

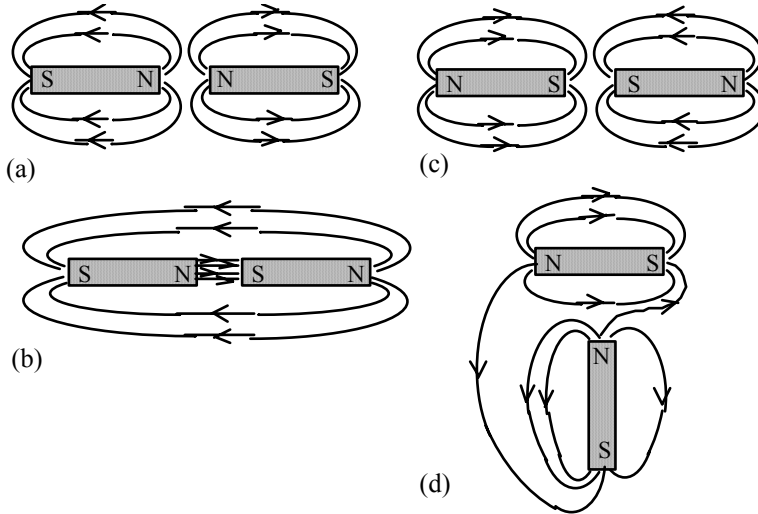
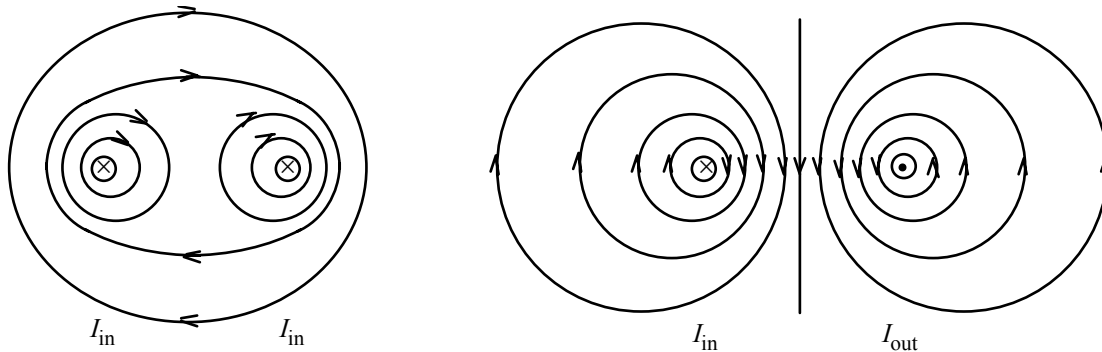
# CHAPTER 28 The Effects of Magnetic Fields

## Answers to Understanding the Concepts Questions

1. The wire is neutral because there are equal numbers of negative and positive charge carriers in each segment. Under the influence of the internal electric field that gives rise to a flowing current, only one of these charges will ordinarily move; if they both move, it will be in opposite directions. In the first case  $q(\vec{v} \times \vec{B})$  is not zero; in the second case, there are two terms, but both  $q_+(\vec{v}_+ \times \vec{B})$  and  $q_-(\vec{v}_- \times \vec{B})$  are in the same direction, and there is no cancellation.
2. If a magnetic monopole existed, then the definition and measurement of a magnetic field would be analogous to those of the electric field. The magnetic field would be defined as the magnetic force exerted on a unit magnetic monopole. To measure the magnetic field at a certain location, we can put a monopole  $q_m$  there and measure the corresponding magnetic force  $\vec{F}$  it experiences. The ratio  $\vec{F}/q_m$  approaches a certain value as  $q_m$  approaches zero, and that is defined as the magnetic field  $\vec{B}$  at that point.
3. If a charged particle moves under the sole influence of a magnetic field, then the force exerted on it is  $\vec{F} = q(\vec{v} \times \vec{B})$ , which is perpendicular to the velocity  $\vec{v}$  of the particle. This force necessarily causes the direction of motion of the particle to change, and so it cannot move in a straight line. Therefore if the electron beam moves across the screen in a straight line then it has to be subject to an electric force.
4. The magnetic force exerted on an electron in the beam is determined from  $\vec{F} = q(\vec{v} \times \vec{B})$ . Note that  $q$  is negative here. It follows from the right-hand-rule that the direction of  $\vec{F}$  is up in the first case so the beam will move up, and it is toward left in the second case so the beam will move to the left.
5. Had we used a left-hand rule, we would just write  $\vec{F} = -q(\vec{v} \times \vec{B})$  and our calculations of physical effects would still be in agreement with experiment.
6. (a) Use the right-hand rule to determine the direction of the magnetic field produced by wire 1. It points into the page at the location of wire 2. (b) The currents in the two wires flow in opposite directions, so the magnetic force between them is repulsive. The force on wire 2 points away from wire 1.
7. No. Velocity selectors work by making the net force on a charged particle zero:  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$ , or  $\vec{E} = -\vec{v} \times \vec{B}$ , which is valid regardless of the sign of the charge.
8. Consider two magnets with the axes from S to N parallel. There is a repulsive force between the magnets; if we rotate one magnet so that the axes are antiparallel, there is an attraction between the magnets. It is hard to see how induced charges would give rise to such an effect.
9. The aurora is the light produced as charged particles from the Sun collide with particles in the atmosphere. Terrestrial magnetic field is the strongest near the two poles. As a charged particle moves in a helical path under the influence of a non-uniform magnetic field that's stronger at the two ends and weaker in the middle region, it is subject to a magnetic force that tends to slow down its forward motion as it moves toward either pole. This is known as the magnetic bottle effect. As a result it is bounced back and forth between the two poles, spending more time in the polar regions. The relatively high concentration of these charged particles in the polar region intensifies the aurora over there.

10. Yes. Even though the wire is charge-neutral, there is a current flowing, and only moving charges (which form a current in the wire) are subject to a magnetic force. For example, in a metal wire, a magnetic force is exerted on the current-carrying drifting electrons, while no magnetic force is acted on the stationary positive ions.
11. The magnetic dipole moment of a current loop is proportional to the product  $NA$ , where  $N$  is the number of turns and  $A$  is the area of the loop. The largest area for a given perimeter is that of a circle, so we conclude we should form circles. Now suppose the wire length is  $L$ , and that we use the wire to form a cylindrical coil of  $N$  turns. Each loop has circumference  $L/N$ , and hence a radius  $L/(2\pi N)$  and area  $\pi[L/(2\pi N)]^2 = L^2/(4\pi N^2)$ . The magnetic moment is therefore proportional to  $NA = L^2/(4\pi N)$ . This decreases with  $N$  for a fixed value of  $L$ . Thus for a given fixed current and a fixed length of wire, a single loop has the largest magnetic dipole moment.
12. For an excellent sketch see Fig. 29-25, in chapter 29 of the textbook.
13. The loops are equivalent to bar magnets, and when these are parallel, there is a repulsion. If the directions of the currents are opposite, the loops attract.
14. The magnetic field above the magnetic north pole points up, so from  $\vec{F} = q(\vec{v} \times \vec{B})$ , where  $q$  is positive for a proton, we see that the direction of  $\vec{F}$  is to the right. The correct answer is (d).
15. The magnetic field lines from the two loops generally point in the same direction instead of opposing each other, so the magnetic force between them is attractive.
16. The magnetic force exerted on a charged particle is  $\vec{F} = q(\vec{v} \times \vec{B})$ , which is perpendicular to the velocity  $\vec{v}$  of the particle. So, as long as  $\vec{F}$  is not zero, it will cause the particle to veer away from its direction of motion and hence move in a curve, not a straight line. However, if  $\vec{F}$  is zero, then the particle will have no acceleration and be able to move in a straight line. This happens if  $\vec{v}$  is parallel to  $\vec{B}$ , whereupon  $\vec{F} = q(\vec{v} \times \vec{B}) = 0$ . Therefore, an electron can move through a magnetic field in a straight line as long as it moves along the magnetic field lines. Another possibility is to introduce an appropriate electric field  $\vec{E}$ , such that  $\vec{E} = -\vec{v} \times \vec{B}$ . The net force exerted on the electron is then  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$ , enabling it to move in a straight line. This is essentially the principle of a velocity selector.
17. When a current flows through a single-turn coil inside a magnetic field, the coil experiences a magnetic torque that forces it to turn. If instead we have an  $N$ -turn coil, each turn being identical to the single turn coil, and the current flowing through each turn is the same as that in the single-turn loop, then each turn in the coil is subject to the same magnetic torque as that on the single-turn coil. So the net torque increases by a factor of  $N$ , as does the rotational inertia of the coil. As a result the  $N$ -turn coil turns just as fast as the single-turn one. The split-ring commutator still works, as the current in each turn is identical.
18. We know geographic N from observations of where the Sun rises. A bar magnet floating on cork in a pail of water will align along Earth's magnetic field lines, which allows us to label the end pointing to geographic north as the magnetic north pole. We now have a compass; that is, a device that points north.
19. No. It repels other magnetic north poles and does not exert any force on a stationary electric charge, positive or negative.
20. Dimensionally  $[F] = [q][v][B]$ . Since  $I = dq/dt$ ,  $[q] = [I][t]$ . Thus  $[F] = [I][vt][B] = [I][L][B]$ , or  $[B] = [F]/([I][L])$ . With  $B$  measured in the Systeme Internationale (SI), this relation reads  $1 \text{ T} = 1 \text{ N/A m}$ . Your classmate is correct.

21. The magnetic force exerted on a charged particle is  $\vec{F} = q(\vec{v} \times \vec{B})$ , which is zero if  $\vec{v}$  and  $\vec{B}$  are parallel to each other; i.e., when the particle moves along the magnetic field lines. So (d) is correct.

**Solutions to Problems****1.****2.**

3. Because the charge is negative, with our fingers pointing in the  $-y$ -direction, we want our thumb to point in the  $+x$ -direction. Thus we have to curl our fingers toward  $-z$ .

The magnetic field is in the  $-z$ -direction.

