with the same data, the Conclusion was different. We will say more about these and other errors later.

- **4. Diagrammatic Representation of the Tests:**

You will notice that there is NO mention of α. How is that? Please note 'p': it is the p-Value and not the Population Proportion. P-Value or p-Val for a Two-tailed Test is defined as follows:

p-Val = $P[Z > Z_{Calc} = 2.5] \times 2 = \{0.5 - A[Z = 2.5]\} \times 2$

= o.oo62 (2) = 0.0124

If p-Val < α ➔ Reject H_0

If p-Val > α ➔ Do NOT Reject H_0.

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1-PropZTest
 prop≠.8000
 z=2.5000
 p=.0124
 p̂=.9000
 n=100.0000
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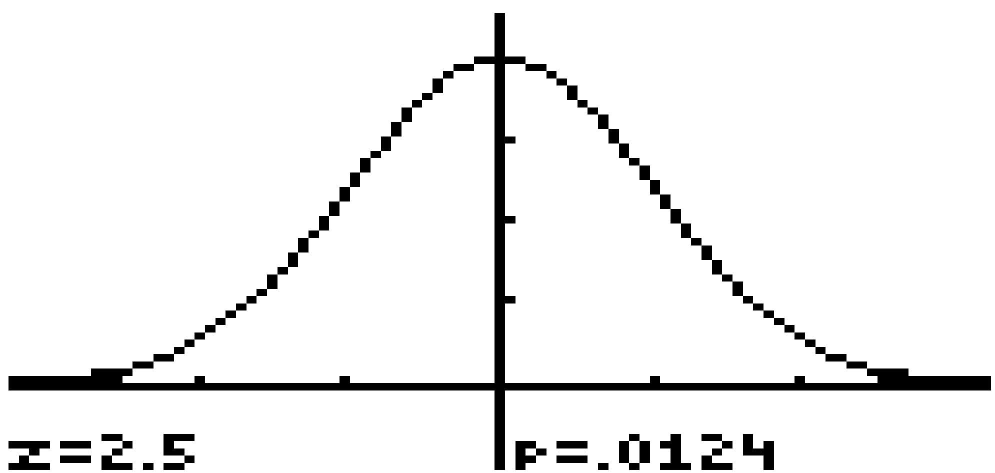


- Please note the flexible nature of p-Value.

If $\alpha = 0.05$, then

$\{p\text{-Val} = 0.0124\} < \{\alpha = 0.05\} \Rightarrow$ Reject H_0.

On the other hand,

if $\alpha = 0.01$, then

$\{p\text{-Val} = 0.0124\} > \{\alpha = 0.01\} \Rightarrow$ Do NOT Reject H_0.

Computers always give the p-Val. It tells rather succinctly, that if α is more than this p-Val, then Reject H_0; if, on the other hand α is less than the p-Val, then Do NOT Reject H_0. However, you must be able to calculate the p-Val manually.

- **5. Hypothesis Test for 1 Proportion:**

Hypothesis Tests for all parameters come in 3 forms.

Test #2: Left Tailed Test

Structure of this Test for One Proportion is as follows:

H_0: $p = p_0$ ➔ Null (Notice '0' in H_0) Hypothesis.

H_A: $p < p_0$ ➔ Alternate (Notice 'A' in H_A) Hypothesis.

Here the assumed value is p_0 and it is a value which we want to disprove. The test is called "Left Tailed" because we want to establish if $p < p_0$ or p is to the left of p_0 .

- **6. Example for Left Tailed Test for 1 Proportion:**

S1 (Step 1): State the Hypothesis for Parameters:

$H_0 : p = 0.93$ & $H_A : p < 0.93$ (Left Tail Test)

S2 (Step 2): Calculate the Test Statistic, 'Z' (Based on CLT)

$$SD(\bar{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.93(0.07)}{100}} = .02552$$

$$Z_{Calc} = Z_{Test} = \frac{(\bar{p} - p)}{SD(\bar{p})} = \frac{(0.90 - 0.93)}{0.02552} = -1.1758$$

