

# CHAPTER 27 Direct-Current Circuits

## Answers to Understanding the Concepts Questions

1. Tap water is an excellent conductor, and if the appliance falls into the tub there is a danger that a large current will flow through the body of the person in the tub, causing burns and sometimes heart failure.
2. The voltmeter measures the terminal voltage  $V$  across the battery, and that depends on the current  $I$  drawn and the internal resistance  $r$  of the battery:  $V = \mathcal{E} - Ir$ . While  $\mathcal{E}$  and  $r$  do not change appreciably,  $I$  can. (Even  $r$  can increase, if the battery gets warmer.)
3. The current that flows into the battery is the same as the current flowing out of the battery. Whether there is a potential drop  $Ir$  just before the current reaches the battery or whether the drop occurs just after the current leaves the battery is irrelevant, since either way there will be the same contribution to the loop rule, and that is all that counts.
4. It makes sense when the resistance of the wire is negligible in comparison with those of the resistors.
5. This is impossible, since by choosing the direction in which the emf is positive, one could create a situation in which one would be creating energy. Such "perpetual motion" machines violate energy conservation.
6. The battery could be all right, of course. Keep in mind, however, that it has to provide the right terminal voltage when hooked up to a working load, not just a voltmeter with presumably a very large resistance. As the resistance of the load decreases the current flowing in the battery increases, and the terminal voltage,  $V = \mathcal{E} - Ir$ , could drop appreciably. Here  $r$  is the internal resistance of the battery.
7. An unfair question. The circuit in a flash does not give the falling exponential characteristic of a pure RC circuit. Rather it uses solid-state devices to tailor the release of energy from a capacitor, typically of size  $1000\ \mu\text{F}$ , so that a current that is basically flat for a period of about  $0.01\ \text{s}$  results. If we work backwards and ask what value of  $R$  in a pure RC circuit would give a time constant  $\tau = RC$  of  $0.01\ \text{s}$  with a capacitor or  $1000\ \mu\text{F}$ , we would find  $R = \tau/C = (0.01\ \text{s})/(10^{-3}\ \text{F}) = 10\ \Omega$ , a very reasonable value.
8. The time constant of an RC circuit is  $\tau = RC$ . To make the discharge time as short as possible we need to minimize  $\tau$ , which means we want the lowest possible value of  $C$  out of the three capacitors. This can be done by connecting the three capacitors in series.
9. There is no disaster. With the new choice of positive direction for the current,  $I$  is related to  $Q$  as  $I = -dQ/dt$ . Thus from  $IR - Q/C = 0$  we get  $-R(dQ/dt) - Q/C = 0$ , which again leads to the finite solution  $Q = Q_0 e^{-t/RC}$ .
10. Let's construct a potential energy diagram with a fluid analogy in mind. The batteries "raise" the liquid, increasing its potential energy. A resistance corresponds to a drop of potential energy given by  $IR$ . When the current goes through resistors in series, it is as if it cascaded down several downward slopes. If current goes through two resistors in parallel, it splits up so "at the bottom" the two currents are reunited at the same potential. Consider now this diagram turned upside down. The batteries (with reversed