Formally, we have

$$\begin{aligned}\vec{F} &= m\vec{a} = q(\vec{v} \times \vec{B}); \\ -F\hat{i} &= (-e)(-v\hat{j}) \times \vec{B}, \text{ which gives } \vec{B} = -B\hat{k}.\end{aligned}$$

4. We find the force from

$$\begin{aligned}\vec{F} &= q(\vec{v} \times \vec{B}) \\ &= (1.60 \times 10^{-19} \text{ C})[(1.7 \times 10^6)\hat{i} + (0.8 \times 10^6)\hat{j} - (4.5 \times 10^5)\hat{k}](\text{m/s}) \times (0.70\hat{i} - 0.50\hat{j} + 0.10\hat{k}) \text{ T} \\ &= (1.60 \times 10^{-19})(10^5)[(0.8 - 2.25)\hat{i} + (-3.15) - 1.7)\hat{j} + (-0.85 - 0.56)\hat{k}] \text{ N} \\ &= \boxed{[-(0.23 \times 10^{-13})\hat{i} - (0.78 \times 10^{-13})\hat{j} - (2.3 \times 10^{-13})\hat{k}] \text{ N}}.\end{aligned}$$

5. We find the speed from

$$K = \frac{1}{2}mv^2, \text{ which gives}$$

$$v = (2K/m)^{1/2}$$

$$= [2(100 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})/(1.67 \times 10^{-27} \text{ kg})]^{1/2} = 4.38 \times 10^6 \text{ m/s}.$$

The magnetic force produces the acceleration:

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a};$$

$$\vec{F} = q(v\hat{i}) \times B\hat{k} = ma\hat{j}.$$

Because the proton is moving perpendicular to the field, and  $\hat{i} \times (-\hat{k}) = \hat{j}$ , we have  $B = -B\hat{k}$ . For the magnitude, we have

$$B = ma/qv$$

$$= (1.67 \times 10^{-27} \text{ kg})(3 \times 10^{12} \text{ m/s}^2)/[(1.60 \times 10^{-19} \text{ C})(4.38 \times 10^6 \text{ m/s})] = 7.2 \times 10^{-3} \text{ T}.$$

Thus, we have  $\vec{B} = \boxed{-(7.3 \times 10^{-3} \text{ T})\hat{k}}$ .

6. To find the direction of the force on the pith ball, we point our fingers vertically down and curl them toward the north; our thumb points east, which is the direction of the force on this positive charge.

Because the vectors are perpendicular, we have

$$F = qvB = (1 \times 10^{-6} \text{ C})(5 \times 10^{-2} \text{ m/s})(0.5 \times 10^{-4} \text{ T}) = \boxed{2.5 \times 10^{-12} \text{ N east}}.$$

7. The time for the proton to cross  $L$ , the width of the magnetic field, is  $t = L/v$ . Because the perpendicular speed is small, we assume the perpendicular acceleration, caused by the field, is constant, so we have

$$F = qvB = ma_{\perp} = mv_{\perp}/t = mv_{\perp}v/L, \text{ which gives}$$

$$B = Lv_{\perp}m/q$$

$$= (1.0 \times 10^{-2} \text{ m})(3.3 \times 10^5 \text{ m/s})(1.67 \times 10^{-27} \text{ kg})/(1.60 \times 10^{-19} \text{ C}) = 3.4 \times 10^{-5} \text{ T} = \boxed{34 \mu\text{T}}.$$

8. The magnetic force produces an acceleration perpendicular to the original motion:

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}, \text{ or } a_{\perp} = qvB/m.$$

The direction of motion is the direction of the velocity. For a small change in direction, we can take the force to be constant, so the perpendicular component of the velocity is

$$v_{\perp} = a_{\perp}t = qvBt/m.$$

The direction of motion is given by

$$\tan \theta = v_{\perp}/v = qBt/m;$$

$$(0.05^\circ)(\pi \text{ rad}/180^\circ) = q(0.007 \text{ T})(3.0 \text{ s})/(0.4 \times 10^{-3} \text{ kg}), \text{ which gives } q = \boxed{1.7 \times 10^{-5} \text{ C}}.$$

9. (a) The magnetic force produces an acceleration perpendicular to the original motion:

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}, \text{ or } a_{\perp} = evB/m.$$

For a small change in direction, we can take the force to be constant, so the perpendicular component of the velocity is

$$v_{\perp} = a_{\perp} \Delta t = (evB \Delta t)/m.$$

If  $D$  is the diameter of the region of the magnetic field, the time the particle spends in the field is  $t = D/v$ .

The direction of motion is given by

$$\tan \theta = v_{\perp}/v = (eB \Delta t)/m.$$

If the angle of deflection is small, we have

$$\tan \theta \approx \theta = \boxed{(eB \Delta t)/m}.$$

- (b) If  $D$  is the diameter of the region of the magnetic field, the time the particle spends in the field is  $\Delta t = D/v$ .

The angle of deflection is

$$\theta = (eB \Delta t)/m = eBD/mv;$$

$$0.1 \text{ rad} = (1.60 \times 10^{-19} \text{ C})B(0.1 \text{ m})/(1.7 \times 10^{-27} \text{ kg})(1.4 \times 10^7 \text{ m/s}), \text{ which gives } B = \boxed{0.15 \text{ T}}.$$

10. (a) From  $\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$ , we see that the force, and thus the deflection, will be to the east.

- (b) If the particle were uncharged, the time to impact the earth from the height of 100 km is

$$\Delta t = H/v = (100 \times 10^3 \text{ m})/(10^8 \text{ m/s}) = 10^{-3} \text{ s}.$$

Because this is so short, we can ignore the rotation of the earth. We assume the acceleration perpendicular to the original direction is constant, with magnitude

$$a_{\perp} = qvB/m.$$

The distance that the particle deflects to the east is

$$\begin{aligned} \Delta x &= \frac{1}{2} a_{\perp} t^2 = \frac{1}{2} (qvB/m)(\Delta t)^2 \\ &= \frac{1}{2} [(1.60 \times 10^{-19} \text{ C})(10^8 \text{ m/s})(10^{-4} \text{ T})/(9.5 \times 10^{-26} \text{ kg})](10^{-3} \text{ s})^2 = \boxed{8.4 \times 10^3 \text{ m} = 8.4 \text{ km}}. \end{aligned}$$

11. We choose vertically up to be the + z-axis. In the northern hemisphere, the magnetic field has a downward component, so

$$\vec{B} = B_x \hat{i} + B_z \hat{k} = (24 \mu\text{T}) \hat{i} - (18 \mu\text{T}) \hat{k}.$$

We find the acceleration of the electron from  $\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$ :

$$\vec{F} = (-e)[(v_0 \hat{i}) \times (B_x \hat{i} + B_z \hat{k})] = m\vec{a}, \text{ which gives } \vec{a} = (ev_0 B_z/m) \hat{j}.$$

The time the electron takes to reach the screen is

$$\Delta t = L/v = (0.40 \text{ m})/(6.0 \times 10^7 \text{ m/s}) = 6.7 \times 10^{-9} \text{ s}.$$

The deflection of the electron is

$$\begin{aligned} \vec{d} &= \frac{1}{2} \vec{a} t^2 = \frac{1}{2} (ev_0 B_z/m)(\Delta t)^2 \hat{j} \\ &= \frac{1}{2} [(1.60 \times 10^{-19} \text{ C})(6 \times 10^7 \text{ m/s})(-18 \times 10^{-6} \text{ T})/(9.1 \times 10^{-31} \text{ kg})](6.7 \times 10^{-9} \text{ s})^2 \hat{j} \\ &= -(4.2 \times 10^{-3} \text{ m}) \hat{j} = \boxed{-(4.2 \text{ mm}) \hat{j} \text{ (horizontal)}}. \end{aligned}$$

This small distance justifies taking the acceleration to be constant.

12. The magnetic force provides the centripetal acceleration:

$$qvB = mv^2/R, \text{ so the radius of the path is}$$

$$R = mv/qB;$$

$$2.40 \text{ m} = (1.67 \times 10^{-27} \text{ kg})(3 \times 10^6 \text{ m/s})/[(1.60 \times 10^{-19} \text{ C})B], \text{ which gives}$$

$$B = 0.013 \text{ T} = \boxed{13 \text{ mT}}.$$

13. (a) The radius of the path is given by

$$R = mv/qB.$$

The kinetic energy acquired by the electron in the gun is  $K = eV$ , so the electron's speed is given by

$$v = (2K/m)^{1/2} = (2eV/m)^{1/2}.$$

When we combine this with the expression for the radius, we get

$$R = (m/eB)(2eV/m)^{1/2} = (2mV/e)^{1/2}/B;$$

$$0.06 \text{ m} = [2(9.1 \times 10^{-31} \text{ kg})(1600 \text{ V})/(1.60 \times 10^{-19} \text{ C})]^{1/2}/B,$$

$$\text{which gives } B = \boxed{2.25 \times 10^{-3} \text{ T}}.$$

- (b) For the alpha particle, we have

$$R = (m/qB)(2K/m)^{1/2} = (2mK)^{1/2}/qB;$$

$$0.20 \text{ m} = [2(7360)(9.1 \times 10^{-31} \text{ kg})(1200 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})]^{1/2}/[2(1.60 \times 10^{-19} \text{ C})B], \text{ which gives } B =$$

$$\boxed{2.5 \times 10^{-2} \text{ T}}.$$

14. If we take the velocity to be perpendicular to the magnetic field, the magnetic force provides the centripetal acceleration:

$$qvB = mv^2/R, \text{ so the radius of the path is}$$

$$R = mv/qB$$

$$= (1.67 \times 10^{-27} \text{ kg})(25 \times 10^3 \text{ m/s})/(1.60 \times 10^{-19} \text{ C})(10^{-10} \text{ T}) = 2.6 \times 10^6 \text{ m}.$$

This is on the order of the earth's radius and much less than interplanetary distances, which are on the order of  $10^{11} \text{ m}$ .

15. The radius of the orbit is

$$R = mv/qB$$

$$= (9.1 \times 10^{-31} \text{ kg})(0.001)(3 \times 10^8 \text{ m/s}) / (1.60 \times 10^{-19} \text{ C})(3 \times 10^7 \text{ T}) = \boxed{5.7 \times 10^{-14} \text{ m}}.$$

Note that this is much smaller than electron orbits in an atom.

The magnitude of the magnetic force is

$$F = qvB$$

$$= (1.60 \times 10^{-19} \text{ C})(0.001)(3 \times 10^8 \text{ m/s})(3 \times 10^7 \text{ T}) = \boxed{1.4 \times 10^{-6} \text{ N}}.$$

16. The cyclotron frequency is

$$f = qB/2\pi m$$

$$= (1.60 \times 10^{-19} \text{ C})(1.2 \text{ T}) / [2\pi(2)(1.67 \times 10^{-27} \text{ kg})] = \boxed{5.2 \times 10^6 \text{ Hz}}.$$

17. Because the cyclotron frequency depends on the charge, mass and magnetic field, and we do not change the particle, we must change the
- magnetic field
- .