S3 (Step 3): Establish the Z* or $Z_{Crit.}$

If Level of Significance, LS = $\alpha = 0.05$, then

$Z_{Crit.} = Z_{\alpha} = 1.645$

S4 (Step 4): Reach an appropriate Conclusion. Since $\{|Z_{Calc}| = 1.1758\} < \{Z_{Crit} = 1.645\}$ ➔ Do NOT Reject H_0

- **By using $|Z_{Calc}|$, only positive Z-Table probability values are needed.**

7. Diagrammatic Representation of the Left Tail Test:

P-Value or p-Val for a Left Tailed Test is defined as follows:

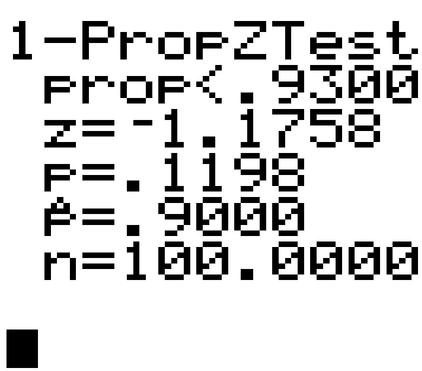
p-Val = $P[Z < Z_{Calc}]$ x 1 = $P[Z < -1.1758]$ x 1

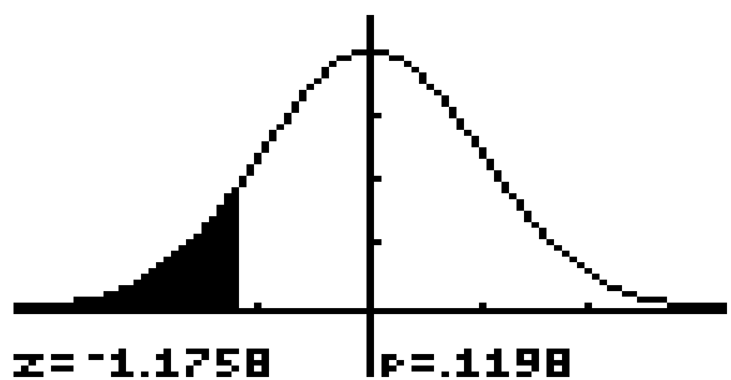
= $P[Z > 1.1758]$x 1 = {0.5 – 0.380}(1) = 0.1200

If p-Val < α ➔ Reject H_0

If p-Val > α ➔ Do NOT Reject H_0. Since,

{p-Val = 0.1200}>{α = 0.05}➔ Do NOT Reject H_0.





- **8. Hypothesis Test for 1 Proportion:**

  Hypothesis Tests for all parameters come in 3 forms.

  Test #3: Right Tailed Test

  Structure of this Test for One Proportion is as follows:

  H_0: $p = p_0$ ➔ Null (Notice '0' in H_0) Hypothesis.

  H_A: $p > p_0$ ➔ Alternate (Notice 'A' in H_A) Hypothesis.

  Here the assumed value is p_0 and it is a value which we want to disprove. The test is called "Right Tailed" because we want to establish if $p > p_0$ or p is to the Right of p_0 .

- **9. Example for Right Tailed Test for 1 Proportion:**

S1 (Step 1): State the Hypothesis for Parameters:

$H_0 : p = 0.85$ & $H_A : p > 0.85$ (Right Tail Test)

S2 (Step 2): Calculate the Test Statistic, 'Z' (Based on CLT)

$$SD(\bar{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.85(0.15)}{100}} = .0357$$

$$Z_{Calc} = Z_{Test} = \frac{(\bar{p} - p)}{SD(\bar{p})} = \frac{(0.90 - 0.85)}{0.0357} = 1.4006$$

S3 (Step 3): Establish the Z* or Z_{Crit}.

If Level of Significance, LS = $\alpha = 0.05$, then

$Z_{Crit.} = Z_{\alpha} = +1.645$ (It is a Right Tailed Test, Remember!)