- polarities) now lower the potential, and  $IR$  must raise them. Since  $R$  is unchanged by the reversal,  $I$  must change sign.
11. The maximum possible emf you can get out of two emf sources is the sum of the two emf values, which is obtained when you connect the two sources in series. To ensure that each light bulb gets that maximum emf we can hook up the light bulbs in parallel and apply the combined emf across each of them simultaneously. This ensures maximum power consumption for each light bulb and therefore maximum brightness -- assuming, of course, that the emf does not exceed the maximum value allowed by each light bulb (so none of them would burn out).
  12. The effective net emf is the difference between the two emf values:  $\mathcal{E}_{\text{net}} = \mathcal{E}_1 - \mathcal{E}_2$ , when they are connected + to + and - to -. The larger emf wins, of course, and the resulting current in the circuit is  $I = \mathcal{E}_{\text{net}}/R_{\text{eq}}$ .
  13. The effective emf that drives the current in the circuit is now  $\mathcal{E}_{\text{net}} = \mathcal{E}_1 + \mathcal{E}_2$ , and the magnitude of the current increases.
  14. Technically speaking this is certainly true. However, it is useless information. The value of the current itself depends on the rest of the circuit as well as on the value of the internal resistance, and so does the value of the "shifted" emf. This is not a very useful way to think about a circuit. In contrast, the original emf is a constant which at least for an ideal battery does not vary with current.
  15. In the circuit diagram depicted in Fig. 27-8(a),  $R_2$  and  $R_3$  are in series, as are  $R_5$  and  $R_6$ , and the two branches are in parallel with  $R_4$ . The equivalent resistance of this three-branch combination is  $R = [1/(R_2 + R_3) + 1/R_4 + 1/(R_5 + R_6)]^{-1}$ . Put this in series with  $R_1$  and we get a single-loop circuit. In general, such reduction is not possible for the circuit diagram depicted in Fig. 27-8(b), unless  $R_2/R_3 = R_5/R_6$ , in which case the voltage difference across  $R_4$  is zero so that it can be removed from the circuit.
  16. The teenagers provide a path for the current parallel to the wire. If the wire has no resistor along it, the resistance is low compared to that of the teenager, and most of the current flows through the wire. With the resistor, more of the current flows through the teenager, with more serious consequences. They are both dumb, but the one with the resistor is dumber.
  17. The steady-state value of  $I_1$  decreases as  $R_3$  is increased.
  18. When the lights are on, a current runs through the battery that powers the lights. Heat is generated as the current flows through the battery due to its internal resistance, and the battery warms up.

**Solutions to Problems**

1. Because the internal resistance of the battery is the only resistance in the single-loop circuit, we have

$$I = \mathcal{E}/r;$$

$$80 \text{ A} = (12 \text{ V})/r, \text{ which gives } r = \boxed{0.15 \Omega}.$$

2. The solar panel is a source of emf, so the power output is

$$P = V_{\text{term}} I;$$

$$1200 \text{ W} = V_{\text{term}}(40 \text{ A}), \text{ which gives } V_{\text{term}} = \boxed{30 \text{ V}}.$$

3. For this single-loop circuit, we have

$$I = \mathcal{E}/(R + r);$$

$$1.99 \text{ A} = (3 \text{ V})/[(1.50 \Omega + r)], \text{ which gives } r = \boxed{0.0075 \Omega}.$$

4. The energy contained in the battery is the total energy output:

$$U = IVt = (It)V = (30 \text{ A} \cdot \text{h})(3600 \text{ s/h})(30 \text{ V}) = \boxed{3.2 \times 10^6 \text{ J}}.$$

5. For this single-loop circuit, we have

$$I = V_{\text{term}}/(R + r) = (5000 \text{ V})/(230 \Omega + 20 \Omega) = \boxed{20 \text{ A}}.$$

6. For this single-loop circuit, we have

$$I = \mathcal{E}/(R + r);$$

$$170 \times 10^{-3} \text{ A} = \mathcal{E}/(15 \Omega + 0.06 \Omega), \text{ which gives } \mathcal{E} = \boxed{2.56 \text{ V}}.$$

The terminal voltage of the battery is

$$V = \mathcal{E} - Ir = 2.56 \text{ V} - (170 \times 10^{-3} \text{ A})(0.06 \Omega) = \boxed{2.55 \text{ V}}.$$

7. The terminal voltage of the battery is

$$V = \mathcal{E} - Ir;$$

$$9.0 \text{ V} = 12 \text{ V} - (100 \text{ A})r, \text{ which gives } r = \boxed{0.030 \Omega}.$$

The power dissipated within the battery is

$$P = I^2 r = (100 \text{ A})^2(0.030 \Omega) = \boxed{300 \text{ W}}.$$

8. The terminal voltage of the battery is the voltage drop across the starter:

$$V_{\text{term}} = \mathcal{E} - Ir = IR;$$

$$8 \text{ V} = 12 \text{ V} - Ir = I(0.11 \Omega), \text{ which gives } I = 73 \text{ A, and } r = 0.05 \Omega.$$