We find the initial magnetic field from

$$f_i = qB_i/2\pi m;$$

$$6.1 \times 10^6 \text{ Hz} = (1.60 \times 10^{-19} \text{ C})B_i / [2\pi(1.67 \times 10^{-27} \text{ kg})], \text{ which gives}$$

$$B_i = \boxed{0.40 \text{ T}}.$$

To triple the frequency, we must triple the magnetic field:  $B_f = \boxed{1.20 \text{ T}}.$

18. We find the speed from the kinetic energy:

$$v = (2K/m)^{1/2}.$$

The radius of the electron's trajectory is

$$R = mv/eB = (2mK)^{1/2}/eB;$$

$$300 \text{ m} = [2(9.1 \times 10^{-31} \text{ kg})(100 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})]^{1/2} / [(1.60 \times 10^{-19} \text{ C})B], \text{ which gives}$$

$$B = \boxed{3.6 \times 10^{-6} \text{ T}}.$$

19. We find the radii from

$$R_e = m_e v_e / eB$$

$$= (9.1 \times 10^{-31} \text{ kg})(10^6 \text{ m/s}) / [(1.60 \times 10^{-19} \text{ C})(10^{-5} \text{ T})] = \boxed{0.57 \text{ m}};$$

$$R_p = m_p v_p / eB$$

$$= (1.67 \times 10^{-27} \text{ kg})(10^4 \text{ m/s}) / [(1.60 \times 10^{-19} \text{ C})(10^{-5} \text{ T})] = \boxed{10.4 \text{ m}}.$$

The proton's orbit is greater because of its much greater mass. If its speed were the same as the electron, the proton's orbit would have a radius 100 times greater:  $\boxed{1.04 \text{ km}}.$

20. In terms of the momentum, the radius is

$$R = p/eB;$$

$$17 \times 10^3 \text{ m} = p / (1.60 \times 10^{-19} \text{ C})(7.0 \text{ T}), \text{ which gives}$$

$$p = 1.9 \times 10^{-14} \text{ kg} \cdot \text{m/s}.$$

The energy of the proton is

$$E = pc$$

$$= (1.9 \times 10^{-14} \text{ kg} \cdot \text{m/s})(3 \times 10^8 \text{ m/s}) / (1.6 \times 10^{-13} \text{ J/MeV})$$

$$= \boxed{3.6 \times 10^7 \text{ MeV}}.$$

21. (a) The speed of the electron is given by

$$v = (2K/m)^{1/2} = [2(10 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})/(9.1 \times 10^{-31} \text{ kg})]^{1/2} = 5.9 \times 10^7 \text{ m/s}.$$

If we assume that the deflection is small, the time the electron takes to reach the screen is

$$\Delta t = L/v = (0.40 \text{ m})/(5.9 \times 10^7 \text{ m/s}) = 6.8 \times 10^{-9} \text{ s}.$$

The magnetic force produces an acceleration perpendicular to the original motion:

$$a_{\perp} = evB/m.$$

For a small deflection, we can take the force to be constant, so the perpendicular component of the velocity is

$$v_{\perp} = a_{\perp} \Delta t = (evB \Delta t)/m = (1.60 \times 10^{-19} \text{ C})(5.9 \times 10^7 \text{ m/s})(5 \times 10^{-5} \text{ T})(6.8 \times 10^{-9} \text{ s})/(9.1 \times 10^{-31} \text{ kg}) = 0.35 \times 10^7 \text{ m/s}.$$

Because this is small compared to the original speed, the speed is essentially constant. We find the angle of the velocity from the original direction from

$$\sin \theta = v_{\perp}/v = (0.35 \times 10^7 \text{ m/s})/(5.9 \times 10^7 \text{ m/s}) = 0.059, \text{ which gives } \theta = 3.4^{\circ}.$$

The final velocity is  $5.9 \times 10^7 \text{ m/s}$ ,  $3.4^{\circ}$  from the original direction.

- (b) Because we have assumed constant acceleration, we find the deflection of the electron from

$$d = \frac{1}{2}(0 + v_{\perp}) \Delta t = \frac{1}{2}(0.35 \times 10^7 \text{ m/s})(6.8 \times 10^{-9} \text{ s}) = 1.2 \times 10^{-2} \text{ m} = 1.2 \text{ cm}.$$

This justifies our assumption of small deflection.

22. (a) The velocity of the particle is perpendicular to the magnetic field and for a negative charge will be to the east, so the magnetic force provides the centripetal acceleration:

$$qvB = mv^2/R, \text{ so the speed of the particle is}$$

$$v = qBR/m = (1.0 \times 10^{-3} \text{ C})(5 \times 10^{-5} \text{ T})(6.4 \times 10^6 \text{ m})/(1.0 \times 10^{-3} \text{ kg}) = 3.2 \times 10^2 \text{ m/s east}.$$

- (b) The ratio of the forces is

$$F_g/F_B = mg/qvB = (1.0 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)/[(1.0 \times 10^{-3} \text{ C})(3.2 \times 10^2 \text{ m/s})(5 \times 10^{-5} \text{ T})] = 610.$$

23. The critical momentum is  $p_c = eBR = 8 \times 10^{-8} \text{ kg} \cdot \text{m/s}$ .

- (a) The energy of the electron is

$$E = p_c c = (8 \times 10^{-8} \text{ kg} \cdot \text{m/s})(3 \times 10^8 \text{ m/s}) = 24 \text{ J } (15 \times 10^{19} \text{ eV}).$$

- (b) Because the alpha particle has a charge of  $2e$ , we have

$$p_c = 2eBR = 2(8 \times 10^{-8} \text{ kg} \cdot \text{m/s}) = 16 \times 10^{-8} \text{ kg} \cdot \text{m/s};$$

$$E = p_c c = (16 \times 10^{-8} \text{ kg} \cdot \text{m/s})(3 \times 10^8 \text{ m/s}) = 48 \text{ J}.$$

- (c) Because the uranium ion has a charge of  $e$ , we have

$$p_c = eBR = 8 \times 10^{-8} \text{ kg} \cdot \text{m/s};$$

$$E = p_c c = (8 \times 10^{-8} \text{ kg} \cdot \text{m/s})(3 \times 10^8 \text{ m/s}) = 24 \text{ J}.$$

24. The two forces are in opposite directions. For their magnitudes to be equal, we have

$$F_g = F_B, \text{ or } mg = qvB;$$

$$(1.67 \times 10^{-27} \text{ kg})(9.8 \text{ m/s}^2) = (1.60 \times 10^{-19} \text{ C})v(0.7 \times 10^{-2} \text{ T}), \text{ which gives } v = 1.5 \times 10^{-5} \text{ m/s}.$$

25. The component of the velocity parallel to the field does not change. The component perpendicular to the field produces a force which causes the circular motion. From the radius of the motion,  $R = mv_{\perp}/qB$ , we find the time for one revolution:

$$T = 2\pi R/v_{\perp} = 2\pi m/qB.$$

In this time, the distance the electron travels along the field is

$$d = v_{\text{parallel}} T = v_{\text{parallel}} 2\pi m/qB = (3.0 \times 10^5 \text{ m/s})(\cos 40^{\circ})2\pi(9.1 \times 10^{-31} \text{ kg})/[(1.60 \times 10^{-19} \text{ C})(0.12 \text{ T})] = 6.8 \times 10^{-5} \text{ m}.$$



26. We find the perpendicular component from the radius of the circular path:

$$R = mv_{\perp}/qB;$$

$$8.5 \times 10^{-3} \text{ m} = (9.1 \times 10^{-31} \text{ kg})v_{\perp}/[(1.60 \times 10^{-19} \text{ C})(0.035 \text{ T})], \text{ which gives}$$

$$v_{\perp} = \boxed{5.2 \times 10^7 \text{ m/s}}.$$

The time for one revolution is

$$T = 2\pi R/v_{\perp} = 2\pi m/qB.$$

We find the parallel component of the velocity from the distance the electron travels along the field:

$$d = v_{\text{parallel}}T = v_{\text{parallel}}2\pi m/qB;$$

$$8.5 \times 10^{-3} \text{ m} = v_{\text{parallel}}2\pi(9.1 \times 10^{-31} \text{ kg})/[(1.60 \times 10^{-19} \text{ C})(0.035 \text{ T})], \text{ which gives}$$

$$v_{\text{parallel}} = \boxed{8.3 \times 10^6 \text{ m/s}}.$$

27. The force produced by the magnetic field will always be perpendicular to the velocity and thus in the plane perpendicular to the axis of the tube. This will change the direction of the velocity but not its magnitude. When an electron reaches the outer cylinder, its kinetic energy must be equal to the decrease in potential energy:

$$K_f = U_i = \boxed{500 \text{ eV}}.$$

28. (a) We find the speed of the particle from the kinetic energy, which is equal to the potential energy change:

$$v = (2K/m)^{1/2} = (2qV/m)^{1/2}.$$

The radius of the orbit is

$$R = mv/qB = (m/qB)(2qV/m)^{1/2} = (2mV/q)^{1/2}/B.$$

For the ratio, we have

$$R_p/R_{\alpha} = [(m_p/m_{\alpha})(q_{\alpha}/q_p)]^{1/2} = [(1/4)(2)]^{1/2} = \boxed{1/\sqrt{2} = 0.707}.$$

- (b) The frequency of the orbit is

$$f = qB/2\pi m.$$

For the ratio, we have

$$f_p/f_{\alpha} = (q_p/q_{\alpha})(m_{\alpha}/m_p) = (1/2)(4) = \boxed{2}.$$

29. We consider the motions created by the fields separately and then see how they are combined. The force from the electric field is

$$\vec{F} = q\vec{E} = -(1.60 \times 10^{-19} \text{ C})(1500 \text{ V/m})\hat{j},$$

which produces a constant acceleration:

$$\vec{a}_E = -[(1.60 \times 10^{-19} \text{ C})(1500 \text{ V/m})/(9.1 \times 10^{-31} \text{ kg})]\hat{j} = -(2.6 \times 10^{14} \text{ m/s}^2)\hat{j}.$$

The velocity is

$$\vec{v}_E = 0 + \vec{a}_E t = -(2.6 \times 10^{14} \text{ m/s}^2)t\hat{j}.$$

Because the initial velocity is perpendicular to the magnetic field, the force from the magnetic field will produce circular motion in the  $xz$ -plane with radius

$$R = mv/qB = (9.1 \times 10^{-31} \text{ kg})(2 \times 10^6 \text{ m/s})/(1.60 \times 10^{-19} \text{ C})(0.2 \text{ T}) = 5.7 \times 10^{-5} \text{ m}.$$

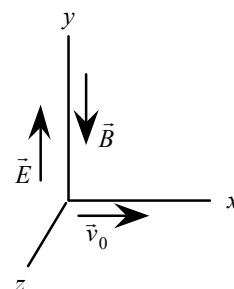
The velocity produced by the electric field is parallel to the magnetic field;

thus, from  $\vec{F} = q\vec{v} \times \vec{B}$ , we see that the electric field does not affect the motion in the  $xz$ -plane.

The motion will be a type of helical motion aligned parallel to the  $y$ -axis.

The circular motion is parallel to the  $xz$ -plane with radius  $5.7 \times 10^{-5} \text{ m}$ .

The motion in the  $y$ -direction has a constant acceleration of  $-2.6 \times 10^{14} \text{ m/s}^2$ .