S4 (Step 4): Reach an appropriate Conclusion. Since $\{Z_{Calc} = 1.4006\} < \{Z_{Crit} = +1.645\}$

➔ Do NOT Reject H_0

10. Diagrammatic Representation of the Right Tail Test:

P-Value or p-Val for a Right Tailed Test is defined as follows:

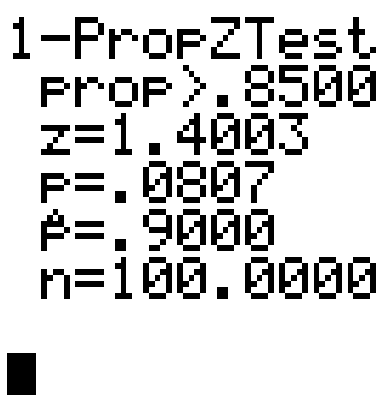
p-Val = $P[Z > Z_{Calc}]$ x 1 = $P[Z > 1.4006]$ x 1

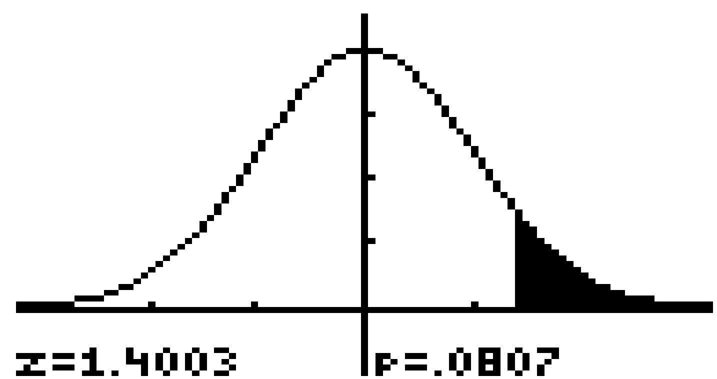
= {0.5 – 0.4192}x (1) = 0.0808

If p-Val < α ➔ Reject H_0

If p-Val > α ➔ Do NOT Reject H_0. Since,

{p-Val = 0.0808}>{α = 0.05}➔ Do NOT Reject H_0.





11. Types of Errors:

Decision or Conclusion	State of Nature H_0	State Of Nature H_A
H_0	Correct Decision	Type II Error: E_{II} $P[E_{II}] = \beta$
H_A	Type I Error: E_I $P[E_I] = \alpha$	Correct Decision

- **Probabilities of the Two Types of Errors:**

P[Committing Type I Error] = α = P[E_I]

Level of Significance, LS = α = P[E_I]

P[Committing Type II Error] = β = P[E_{II}]

P[Reject H_0 | H_0 is True] = α =P[Accept H_A| H_0True]
P[Reject H_A | H_A is True] = β = P[Accept H_0| H_ATrue]

In this Course, we will emphasize the Type I Error and the Probability of making this Type I Error, called α. It must be stressed that in advanced Courses, both α, and β, the two Error Probabilities can be controlled simultaneously with attention to some mathematical detail.

Remember: P[Z > Z_α] = α where Z_α is a Positive #.

12. The Three Tests at a Glance: LS = α

- **1. Two-Tailed Test (with H_A:<,>):**

If $\{|Z_{Calc}|\} > \{Z_{Crit} = Z_{\alpha/2}\}$ ➔ Reject H_0

If $\{|Z_{Calc}|\} < \{Z_{Crit} = Z_{\alpha/2}\}$ ➔ Do NOT Reject H_0

- **2. Left-Tailed Test (with H_A: <):**

If $\{|Z_{Calc}|\} > \{Z_{Crit} = Z_{\alpha}\}$ ➔ Reject H_0

If $\{|Z_{Calc}|\} < \{Z_{Crit} = Z_{\alpha}\}$ ➔ Do NOT Reject H_0

- **3. Right-Tailed Test (with H_A: >):**

If $\{Z_{Calc}\} > \{Z_{Crit} = Z_{\alpha}\}$ ➔ Reject H_0

If $\{Z_{Calc}\} < \{Z_{Crit} = Z_{\alpha}\}$ ➔ Do NOT Reject H_0

In General, for Any of the Above Three Tests:

If p-Val < α ➔ Reject H_0

If p-Val > α ➔ Do NOT Reject H_0.