The rate at which heat is produced is the power dissipated within the battery. For 10 s we have

$$W = I^2 r t = (73 \text{ A})^2(0.05 \Omega)(10 \text{ s})/(4.185 \text{ J/cal}) = 0.6 \times 10^3 \text{ cal} = \boxed{0.6 \text{ kcal}}.$$

This would raise the temperature of a liter of electrolyte (water) by  $\sim 0.6^\circ\text{C}$ , which may decrease the internal resistance slightly. It is more important to raise the temperature of the oil.

9. The terminal voltage of the battery is

$$V_{\text{term}} = \mathcal{E} - Ir = \mathcal{E} - I(\alpha + \beta I).$$

The power dissipated within the battery is

$$P = I^2 r = I^2(\alpha + \beta I).$$

For  $I = 1.0 \text{ A}$ , we have

$$V_1 = (12.0 \text{ V}) - (1.0 \text{ A})[0.15 \Omega + (0.018 \Omega/\text{A})(1.0 \text{ A})] = \boxed{11.8 \text{ V}}.$$

$$P_1 = (1.0 \text{ A})^2[0.15 \Omega + (0.018 \Omega/\text{A})(1.0 \text{ A})] = \boxed{0.17 \text{ W}}.$$

For  $I = 10.0 \text{ A}$ , we have

$$V_{10} = (12.0 \text{ V}) - (10.0 \text{ A})[0.15 \Omega + (0.018 \Omega/\text{A})(10.0 \text{ A})] = \boxed{8.7 \text{ V}}.$$

$$P_{10} = (10.0 \text{ A})^2[0.15 \Omega + (0.018 \Omega/\text{A})(10.0 \text{ A})] = \boxed{33 \text{ W}}.$$

10. The resistance of each bulb can be found from its rating:  $R_1 = 2.5 \text{ V}/0.5 \text{ A} = 5.0 \Omega$ , and  $R_2 = (110 \text{ V})^2/10 \text{ W} = 1.21 \text{ k}\Omega$  (as  $P_2 = V_2^2/R_2$ ). When connected in series the equivalent resistance of the two bulbs is  $R =$

$R_1 + R_2$ , and when hooked up to a power supply of  $\mathcal{E} = 110 \text{ V}$  the current through each bulb is

$I = \mathcal{E}/(R_1 + R_2) = 110 \text{ V}/(5.0 \Omega + 1.21 \text{ k}\Omega) = 0.0905 \text{ A}$ . The power consumed on each bulb is then

$$P_1 = I^2 R_1 = (0.0905 \text{ A})^2(5.0 \Omega) = \boxed{0.041 \text{ W}} \text{ and}$$

$$P_2 = I^2 R_2 = (0.0905 \text{ A})^2(1.21 \text{ k}\Omega) = \boxed{9.9 \text{ W}}.$$

11. If we go around the single loop in the direction shown, starting at point  $a$ , we have

$$\sum \Delta V = -IR_2 + \mathcal{E} - IR_1 + \mathcal{E} = 0, \text{ which gives}$$

$$I = 2\mathcal{E}/(R_1 + R_2).$$

If we go from  $a$  to  $b$ , we have

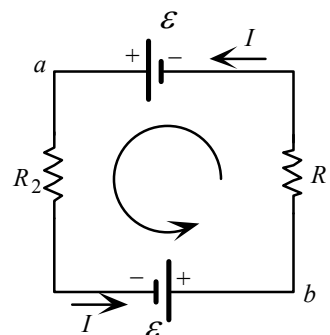
$$V_a - IR_2 + \mathcal{E} = V_b = V_a, \text{ or } \mathcal{E} = IR_2.$$

When we combine this with the expression for the current, we get

$$\mathcal{E} = 2\mathcal{E}R_2/(R_1 + R_2), \text{ which gives } \boxed{R_2 = R_1}.$$