30. For the velocity to be constant, the electric force must balance the magnetic force:

$$eE = evB, \text{ or } v = E/B.$$

We find the speed acquired from the accelerating potential from

$$v = (2eV/m)^{1/2}.$$

For a fixed magnetic field, the necessary electric field is

$$E = B(2eV/m)^{1/2}, \text{ which gives}$$

$$E_{\min} = (0.40 \text{ T})[2(1.60 \times 10^{-19} \text{ C})(1.5 \times 10^3 \text{ V})/(9.1 \times 10^{-31} \text{ kg})]^{1/2} = \boxed{9.2 \times 10^6 \text{ V/m}};$$

$$E_{\max} = (0.40 \text{ T})[2(1.60 \times 10^{-19} \text{ C})(1.5 \times 10^4 \text{ V})/(9.1 \times 10^{-31} \text{ kg})]^{1/2} = \boxed{2.9 \times 10^7 \text{ V/m}}.$$

The range of electric field strengths is  $\boxed{9.2 \times 10^6 \text{ V/m} < E < 2.9 \times 10^7 \text{ V/m}}.$

For a fixed electric field, the necessary magnetic field is

$$B = E/(2eV/m)^{1/2}, \text{ which gives}$$

$$B_{\min} = (15 \times 10^2 \text{ V/m})/[2(1.60 \times 10^{-19} \text{ C})(1.5 \times 10^3 \text{ V})/(9.1 \times 10^{-31} \text{ kg})]^{1/2} = \boxed{2.1 \times 10^{-5} \text{ T}};$$

$$B_{\max} = (15 \times 10^2 \text{ V/m})/[2(1.60 \times 10^{-19} \text{ C})(1.5 \times 10^4 \text{ V})/(9.1 \times 10^{-31} \text{ kg})]^{1/2} = \boxed{6.5 \times 10^{-5} \text{ T}}.$$

The range of magnetic field strengths is  $\boxed{2.1 \times 10^{-5} \text{ T} < B < 6.5 \times 10^{-5} \text{ T}}.$

31. (a) The cyclotron frequency is

$$f = qB/2\pi m = (1.60 \times 10^{-19} \text{ C})(1.0 \text{ T})/[2\pi(1.7 \times 10^{-27} \text{ kg})] = \boxed{1.5 \times 10^7 \text{ Hz}}.$$

- (b) We find the maximum velocity from

$$R_{\max} = mv_{\max}/qB = v_{\max}/2\pi f;$$

$$0.50 \text{ m} = v_{\max}/[2\pi(1.5 \times 10^7 \text{ Hz})], \text{ which gives } v_{\max} = \boxed{4.8 \times 10^7 \text{ m/s tangential}}.$$

- (c) The maximum kinetic energy is

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}(1.7 \times 10^{-27} \text{ kg})(4.8 \times 10^7 \text{ m/s})^2 = \boxed{1.9 \times 10^{-12} \text{ J } (1.2 \times 10^7 \text{ eV})}.$$

- (d) In a full circle, the proton crosses the gap twice, so the energy gain in one cycle is

$$\Delta E = 2e \Delta V.$$

The number of circles is

$$n = K_{\max}/\Delta E = (1.2 \times 10^7 \text{ eV})/[2(1 \text{ e})(50 \times 10^3 \text{ V})] = \boxed{120}.$$

- (e) The time the proton spends in the accelerator is

$$t = nT = n/f = (120)/(1.5 \times 10^7 \text{ Hz}) = \boxed{8.0 \times 10^{-6} \text{ s}}.$$

32. (a) The cyclotron frequency is

$$f_p = qB/2\pi m = eB/2\pi m_p = (1.60 \times 10^{-19} \text{ C})(1.7 \text{ T})/[2\pi(1.67 \times 10^{-27} \text{ kg})] = \boxed{2.6 \times 10^7 \text{ Hz}}.$$

- (b) The maximum kinetic energy is

$$K_{p,\max} = \frac{1}{2}m_p v_{p,\max}^2 = \frac{1}{2}(m_p v_{p,\max})^2/m_p = \frac{1}{2}(qBR)^2/m_p = \frac{1}{2}(eBR)^2/m_p = \frac{1}{2}[(1.60 \times 10^{-19} \text{ C})(1.7 \text{ T})(0.40 \text{ m})]^2/(1.67 \times 10^{-27} \text{ kg}) = \boxed{3.5 \times 10^{-12} \text{ J } (22 \text{ MeV})}.$$

- (c) The cyclotron frequency is

$$f = qB/2\pi m = 2eB/[2\pi 4m_p] = \frac{1}{2}f_p = \boxed{1.3 \times 10^7 \text{ Hz}}.$$

The maximum kinetic energy is

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}(mv_{\max})^2/m = \frac{1}{2}(qBR)^2/m = \frac{1}{2}(2eBR)^2/4m_p = K_{p,\max} = \boxed{3.5 \times 10^{-12} \text{ J } (22 \text{ MeV})}.$$

33. We relate the radius to the momentum:

$$R = mv/qB = p/qB, \text{ so we have}$$

$$p_{\min} = qB_{\min}R = (1.60 \times 10^{-19} \text{ C})(1 \text{ T})(6.2 \times 10^3 \text{ m}) = 9.9 \times 10^{-16} \text{ kg} \cdot \text{m/s};$$

$$p_{\max} = qB_{\max}R = (1.60 \times 10^{-19} \text{ C})(4.5 \text{ T})(6.2 \times 10^3 \text{ m}) = 45 \times 10^{-16} \text{ kg} \cdot \text{m/s}.$$

The range of momenta is  $9.9 \times 10^{-16} \text{ kg} \cdot \text{m/s} < p < 45 \times 10^{-16} \text{ kg} \cdot \text{m/s}$ .

For the energy, we have

$$E_{\min} = p_{\min}c = (9.9 \times 10^{-16} \text{ kg} \cdot \text{m/s})(3 \times 10^8 \text{ m/s}) = 3.0 \times 10^{-7} \text{ J} = 1.9 \text{ TeV};$$

$$E_{\max} = p_{\max}c = (45 \times 10^{-16} \text{ kg} \cdot \text{m/s})(3 \times 10^8 \text{ m/s}) = 1.34 \times 10^{-6} \text{ J} = 8.4 \text{ TeV}.$$

The range of energies is  $1.9 \text{ TeV} < E < 8.4 \text{ TeV}$ .

We find the speed of the baseball from

$$K = \frac{1}{2}mv^2;$$

$$1.34 \times 10^{-6} \text{ J} = \frac{1}{2}(0.15 \text{ kg})v^2, \text{ which gives } v = 4.2 \times 10^{-3} \text{ m/s}.$$

34. We find the speed acquired from the accelerating potential from

$$v = (2qV/m)^{1/2}.$$

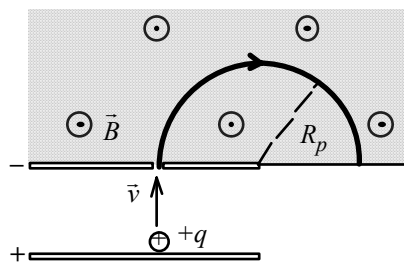
We combine this with the expression for the radius of the path,

$$R = mv/qB, \text{ to get}$$

$$R = (2mV/q)^{1/2}/B.$$

With  $V$ ,  $q$ , and  $B$  the same,

$$R_x/R_p = 1.4 = (m_x/m_p)^{1/2}, \text{ which gives } m_x/m_p = 2.0.$$



35. We find the speed of the particle from the kinetic energy:

$$v = (2K/m)^{1/2}.$$

The required magnetic field for the orbit of the particle with the maximum energy is

$$\begin{aligned} B_{\max} &= m_{\alpha}v_{\max}/q_{\alpha}R = (2K_{\max}m_{\alpha})^{1/2}/q_{\alpha}R \\ &= [2(6 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})4(1.67 \times 10^{-27} \text{ kg})]^{1/2}/[2(1.60 \times 10^{-19} \text{ C})(0.10 \text{ m})] = 3.54 \text{ T}. \end{aligned}$$

Smaller kinetic energies require a smaller field.

36. We find the speed of the proton from the kinetic energy:

$$v = (2K/m)^{1/2} = [2(150 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})/(1.67 \times 10^{-27} \text{ kg})]^{1/2} = 1.7 \times 10^8 \text{ m/s}.$$

Because the acceleration in the  $y$ -direction is perpendicular the magnetic field, we can write

$$\vec{B} = B_x \hat{i} + B_z \hat{k}.$$

The force produces the acceleration; with  $\vec{v} = v \hat{i}$  we have

$$m_p a \hat{j} = e \vec{v} \times \vec{B} = ev \hat{i} \times (B_x \hat{i} + B_z \hat{k}) = evB_z(-\hat{j}), \text{ from which we get}$$

$$B_z = -m_p a / ev = - (1.67 \times 10^{-27} \text{ kg})(7.0 \times 10^{12} \text{ m/s}^2) / [(1.60 \times 10^{-19} \text{ C})(9.8 \times 10^7 \text{ m/s})] = -4.3 \times 10^{-4} \text{ T}.$$

Because  $B_x$  cannot be determined from the given data, we have  $\vec{B} = B_x \hat{i} - (4.3 \times 10^{-4} \text{ T}) \hat{k}$ .

37. The kinetic energy of the electron is

$$K = \frac{1}{2}mv^2 = 2.3 \text{ keV} = (2.3 \times 10^3 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = 3.68 \times 10^{-16} \text{ J}. \text{ Thus its speed is}$$

$$v = (2K/m)^{1/2} = [2(3.68 \times 10^{-16} \text{ J})/(9.11 \times 10^{-31} \text{ kg})]^{1/2} = 2.84 \times 10^7 \text{ m/s}.$$

The magnetic force exerted on the electron follows from

$$F_B = evB = (1.6 \times 10^{-19} \text{ C})(2.84 \times 10^7 \text{ m/s})(0.52 \times 10^{-4} \text{ T}) = 2.4 \times 10^{-16} \text{ N}.$$

The ratio of this force to the gravitational force is

$$F_B/F_g = (2.36 \times 10^{-16} \text{ N}) / [(9.11 \times 10^{-31} \text{ kg})(9.8 \text{ m/s}^2)] = 2.6 \times 10^{13}.$$

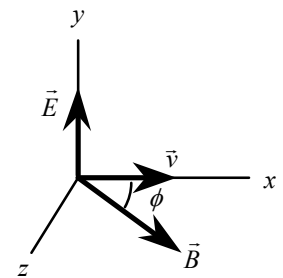
The magnetic force as a function of the kinetic energy of the electron is

$$F_B = evB = eB(2K/m)^{1/2}, \text{ so for } F_B = F_g = mg$$

$$\begin{aligned} K &= \frac{1}{2}m(mg/eB)^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})[(9.11 \times 10^{-31} \text{ kg})(9.8 \text{ m/s}^2)/(1.6 \times 10^{-19} \text{ C})(0.52 \times 10^{-4} \text{ T})]^2 \\ &= 5.2 \times 10^{-43} \text{ J} = 3.2 \times 10^{-24} \text{ eV}. \end{aligned}$$