From the expression for the current, we see that

$$I \rightarrow 0 \text{ when } \boxed{R_2 \rightarrow \infty}.$$



12. (a) Without the series resistor in the single-loop circuit, we have

$$I = (\mathcal{E}_{\text{gen}} - N\mathcal{E}_{\text{batt}})/(r_{\text{gen}} + Nr_{\text{batt}}) \\ = [(110 \text{ V}) - 20(2.2 \text{ V})]/[(0.50 \Omega) + 20(0.06 \Omega)] = 38.8 \text{ A}.$$

The terminal voltage of the generator is

$$V_a - V_b = \mathcal{E}_{\text{gen}} - Ir_{\text{gen}} = (110 \text{ V}) - (38.8 \text{ A})(0.50 \Omega) = \boxed{91 \text{ V}}.$$

- (b) The terminal voltage of the bank of batteries is  $V_d - V_c$ .

Because the batteries are being charged, this must be greater than the total emf of the bank:

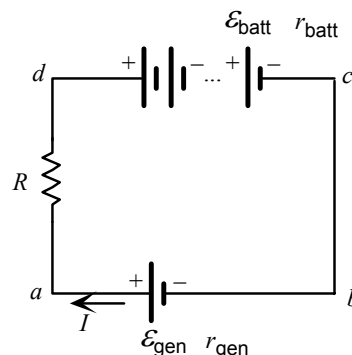
$$V_d - V_c = N(\mathcal{E}_{\text{batt}} + Ir_{\text{batt}}) \\ = 20[2.2 \text{ V} + (38.8 \text{ A})(0.06 \Omega)] = \boxed{91 \text{ V}}.$$

- (c) With the series resistor in the single-loop circuit, we have

$$I = (\mathcal{E}_{\text{gen}} - N\mathcal{E}_{\text{batt}})/(r_{\text{gen}} + Nr_{\text{batt}} + R); \\ 15 \text{ A} = [110 \text{ V} - 20(2.2 \text{ V})]/[0.50 \Omega + 20(0.06 \Omega) + R], \\ \text{which gives } R = \boxed{2.7 \Omega}.$$

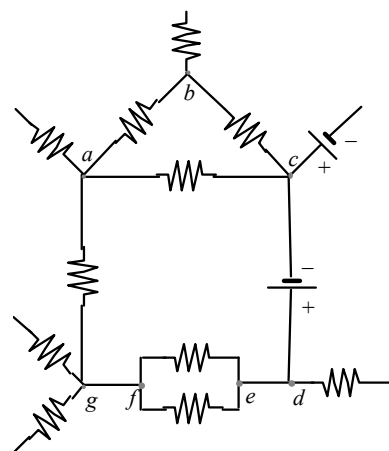
- (d) The power dissipated in all the resistors is

$$P = I^2(\sum R) = (15 \text{ A})^2[0.50 \Omega + 2.7 \Omega + 20(0.06 \Omega)] = \boxed{9.9 \times 10^2 \text{ W}}.$$



13. Because there are no resistors between points  $f$  and  $g$ , they must be at the same potential:

$$V_{gf} = \boxed{0}.$$



If we choose a path between two points, the potential difference is the sum of the potential differences of the segments of the path. Thus, we have

$$\begin{aligned}
 V_{ag} &= V_{af} = V_{ab} + V_{bc} + V_{cd} + V_{df} \\
 &= -V_{ba} - V_{cb} + V_{cd} + V_{df} \\
 &= -2\text{ V} - 3.5\text{ V} + 2\text{ V} - 0.5\text{ V} = \boxed{-4.0\text{ V}}; \\
 V_{ca} &= V_{cb} + V_{ba} \\
 &= 3.5\text{ V} + 2\text{ V} = \boxed{5.5\text{ V}}.
 \end{aligned}$$

14. (a) If we assume initially there is no internal resistance, we have

$$I_0 = 2\mathcal{E}/R = 2(1.5 \text{ V})/(10 \Omega) = 0.30 \text{ A}.$$

The power delivered to the bulb is the power dissipated in the bulb:

$$P_0 = I_0^2 R = (0.30 \text{ A})^2 (10 \Omega) = \boxed{0.90 \text{ W}}.$$

- (b) If the power delivered to the bulb, which is also the power dissipated in the bulb, decreases by one-third, we have

$$P = I^2 R;$$

$$\frac{2}{3}(0.90 \text{ W}) = I^2 (10 \Omega), \text{ which gives } I = 0.25 \text{ A}.$$

For the single-loop circuit, we have

$$I = 2\mathcal{E}/(R + 2r);$$

$$0.25 \text{ A} = 2(1.5 \text{ V})/[(10 \Omega) + 2r], \text{ which gives } r = \boxed{1.0 \Omega/\text{battery}}.$$

15. For the conservation of current, we have

$$\sum I_{\text{in}} = 0;$$

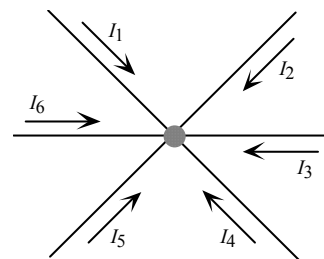
$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = 0;$$

$$(2 \text{ A}) + (0.5 \text{ A}) - (3 \text{ A}) - 0.5I_6 - I_6 + I_6 = 0,$$

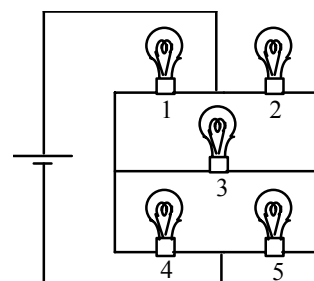
which gives  $I_6 = -1.0 \text{ A}$ .

Thus we have

$$I_4 = \boxed{+0.5 \text{ A}}, \quad I_5 = \boxed{+1.0 \text{ A}}, \quad I_6 = \boxed{-1.0 \text{ A}}.$$



16. From symmetry, the current will be the same in bulbs 1, 2, 4, and 5. Thus there will be no current through bulb 3, which will have zero brightness.



17. (a) With the switch open, the circuit consists of two branches in parallel, one with a resistance of  $R_1 = (4 \Omega + 12 \Omega) = 16 \Omega$ , and the other with  $R_2 = (8 \Omega + 6 \Omega) = 14 \Omega$ . The equivalent resistance is then  $R_{\text{eq}} = R_1 R_2 / (R_1 + R_2) = (16 \Omega)(14 \Omega) / (16 \Omega + 14 \Omega) = \boxed{7.5 \Omega}$ .

- (b) We assume the current directions shown in the diagram.

We use conservation of current at points  $a$  and  $b$ :

$$\sum I_{\text{in}} = 0;$$

$$I_1 - I_3 - I_4 = 0; \quad I_2 + I_3 - I_5 = 0.$$

We apply the loop rule for the two loops indicated in the diagram:

$$\text{loop 1: } I_1(4 \Omega) - I_2(8 \Omega) + I_3(5 \Omega) = 0;$$

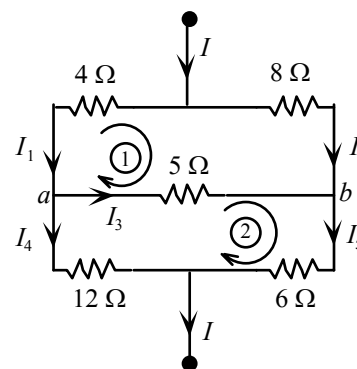
$$\text{loop 2: } -I_3(5 \Omega) - I_5(6 \Omega) + I_4(12 \Omega) = 0.$$

Also, when an emf  $\mathcal{E}$  is applied across the top and bottom of the circuit

$$\mathcal{E} = I_1(4 \Omega) + I_4(12 \Omega) = I_2(8 \Omega) + I_5(6 \Omega).$$