38. (a) The kinetic energy of the proton is

- $K = \frac{1}{2}mv^2$ , which gives  $v = (2K/m)^{1/2}$ . The radius  $R$  of the orbit of the proton is  
 $R = mv/eB = m(2K/m)^{1/2}/eB$ , so  
 $B = (2Km)^{1/2}/eR$   
 $= [2(55 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})(1.67 \times 10^{-27} \text{ kg})]^{1/2} / [(1.6 \times 10^{-19} \text{ C})(44 \text{ in.})(0.0254 \text{ m/in.})]$   
 $= \boxed{0.96 \text{ T}}$ .
- (b) Solve for  $K$  from the expression for  $B$  above to yield  
 $K = (qRB)^2/2m$ , where we replaced  $e$  with a general value  $q$ . So for the Kr and the proton cases  
 $K_{\text{Kr}}/K_p = (q_{\text{Kr}}/q_p)^2(m_p/m_{\text{Kr}}) = (19e/e)^2(m_p/86m_p) = 4.2$ , and  
 $K_{\text{Kr}} = 4.2 K_p = 4.2(55 \text{ MeV}) = 230 \text{ MeV} = \boxed{0.23 \text{ GeV}}$ .
39. (a)  $\vec{F}_E = q\vec{E}$ , in the direction of  $\vec{E}$ , i.e., from left to right on the page.  
 $\vec{F}_B = q\vec{v} \times \vec{B}$ , so the direction of  $\vec{F}_B$  is "out of page"  $\times$  "up" = from right to left on the page.  
 $\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B}$ , where  $\vec{F}_E$  and  $\vec{F}_B$  are opposite to each other so  
 $F = |F_E - F_B| = \boxed{q|E - vB|}$ .
- (c)  $F = 0$  if  $E - vB = 0$ , or  $E = vB$ .
40. (a) The force produced by the magnetic field will be perpendicular to the plane formed by  $\vec{v}$  and  $\vec{B}$ . To form the velocity selector, we want the force produced by the electric field to be opposite to the magnetic force, so  $\vec{E}$  will be perpendicular to the plane formed by  $\vec{v}$  and  $\vec{B}$ . For the forces to have equal magnitudes, we have  
 $evB_{\perp} = eE$ , where  $B_{\perp}$  is the component of  $\vec{B}$  perpendicular to  $\vec{v}$ .  
 We orient  $\vec{E}$  along the  $y$ -axis,  $\vec{v}$  along the  $x$ -axis, and  $\vec{B}$  in the  $xz$ -plane.  
 If  $\phi$  is the angle between  $\vec{v}$  and  $\vec{B}$ , we have  
 $v = E/B_{\perp} = E/(B \sin \phi)$ ;  
 $2 \times 10^4 \text{ m/s} = (2000 \text{ N/C})/[(0.3 \text{ T}) \sin \phi]$ , which gives  $\phi = 19.5^\circ$ .
- (b) The minimum value of  $v$  occurs for the maximum value of  $\sin \phi$ .  
 $v_{\min} = E/B \sin 90^\circ = (2000 \text{ N/C})/[(0.3 \text{ T})(1)] = \boxed{6.7 \times 10^3 \text{ m/s}}$ .  
 The maximum value of  $v$  occurs for the minimum value of  $\sin \phi$ .  
 $v_{\max} = E/B \sin 0^\circ = \boxed{\infty}$ .



41. The force of attraction between the two charges provides the centripetal acceleration:

$$F_0 = mR\omega_0^2.$$

A small uniform magnetic field perpendicular to the plane of the orbit, and thus the velocity, will add an additional radial force, with a magnitude

$$F_M = qvB = qR\omega B, \text{ where } \omega = \omega_0 + d\omega.$$

If the direction is such that this force is toward the center, we have

$$F_0 + F_M = mR\omega^2;$$

$$mR\omega_0^2 + qR(\omega_0 + d\omega)B = mR(\omega_0 + d\omega)^2.$$

If we can neglect the  $B d\omega$  term, we get

$$mR\omega_0^2 + qRB\omega_0 = mR(\omega_0 + d\omega)^2 = mR\omega_0^2 + 2mR\omega_0 d\omega + mR(d\omega)^2$$

$$qRB\omega_0 = 2mR\omega_0 d\omega,$$

which gives  $d\omega = qB/2m$ .

42. The magnetic force exerted on a current-carrying wire is  $\vec{F} = I\vec{L} \times \vec{B}$ .

When the wire is placed along the  $x$ -axis

$$-(0.02 \text{ N})\hat{j} = (1.3 \text{ A})[(0.12 \text{ m})\hat{i}] \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = (0.156 \text{ A} \cdot \text{m})(B_y\hat{k} - B_z\hat{j});$$

which gives  $B_y = 0$  and  $B_z = 0.13 \text{ T}$ .

When the wire is placed along the  $x$ -axis

$$-(0.02 \text{ N})\hat{j} = (1.3 \text{ A})[(0.12 \text{ m})\hat{i}] \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = (0.156 \text{ A} \cdot \text{m})(B_y\hat{k} - B_z\hat{j}); \text{ which gives } B_y = 0 \text{ and } B_z = 0.13 \text{ T}.$$

When the wire is placed along the  $y$ -axis

$$(0.02 \text{ N})\hat{i} = (1.3 \text{ A})[(0.12 \text{ m})\hat{j}] \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = (0.156 \text{ A} \cdot \text{m})(-B_x\hat{k} + B_z\hat{i});$$

which gives  $B_x = 0$  and  $B_z = 0.13 \text{ T}$ . Thus  $\vec{B} = \boxed{(0.13 \text{ T})\hat{k}}$ .

43. The maximum force will be produced when the wire and the magnetic field are perpendicular, so we have

$$F = ILB, \text{ or } F/L = IB$$

$$0.18 \text{ N/m} = (6 \text{ A})B, \text{ which gives } B = \boxed{0.03 \text{ T}}.$$

44. (a) Because the velocity of the electron and the magnetic field are perpendicular, we have

$$F = evB$$

$$= (1.60 \times 10^{-19} \text{ C})(1.7 \times 10^{-2} \text{ m/s})(0.50 \text{ T}) = \boxed{1.4 \times 10^{-21} \text{ N}} \text{ perpendicular to the wire and to } \vec{B}.$$

- (b) Because the current and the magnetic field are perpendicular, we have

$$F = ILB$$

$$= (6.5 \text{ A})(1 \text{ m})(0.50 \text{ T}) = \boxed{3.3 \text{ N}} \text{ perpendicular to the wire and to } \vec{B}.$$

45. From  $\vec{F} = I\vec{L} \times \vec{B}$ , we see that the force on the wire is produced by the component of the magnetic field perpendicular to the wire:

$$F = ILB \sin 60^\circ$$

$$= (10 \times 10^{-3} \text{ A})(0.10 \text{ m})(10^{-6} \text{ T}) \sin 60^\circ$$

$$= \boxed{8.7 \times 10^{-10} \text{ N}} \text{ perpendicular to the wire and to the magnetic field}.$$

46. Because the current and the magnetic field are perpendicular, we have

$$F = ILB$$

$$= (10^4 \text{ A})(20 \text{ m})(0.5 \times 10^{-4} \text{ T}) = \boxed{10 \text{ N horizontal and perpendicular to the magnetic field}}.$$

47. From  $\vec{F} = I\vec{L} \times \vec{B}$ , we see that the force on the wire is produced by the component of the magnetic field perpendicular to the wire:

$$F = ILB \sin 30^\circ$$

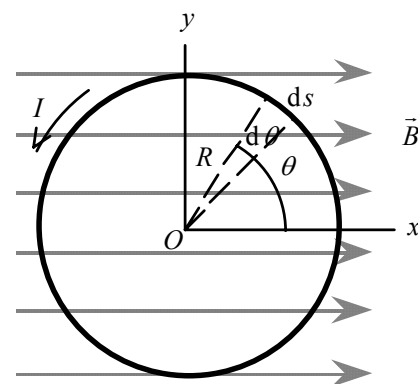
$$= (10 \text{ A})(1.0 \text{ m})(0.010 \text{ T}) \sin 30^\circ = \boxed{5.0 \times 10^{-2} \text{ N}} \text{ perpendicular to the wire and to the magnetic field}.$$

48. The element has length  $ds = R d\theta$  and makes an angle of  $\theta$  from the  $y$ -axis toward the  $-x$ -axis. The force on the element is

$$d\vec{F} = I d\vec{s} \times \vec{B} = IR d\theta (-\sin \theta \hat{i} + \cos \theta \hat{j}) \times (B\hat{i}),$$

which gives

$$\boxed{dF = -IRB \cos(\theta) d\theta}.$$



49. From  $\vec{F} = I\vec{L} \times \vec{B}$ , we see that the force on the wire produced by the magnetic field will be down, so the wire will move down. At the new equilibrium position, the magnetic force will be balanced by the increased elastic forces of the springs:

$$F_B = F_{\text{elastic}};$$

$$ILB = 2k \Delta y, \text{ which gives } \Delta y = \boxed{ILB/2k}.$$

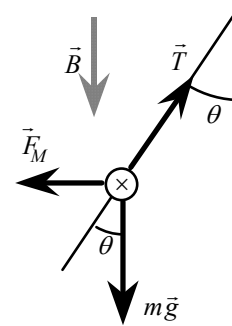
50. Because the magnetic field is perpendicular to the wire, the magnetic force is horizontal. If we look along the support axis, we have the three forces shown in the diagram.

In the equilibrium condition, the resultant force is 0, so we have

$$\tan \theta = F_M/mg = ILB/mg$$

$$= (0.55 \text{ A})(0.8 \text{ m})(0.03 \text{ T})/[(0.070 \text{ kg})(9.8 \text{ m/s}^2)] = 0.017, \text{ so}$$

$$\theta = \boxed{1^\circ}.$$



51. The forces on the two vertical segments of the loop that are in the magnetic field will be horizontal and away from the loop, so their net force is zero. The force on the horizontal segment of the loop in the magnetic field is directed down. At the equilibrium position, the magnetic force will be balanced by the elastic force of the spring:

$$F_B = F_{\text{elastic}};$$

$$I\vec{L} \times \vec{B} = k \Delta y, \text{ which gives}$$

$$B = k \Delta y / I\vec{L} = (5 \times 10^{-2} \text{ N/m})(0.6 \times 10^{-2} \text{ m})/[(100 \times 10^{-3} \text{ A})(1.2 \times 10^{-2} \text{ m})] = \boxed{0.25 \text{ T}}.$$

One way to measure a variable field is to use a variable current and measure  $I$  as a function of  $y$ .

52. The torque on a coil is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \text{ where } \vec{\mu} = IN\vec{A}.$$

The maximum torque occurs when  $\vec{\mu}$  and  $\vec{B}$  are perpendicular:

$$\tau_{\text{max}} = INAB;$$

$$2 \times 10^{-2} \text{ N} \cdot \text{m} = I(180)(20 \times 10^{-4} \text{ m}^2)(0.30 \text{ T}), \text{ which gives } I = \boxed{0.19 \text{ A}}.$$

53. The magnetic dipole moment of the loop is

$$\mu = INA = (1.2 \text{ A})(60)(0.05 \text{ m})(0.03 \text{ m}) = \boxed{0.11 \text{ A} \cdot \text{m}^2 \text{ perpendicular to the loop}}.$$

From  $\vec{\tau} = \vec{\mu} \times \vec{B}$ , we have

$$\tau = \mu B \sin \theta = (0.11 \text{ A} \cdot \text{m}^2)(0.50 \text{ T}) \sin 26^\circ = \boxed{0.024 \text{ N} \cdot \text{m} \text{ parallel to the loop and perpendicular to } \vec{B}}.$$

54. We find the work required to rotate the coil from

$$W = \Delta U = (-\vec{\mu} \cdot \vec{B})_f - (-\vec{\mu} \cdot \vec{B})_i$$

$$= \mu B (-\cos \theta_f + \cos \theta_i)$$

$$= IN\pi R^2 B (-\cos \theta_f + \cos \theta_i) = IN\pi R^2 B (\cos \theta_i - \cos \theta_f)$$

$$= (0.050 \text{ A})(1500)(3.14)(1.25 \times 10^{-2} \text{ m})^2(0.75 \text{ T})(\cos 50^\circ - \cos 230^\circ) = \boxed{3.5 \times 10^{-2} \text{ J}}.$$

55. We find the work required to rotate the coil from

$$W = \Delta U = (-\vec{\mu} \cdot \vec{B})_f - (-\vec{\mu} \cdot \vec{B})_i$$

$$= \mu B (-\cos \theta_f + \cos \theta_i)$$

$$= IN\pi R^2 B (-\cos \theta + \cos 0^\circ) = \boxed{IN\pi R^2 B (1 - \cos \theta)}.$$

56. The loop will oscillate with decreasing amplitude until the magnetic dipole moment is aligned with the magnetic field. The dissipated energy equals the loss in potential energy:

$$U_{\text{dissipated}} = -\Delta U = -[(-\vec{\mu} \cdot \vec{B})_f - (-\vec{\mu} \cdot \vec{B})_i] = \mu B - 0 = IAB \\ = (5.0 \text{ A})(3.0 \times 10^{-4} \text{ m}^2)(0.25 \text{ T}) = \boxed{3.8 \times 10^{-4} \text{ J}}.$$

57. The potential energy is

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta.$$

Because the extreme values of  $\cos \theta$  are  $\pm 1$ , the extreme values of the potential energy are

$$U = \pm \mu B = \pm (10^{-22} \text{ J/T})(10 \text{ T}) = \pm 10^{-22} \text{ J}.$$

The range of potential energies is  $\boxed{-10^{-22} \text{ J} \leq U \leq +10^{-22} \text{ J}}.$

58. We find the current in each side of the loop. Because the electric potential across each side of the loop is the same,

$$I_1 R_1 = I_2 R_2.$$

From the conservation of current at the junction, we have

$$I = I_1 + I_2.$$

Combining these two equations, we get

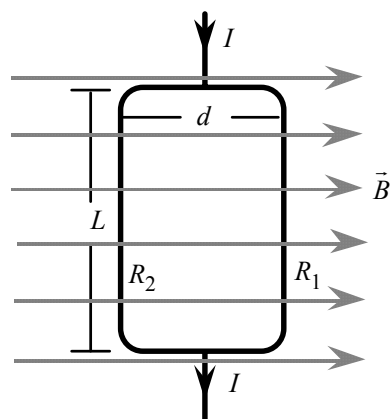
$$I_1 = [R_2 / (R_1 + R_2)]I \quad \text{and} \quad I_2 = [R_1 / (R_1 + R_2)]I.$$

The force on each side of the loop produced by the magnetic field is directed out of the paper. Because the current and magnetic field are perpendicular, we have

$$F_1 = I_1 L B \quad \text{and} \quad F_2 = I_2 L B.$$

Because the moment arm for each force is  $d$ , the net torque in the direction of the current is

$$\tau = (F_1 d) - (F_2 d) \\ = (I_1 - I_2) L B d = \boxed{[(R_2 - R_1) / (R_2 + R_1)] I L B d \text{ in the direction of the current}}.$$



59. (a) We find the current from the maximum torque:

$$\tau_{\text{max}} = \mu B = I N A B;$$

$$3 \times 10^{-5} \text{ N} \cdot \text{m} = I(50)(6 \times 10^{-4} \text{ m}^2)(0.2 \text{ T}), \text{ which gives } I = 5.0 \times 10^{-3} \text{ A} = \boxed{5.0 \text{ mA}}.$$

- (b) We find the work required to rotate the coil from

$$W = \Delta U = (-\vec{\mu} \cdot \vec{B})_f - (-\vec{\mu} \cdot \vec{B})_i = \mu B(-\cos \theta_f + \cos \theta_i).$$

For a rotation of  $180^\circ$ , we have

$$W = \mu B[-\cos(180^\circ + \theta_i) + \cos \theta_i] = 2\mu B \cos \theta_i = 2\tau_{\text{max}} \cos \theta_i = \boxed{6 \times 10^{-5} \cos \theta_i \text{ J}}.$$

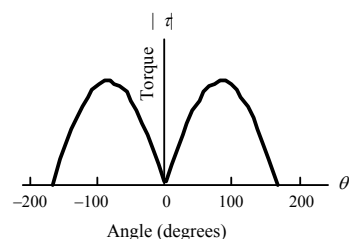
The work does depend on the initial angle.

60. The torque produced by the magnetic field is  $\vec{\tau} = \vec{\mu} \times \vec{B}$ , so its magnitude is

$$\tau = \mu B \sin \theta.$$

At  $\theta = -180^\circ$ , the torque is zero. It will increase to  $\mu B$  at  $\theta = 90^\circ$  and then decrease to zero at  $\theta = 0^\circ$ . At this point, the commutator changes the direction of  $\vec{\mu}$ . The new effective angle between  $\vec{\mu}$  and  $\vec{B}$  is  $-180^\circ$ , so the torque during the next half turn will be the same as during the first half turn. We find the average value of  $\tau$  during a half turn from

$$\tau_{\text{average}} = \frac{\int_0^\pi \tau d\theta}{\int_0^\pi d\theta} = \frac{\mu B \int_0^\pi \sin \theta d\theta}{\pi} = \frac{\mu B}{\pi} (-\cos \theta) \Big|_0^\pi = \frac{2\mu B}{\pi}.$$



Using the given data, we get

$$\tau_{\text{average}} = 2\mu B / \pi = 2INAB / \pi = 2(6.2 \text{ A})(1)(54 \times 10^{-4} \text{ m}^2)(0.45 \text{ T}) / \pi = \boxed{9.6 \times 10^{-3} \text{ N} \cdot \text{m}}.$$

61. (a) For the magnetic moment of a charge moving in a circle with speed  $v$ , we have

$$\begin{aligned}\mu &= IA = (q/T)A = q(v/2\pi r)(\pi r^2) = \frac{1}{2}qvr; \\ 10^{-23} \text{ A} \cdot \text{m}^2 &= \frac{1}{2}(1.6 \times 10^{-19} \text{ C})v(3 \times 10^{-15} \text{ m}), \text{ which gives} \\ v &= \boxed{4 \times 10^{10} \text{ m/s}}.\end{aligned}$$

Note that this is greater than the speed of light.

- (b) When the magnetic moment is perpendicular to the magnetic field, we have

$$\tau = \mu B = (10^{-23} \text{ A} \cdot \text{m}^2)(1 \text{ T}) = \boxed{10^{-23} \text{ N} \cdot \text{m}}.$$

62. (a) For the magnetic moment of an electron moving in a circle with speed  $v$ , we have

$$\begin{aligned}\mu &= IA = (e/T)A = e(v/2\pi r)(\pi r^2) = \frac{1}{2}evr; \\ &= \frac{1}{2}(1.6 \times 10^{-19} \text{ C})(2.2 \times 10^6 \text{ m/s})(0.5 \times 10^{-10} \text{ m}) \\ &= \boxed{8.8 \times 10^{-24} \text{ A} \cdot \text{m}^2}.\end{aligned}$$

- (b) If we have one electron in a cube  $10^{-10} \text{ m}$  on a side, the number of electrons, and thus the number of dipoles, is

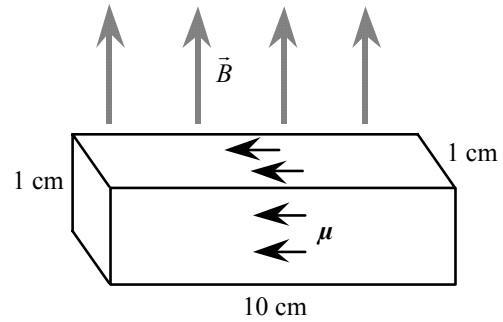
$$N = \text{volume} / \text{volume}_{\text{cube}} = AL/d^3.$$

If a fraction  $f$  of the dipoles are aligned, we have

$$\begin{aligned}\mu_{\text{net}} &= fN\mu = f(AL/d^3)\mu \\ &= f[(1 \times 10^{-4} \text{ m}^2)(0.10 \text{ m}) / (10^{-10} \text{ m})^3](8.8 \times 10^{-24} \text{ A} \cdot \text{m}^2) = \boxed{88f \text{ A} \cdot \text{m}^2}.\end{aligned}$$

- (c) The torque on the material is  $\vec{\tau} = \vec{\mu}_{\text{net}} \times \vec{B}$ . Because the magnetic moment and the field are perpendicular, we have

$$\tau = \mu_{\text{net}}B = (88f \text{ A} \cdot \text{m}^2)(10^{-3} \text{ T}) = \boxed{8.8 \times 10^{-2} f \text{ N} \cdot \text{m} \text{ perpendicular to } \vec{\mu} \text{ and } \vec{B}}.$$



63. (a) For the segment  $\alpha$ , we have

$$\vec{F}_\alpha = I\vec{L} \times \vec{B} = I(2R\hat{i}) \times (-B\hat{k}) = \boxed{2IRB\hat{j}}.$$

- (b) For the segment  $\beta$ , we choose a differential element  $d\vec{s}$  at an angle  $\theta$  from the  $x$ -axis. The force on every element will be directed toward the center of the arc, along the radius. By pairing elements symmetrically placed from the  $y$ -axis, we see that the resultant force will be along the  $-y$ -axis. The force on the element is

$$\begin{aligned}d\vec{F}_\beta &= I d\vec{s} \times \vec{B} = I(-\sin\theta ds \hat{i} + \cos\theta ds \hat{j}) \times (-B\hat{k}) \\ &= IB ds (-\sin\theta \hat{j} - \cos\theta \hat{i}).\end{aligned}$$

The resultant force is

$$\vec{F}_\beta = IB \int (-\sin\theta ds) \hat{j}.$$

We could use  $ds = R d\theta$  and perform the integration over  $\theta$ . If we recognize that  $\sin\theta ds = dx$ , we can

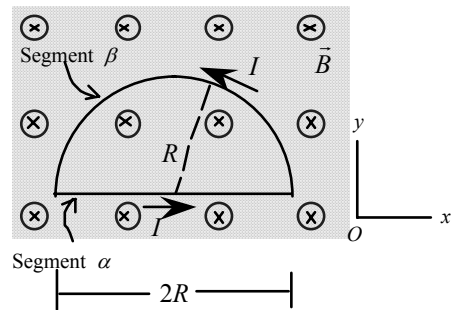
simplify the integral:

$$\vec{F}_\beta = -IB \int dx \hat{j} = -IB \Delta x \hat{j} = -2IRB\hat{j}.$$

- (c) For the net force, we have

$$\vec{F}_{\text{net}} = \vec{F}_\alpha + \vec{F}_\beta = 2IRB\hat{j} - 2IRB\hat{j} = \boxed{0}.$$

- (d) From the analysis of part (b), which did not use the shape of the wire, we see that the net force in a uniform field will be zero for a loop of any shape.