These equations yield  $I_1 = 0.084 \mathcal{E}/\Omega$  and  $I_2 = 0.059 \mathcal{E}/\Omega$ . Thus

$$R_{\text{eq}} = \mathcal{E} / (I_1 + I_2) = \mathcal{E} / (0.084 \mathcal{E}/\Omega + 0.059 \mathcal{E}/\Omega) = \boxed{7.0 \Omega}.$$



18. Let the current in the  $8\text{-}\Omega$  resistor be  $I_1$ , to the right, and that in the  $10\text{-}\Omega$  resistor be  $I_2$ , to the right. Then the current in the remaining two resistors is  $I_1 + I_2$ , to the left.

Apply the loop rule to the loop containing  $I_1$  and  $I_2$ :

$$-I_1(8\ \Omega) - 6\text{ V} + I_2(10\ \Omega) - 10\text{ V} = 0.$$

Now apply the loop rule to the circumference of the circuit, starting clockwise from point  $a$ :

$$-I_1(8\ \Omega) - 6\text{ V} - (I_1 + I_2)(4\ \Omega) + 20\text{ V} - (I_1 + I_2)(16\ \Omega) = 0.$$

Solve the two equations above to obtain

$$I_1 = -0.409\text{ A}, \quad I_2 = +1.27\text{ A}. \text{ The current in the } 16\text{-}\Omega \text{ resistor is then}$$

$$I_1 + I_2 = -0.409\text{ A} + 1.27\text{ A} = \boxed{+0.86\text{ A, to the left.}}$$

The voltage difference between  $a$  and  $b$  is

$$V_b - V_a = 10\text{ V} - (1.27\text{ A})(10\ \Omega) = \boxed{-2.7\text{ V}}, \text{ with } V_b < V_a.$$

19. We assume the current directions shown in the diagram.

We use conservation of current at point  $a$ :

$$\sum I_{\text{in}} = 0;$$

$$I_1 - I_2 + I_3 = 0.$$

We apply the loop rule for the two loops indicated in the diagram:

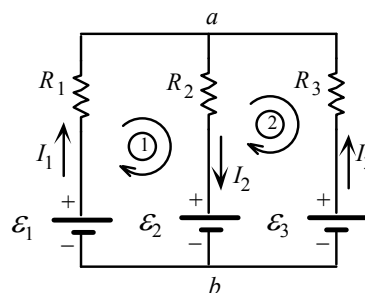
$$\text{loop 1: } -I_2 R_2 - \mathcal{E}_2 + \mathcal{E}_1 - I_1 R_1 = 0;$$

$$-I_2(10\ \Omega) - 6\text{ V} + 12\text{ V} - I_1(5\ \Omega) = 0;$$

$$\text{loop 2: } +I_3 R_3 - \mathcal{E}_3 + I_2 R_2 + \mathcal{E}_2 = 0;$$

$$+I_3(12\ \Omega) - 9\text{ V} + I_2(10\ \Omega) + 6\text{ V} = 0.$$

When we combine these equations, we get  $I_1 = \boxed{+0.45\text{ A}}$ ,  $I_2 = \boxed{+0.38\text{ A}}$ ,  $I_3 = \boxed{-0.068\text{ A}}$ .



20. We assume the current directions shown in the diagram.

We use conservation of current at point  $a$ :

$$\sum I_{\text{in}} = 0;$$

$$I_1 - I_2 - I_3 = 0.$$

We apply the loop rule for the two loops indicated in the diagram:

$$\text{loop 1: } \mathcal{E}_2 - I_2 R_2 + \mathcal{E}_1 - I_1 R_1 = 0;$$

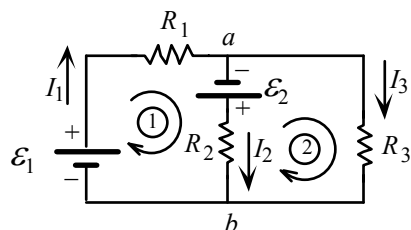
$$5\text{ V} - I_2(3\ \Omega) + 3\text{ V} - I_1(2\ \Omega) = 0;$$

$$\text{loop 2: } -I_3 R_3 + I_2 R_2 - \mathcal{E}_2 = 0;$$

$$-I_3(4\ \Omega) + I_2(3\ \Omega) - 5\text{ V} = 0.$$

When we combine these equations, we get  $I_1 = +1.577\text{ A}$ ,  $I_2 = +1.616\text{ A}$ , and

$$I_3 = \boxed{-0.039\text{ A}}. \text{ The negative sign means that the current is up.}$$