64. For small displacements from alignment, we can write the angular position of the dipole as

$$\theta = \theta_0 \cos(\omega t), \text{ from which we get}$$

$$d\theta/dt = -\theta_0 \omega \sin(\omega t).$$

The kinetic energy is

$$K = \frac{1}{2} I_M (d\theta/dt)^2 = \frac{1}{2} I_M \theta_0^2 \omega^2 \sin^2(\omega t) = \boxed{\frac{1}{2} \mu B \theta_0^2 \sin^2(\omega t)}.$$

The potential energy of the dipole is

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta.$$

For small displacements, we use the approximation  $\cos \theta \approx 1 - \frac{1}{2} \theta^2$ :

$$U \approx -\mu B (1 - \frac{1}{2} \theta^2) = \frac{1}{2} \mu B \theta_0^2 \cos^2(\omega t) - \mu B.$$

The total energy is

$$E = K + U$$

$$= \frac{1}{2} \mu B \theta_0^2 \sin^2(\omega t) + \frac{1}{2} \mu B \theta_0^2 \cos^2(\omega t) - \mu B = \frac{1}{2} \mu B \theta_0^2 - \mu B, \text{ which is constant.}$$

65. We consider  $5^\circ$  to be a small angle. The torque produced by the magnetic field will provide an angular acceleration to align the dipole with the magnetic field:

$$\tau = -\mu B \sin \theta \approx -\mu B \theta = I_M (d^2\theta/dt^2).$$

This is the equation of motion for angular harmonic motion, with angular frequency

$$\omega = (\mu B / I_M)^{1/2} = (IAB / I_M)^{1/2} \\ = [(50 \times 10^{-3} \text{ A})(6.0 \times 10^{-4} \text{ m}^2)(0.60 \text{ T}) / (7.5 \times 10^{-7} \text{ kg} \cdot \text{m}^2)]^{1/2} = 4.9 \text{ rad/s}.$$

If we choose  $t = 0$  at the moment of release, we have

$$\theta = \theta_0 \cos(\omega t), \text{ from which we get}$$

$$d\theta/dt = -\theta_0 \omega \sin(\omega t), \quad \text{and} \quad (d^2\theta/dt^2) = -\theta_0 \omega^2 \cos(\omega t) = -\omega^2 \theta.$$

The maximum angular speed of the coil is

$$(d\theta/dt)_{\max} = \theta_0 \omega = (5^\circ)[(\pi \text{ rad}) / (180^\circ)](4.9 \text{ rad/s}) = \boxed{0.43 \text{ rad/s}}.$$

66. For a length  $L$  of the metal, the number of charge carriers is  $N = nLwd_0$ . The magnetic force on the charges is balanced by the electric force:

$$ILB = nLwd_0 eE = nLwd_0 e \Delta V / w, \text{ which gives}$$

$$n = IB / (d_0 e \Delta V).$$

We use the current density to find the drift speed:

$$J = I/A = env;$$

$$I/wd_0 = e[IB / (d_0 e \Delta V)]v, \text{ which gives } v = \boxed{\Delta V / wB}.$$

- 67.** Using the result from Problem 66 for the density of carriers, we have

$$n = IB / (d_0 e \Delta V) \\ = (1.8 \text{ A})(1.2 \text{ T}) / (1.5 \times 10^{-3} \text{ m})(1.6 \times 10^{-19} \text{ C})(6.2 \times 10^{-6} \text{ V}) = \boxed{1.5 \times 10^{27} \text{ carriers/m}^3}.$$

68. The drift speed is

$$v = \Delta V / wB.$$

Using the same current means that the current densities and thus the drift speeds are the same.

When we form the ratio of the two readings, we have

$$B_2/B_1 = \Delta V_2/\Delta V_1;$$

$$B_2/(7500 \text{ G}) = (390 \text{ mV})/(165 \text{ mV}), \text{ which gives } B_2 = \boxed{1.77 \times 10^4 \text{ G} \quad (1.77 \text{ T})}.$$

69. Initially the magnetic moment is perpendicular to the field. When the coil deflects an angle  $\theta$ , the angle between  $\vec{\mu}$  and  $\vec{B}$  is  $90^\circ - \theta$ . When the coil comes to rest, the net torque is zero:

$$\mu B \sin(90^\circ - \theta) - k\theta = 0, \quad \text{or} \quad IANB \cos \theta = k\theta;$$

$$I(2 \times 10^{-4} \text{ m}^2)(500)(0.18 \text{ T}) \cos 70^\circ = (10^{-8} \text{ N} \cdot \text{m}/^\circ)(70^\circ), \text{ which gives}$$

$$I = 1.1 \times 10^{-4} \text{ A} = \boxed{0.11 \text{ mA}}$$

70. The radius of the circular motion is

$$R = mv/eB.$$

From the period of the motion,  $T = 2\pi R/v$ , we get

$$m = eBT/2\pi.$$

If the uncertainties are small, we can approximate them as differentials:

$$\begin{aligned} dm &= (eB/2\pi) dT \\ &= [(1.6 \times 10^{-19} \text{ C})(3 \text{ T})/2\pi](10^{-9} \text{ s}) = \boxed{7.6 \times 10^{-29} \text{ kg}}. \end{aligned}$$

The time for  $N$  revolutions is  $NT$ , so we have

$$dt = N dT = N(2\pi/eB) dm;$$

$$dm = [(eB/2\pi) dT]/N;$$

$$5 \times 10^{-31} \text{ kg} = (7.6 \times 10^{-29} \text{ kg})/N, \text{ which gives } N = \boxed{150 \text{ rev}}.$$

71. We find the magnitude of the electric field from

$$\begin{aligned} E &= (1/4\pi\epsilon_0)e/r^2 \\ &= (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})/(0.5 \times 10^{-10} \text{ m})^2 = \boxed{5.8 \times 10^{11} \text{ N/C}}. \end{aligned}$$

The force produced by the electric field provides the centripetal acceleration:

$$eE = mv^2/r, \text{ which gives } v = (reE/m)^{1/2}.$$

For the magnetic field to provide the same centripetal acceleration, we have

$$evB = mv^2/r = eE, \text{ from which we get}$$

$$\begin{aligned} B &= E/v = (mE/er)^{1/2} \\ &= [(9.1 \times 10^{-31} \text{ kg})(5.8 \times 10^{11} \text{ N/C})/(1.6 \times 10^{-19} \text{ C})(0.5 \times 10^{-10} \text{ m})]^{1/2} = \boxed{2.6 \times 10^5 \text{ T}}. \end{aligned}$$

72. The orbit must be in the  $xy$ -plane. Because the electric force is repulsive, the magnetic force must be toward the origin, so the net force provides the centripetal acceleration:

$$qvB - (1/4\pi\epsilon_0)qQ/r^2 = mv^2/r.$$

The angular momentum of the particle is

$$L = mvr, \text{ which gives } v = L/mr.$$

When we use this to eliminate  $v$ , we get the magnitude of the magnetic field from

$$q(L/mr)B - (1/4\pi\epsilon_0)qQ/r^2 = m(L/mr)^2/r, \text{ which gives}$$

$$\vec{B} = \boxed{(L/qr^2 + Qm/4\pi\epsilon_0 Lr) \vec{k}}.$$

73. To produce a force opposite to the electric field, the magnetic field must be directed in the  $x$ -direction.

The speed of the deuteron is

$$v = (2K/m)^{1/2}.$$

The electric field of the capacitor is

$$E = \sigma/\epsilon_0.$$

For the velocity selector, we have

$$v = E/B;$$

$$(2K/m)^{1/2} = \sigma/\epsilon_0 B;$$

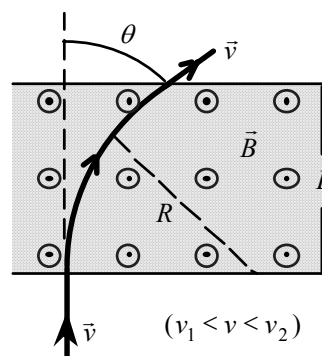
$$[2(60 \text{ keV})(1.6 \times 10^{-16} \text{ J/keV})/(3.2 \times 10^{-27} \text{ kg})]^{1/2} = (8.0 \times 10^{-7} \text{ C/m}^2)/[(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)B],$$

$$\text{which gives } B = \boxed{0.037 \text{ T}}.$$

74. The radius of the path in the magnetic field is

$$R = mv/qB.$$

Because the exit velocity is perpendicular to the radial line from the center of curvature, the exit angle is also the angle the radial line makes with the boundary of the field:



$$\sin \theta = L/R = qBL/mv.$$

Thus the range of angles is

$$\sin^{-1}(qBL/mv_2) \leq \theta \leq \sin^{-1}(qBL/mv_1).$$

75. The radius of the path in the magnetic field is

$$r = mv/eB, \text{ or } mv = eBr.$$

The kinetic energy of the electron is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(eBr)^2/m.$$

If the energy changes to  $K - \Delta K$ , the radius will change to  $r - \Delta r$ . If we form the ratio for the two energies, we have

$$(K - \Delta K)/K = [(r - \Delta r)/r]^2;$$

$$1 - \Delta K/K = (1 - \Delta r/r)^2;$$

$$1 - 0.10 = (1 - \Delta r/r)^2, \text{ which gives } \Delta r/r = 0.05 \text{ (decrease).}$$

76. (a) The radius of the path in the magnetic field is

$$r = mv/eB, \text{ and}$$

the period of the motion is

$$T = 2\pi r/v = 2\pi m/eB.$$

The angular momentum of the electron is

$$L = mvr = (mv)^2/eB = 2mK/eB.$$

If the energy changes to  $K - \Delta K$ , the angular momentum will change to  $L - \Delta L$ . If we form the ratio for the two energies, we have

$$(L - \Delta L)/L = (K - \Delta K)/K, \text{ which gives}$$

$$\Delta L/L = \Delta K/K = 0.10 \text{ (decrease).}$$

- (b) The torque exerted by the frictional forces reduces the angular momentum of the electron:

$$\tau = -\Delta L/\Delta t = -(L/\Delta t)(\Delta L/L).$$

For 20 turns,  $\Delta t = 20T$ , so we have

$$\begin{aligned} \tau &= -[(2mK/eB)/20(2\pi m/eB)](\Delta L/L) = -(K/20\pi)(\Delta L/L) \\ &= -(K/20\pi)(0.10) = -K/200\pi. \end{aligned}$$

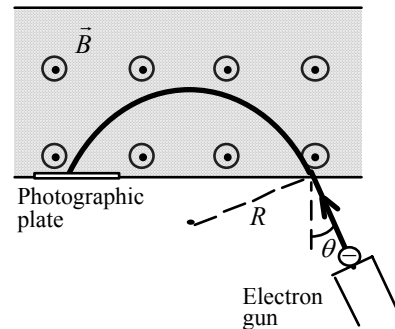
77. The speed of the electron in the magnetic field is determined by the electron gun voltage:

$$v = (2K/m)^{1/2} = (2eV/m)^{1/2}.$$

The radius of the path in the magnetic field is

$$R = mv/eB = m(2eV/m)^{1/2}/eB = (2mV/e)^{1/2}/B,$$

which gives  $e/m = 2V/B^2R^2$ .



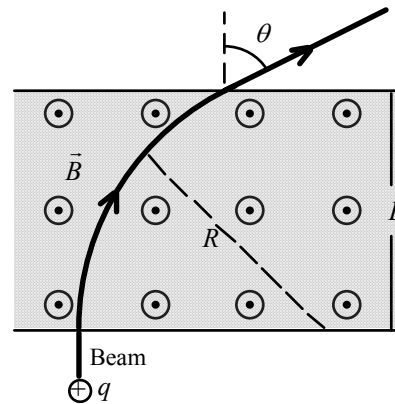
78. The speed of the particle in the magnetic field is determined by the potential difference  $V$ :

$$v = (2K/M_A)^{1/2} = (2qV/M_A)^{1/2}.$$

The radius of the path in the magnetic field is

$$R = M_A v/qB = M_A(2qV/M_A)^{1/2}/qB = (2M_A V/q)^{1/2}/B.$$

Because the exit velocity is perpendicular to the radial line from the center of curvature, the exit angle is also the angle the radial line makes with the boundary of the field. If the depth of the field is  $L$ , we have



$$\begin{aligned}\sin \theta &= L/R = LB(q/2M_A V)^{1/2}; \\ &= (0.35 \text{ m})(2.2 \text{ T})\{(1.6 \times 10^{-19} \text{ C})/ \\ &\quad [2A(1.6 \times 10^{-27} \text{ kg})(6.0 \times 10^4 \text{ V})]\}^{1/2} \\ &= 22/\sqrt{A}.\end{aligned}$$

Thus the angular deflection is  $\theta = \sin^{-1} (22/\sqrt{A})$ .

79. (a) The angular momentum of the electrons is  
 $L = NmvR$  perpendicular to the orbit.  
 (b) The period of the orbit is  $T = 2\pi R/v$ . The magnetic dipole moment is  
 $\mu = IA = (Ne/T)\pi R^2 = [Ne/(2\pi R/v)]\pi R^2 = \frac{!NevR}{2}$   
 (c) The ratio is  
 $L/\mu = NmvR / \frac{!NevR}{2} = \frac{2m}{e}$

80. The magnetic dipole moment of the loop is

$$\mu = IA = Iab \text{ perpendicular to the loop.}$$

The diagram shows the loop as viewed along the pivot axis.

At equilibrium, the net torque is zero:

$$\vec{\tau}_{\text{net}} = \vec{\mu} \times \vec{B} + \vec{b} \times (2m\vec{g}) = 0, \text{ or}$$

$$IabB \sin(90^\circ - \theta) - 2mgb \sin \theta = 0, \text{ which gives}$$

$$\theta = \tan^{-1}(IaB/2mg).$$

We choose the horizontal plane through the pivot as the reference

level for the gravitational potential energy. The total potential energy is

$$U = -\vec{\mu} \cdot \vec{B} + 2mgh = -IabB \sin \theta - 2mgb \cos \theta.$$

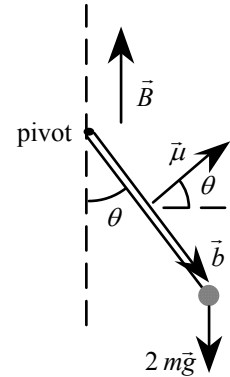
To find the angle which minimizes the potential energy, we set  $dU/d\theta = 0$ :

$$dU/d\theta = -IabB \cos \theta + 2mgb \sin \theta = 0, \text{ which gives}$$

$$\tan \theta = IaB/2mg.$$

If the current is reversed, the magnetic dipole moment reverses.

The loop will rise to the same angle on the other side of the vertical.



81. (a) The gravitational force is downward, on the page. Since the initial velocity of the particle is zero it is subject to no magnetic force at  $t = 0$ , so it starts off moving downward under the influence of gravity. With  $B$  pointing into the page the magnetic force,  $\vec{F} = q\vec{v} \times \vec{B}$ , points to the right if  $q$  is positive and to the left if it is negative. Either way, the magnetic force is also on the page. So the particle, with neither an initial velocity perpendicular to the page nor any force exerted on it that's perpendicular to the page, can only move in the plane of the page.

- (c) Set up a coordinate system with the  $+x$ -axis pointing into the page, along the magnetic field,  $y$ -axis pointing to the left on the page, and  $z$ -axis downward. The equation of motion of the particle is

$m d\vec{v}/dt = m\vec{g} + q\vec{v} \times \vec{B}$ , where  $\vec{g} = g\hat{k}$ ,  $\vec{B} = B\hat{i}$ , and  $\vec{v} = v_y\hat{i} + v_z\hat{j}$ . Plug these into the equation of motion to obtain

$$mg\hat{k} + qB(v_y\hat{i} + v_z\hat{j}) \times \hat{i} = m(dv_y/dt)\hat{j} + m(dv_z/dt)\hat{k}, \text{ or}$$

$$mg - qBv_z = m(dv_z/dt); \quad qBv_y = m(dv_y/dt). \text{ Separate the variables into two equations:}$$

$$mg - qBv_z = (m^2/qB)(d^2v_y/dt^2); \quad -(qB/m)v_z = d^2v_y/dt^2.$$

The last equation resembles that of a simple-harmonic oscillator with angular frequency

$\omega = qB/m$ , so  $v_z = C \sin \omega t$ . To determine  $C$ , note that at  $t = 0$  the acceleration in the vertical direction is  $g$ :

$$a_z = dv_z/dt = \omega C \cos \omega t = g \quad \text{at } t = 0, \text{ so } C = g/\omega = mg/qB. \text{ So}$$

$$v_z = (mg/qB) \sin \omega t. \text{ Plug this into the equation}$$

$$mg - qBv_z = m(dv_y/dt) \text{ to obtain } v_y = (mg/qB)(1 - \cos \omega t). \text{ Therefore}$$

$$\vec{v} = v_y\hat{i} + v_z\hat{j} = (mg/qB)(1 - \cos \omega t)\hat{j} + (mg/qB)\sin \omega t \hat{k}$$

$$= \left[ \frac{mg}{qB}\hat{j} + \frac{mg}{qB}(\sin \omega t \hat{k} - \cos \omega t \hat{j}) \right].$$

The first term above represents a constant velocity in the horizontal ( $y$ ) direction, while the second one is for a uniform circular motion with a speed of  $mg/qB$ . The average value of the velocity in a uniform circular motion is always zero, so the average velocity of the particle is

$\vec{v} = (mg/qB)\hat{j}$ , in the horizontal direction. This is also the direction of the average current. Note that, at this velocity, the particle is subject to zero net force, as the magnetic force cancels with

gravitational force.

82. From the first condition, we have

$$\vec{F}_1 = q(\vec{E} + \vec{v}_1 \times \vec{B}) = 0, \quad \text{or} \quad \vec{E} = -\vec{v}_1 \times \vec{B},$$

which means that  $\vec{E}$  and  $\vec{v}_1$  are perpendicular.

Because  $\vec{v}_1 = v_1 \hat{k}$ , we can write the fields as

$$\vec{E} = E_x \hat{i} + E_y \hat{j} \quad \text{and} \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}.$$

From the second condition, we have

$$\vec{F}_2 = q[\vec{E} + (\vec{v}_2 \times \vec{B})] = m\vec{a};$$

$$q[E_x \hat{i} + E_y \hat{j} + (v_2 \sin 42^\circ \hat{j} + v_2 \cos 42^\circ \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})] = ma(-\hat{i});$$

$$q[E_x \hat{i} + E_y \hat{j} - (v_2 B_x \sin 42^\circ) \hat{k} + (v_2 B_z \sin 42^\circ) \hat{i} + (v_2 B_x \cos 42^\circ) \hat{j} - (v_2 B_y \cos 42^\circ) \hat{i}] = -ma \hat{i}.$$

From the  $\hat{k}$  term, we have

$$B_x = 0, \text{ so } \vec{B} = B_y \hat{j} + B_z \hat{k}.$$

From the  $\hat{j}$  terms, we have

$$E_y = -v_2 B_x \cos 42^\circ = 0, \text{ so } \vec{E} = E \hat{i}.$$

From the  $\hat{i}$  terms, we have

$$q(E - v_2 B_z \sin 42^\circ - v_2 B_y \cos 42^\circ) = -ma.$$

For the third condition of circular motion in the  $xy$ -plane, we can write the velocity as

$$\vec{v}_3 = v_3 \cos(\omega t) \hat{i} + v_3 \sin(\omega t) \hat{j}.$$

The force must lie in the  $xy$ -plane:

$$\vec{F}_3 = q(\vec{E} + \vec{v}_3 \times \vec{B});$$

$$F_{3x} \hat{i} + F_{3y} \hat{j} = q[E \hat{i} + (v_3 \cos \omega t) \hat{i} + (v_3 \sin \omega t) \hat{j} \times (B_y \hat{j} + B_z \hat{k})];$$

$$F_{3x} \hat{i} + F_{3y} \hat{j} = q[E \hat{i} + (v_3 B_y \cos \omega t) \hat{k} - (v_3 B_z \cos \omega t) \hat{j} + (v_3 B_z \sin \omega t) \hat{i}];$$

From the  $\hat{k}$  term, we have

$$v_3 B_y \cos(\omega t) = 0, \text{ which gives } B_y = 0, \text{ so } \vec{B} = B \hat{k}.$$

If we use the result of the first condition, we get

$$\vec{E} = -\vec{v}_1 \times \vec{B} = -(v_1 \hat{k}) \times (B \hat{k}) = 0.$$

Using the result from the  $\hat{i}$  term of the second condition, we have

$$q(E - v_2 B_z \sin 42^\circ - v_2 B_y \cos 42^\circ) = q(0 - v_2 B \sin 42^\circ - 0) = -ma;$$

$$-(1.60 \times 10^{-19} \text{ C})(8.0 \times 10^2 \text{ m/s})B \sin 42^\circ = -(1.67 \times 10^{-27} \text{ kg})(3.5 \times 10^8 \text{ m/s}^2),$$

which gives  $B = 6.8 \times 10^{-3} \text{ T}$ .

Thus the values of the fields are  $\vec{E} = \boxed{0}$ ,  $\vec{B} = \boxed{(6.8 \times 10^{-3} \text{ T}) \hat{k}}$ .