21. We combine the three resistors, which are in parallel:

$$1/R_{\text{eq}} = 1/R_1 + 1/R_2 + 1/R_3$$

$$= 1/(250\ \Omega) + 1/(420\ \Omega) + 1/(510\ \Omega), \text{ which gives } R_{\text{eq}} = 120\ \Omega.$$

The potential difference across the equivalent resistance is

$$V_{ab} = IR_{\text{eq}} = (0.020\text{ A})(120\ \Omega) = \boxed{2.4\text{ V}}.$$

Because this is the potential difference across each resistor, we have

$$I_1 = V_{ab}/R_1 = (2.4\text{ V})/(250\ \Omega) = 9.6 \times 10^{-3}\text{ A} = \boxed{9.6\text{ mA}};$$

$$I_2 = V_{ab}/R_2 = (2.4\text{ V})/(420\ \Omega) = 5.7 \times 10^{-3}\text{ A} = \boxed{5.7\text{ mA}};$$

$$I_3 = V_{ab}/R_3 = (2.4\text{ V})/(510\ \Omega) = 4.7 \times 10^{-3}\text{ A} = \boxed{4.7\text{ mA}}.$$

Note that we have conservation of current:

$$I = I_1 + I_2 + I_3 = (9.6 \times 10^{-3} \text{ A}) + (5.7 \times 10^{-3} \text{ A}) + (4.7 \times 10^{-3} \text{ A}) = 0.020 \text{ A}.$$



22. Because no two resistors have the same current and no two resistors have the same potential difference across them, we cannot combine them in series or parallel.

We assume the current directions shown in the diagram.

We use conservation of current at point  $a$ :

$$\sum I_{\text{in}} = 0;$$

$$I_1 - I_2 + I_3 = 0.$$

We apply the loop rule for the two loops indicated in the diagram:

$$\text{loop 1: } -\mathcal{E} + I_1 R_1 + I_2 R_2 = 0;$$

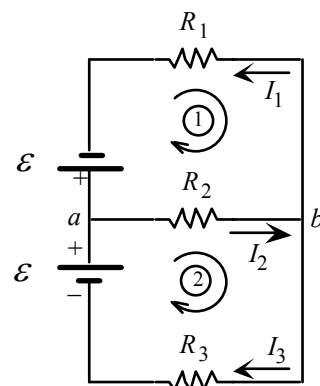
$$\text{loop 2: } -I_2 R_2 - I_3 R_3 + \mathcal{E} = 0.$$

When we combine these equations, we get

$$I_1 = R_3 \mathcal{E} / (R_1 R_2 + R_1 R_3 + R_2 R_3),$$

$$I_2 = (R_1 + R_3) \mathcal{E} / (R_1 R_2 + R_1 R_3 + R_2 R_3),$$

$$I_3 = R_1 \mathcal{E} / (R_1 R_2 + R_1 R_3 + R_2 R_3).$$



23. With identical batteries, the terminal voltage and the current through each battery are the same. When the batteries are connected in parallel, the terminal voltage is the voltage across the resistance, so we have

$$\sum I_i = N I_i = I_a;$$

$$V_{ab} = \mathcal{E} - I_i r = I_a R.$$

If we eliminate  $I_i$  from these equations, we get

$$I_a = \frac{\mathcal{E}}{[R + (r/N)]}.$$

When the batteries are connected in series, the current through each battery is the current through the resistance, so we have

$$\sum V_i = N V_i = V_{cd}; \quad (\mathcal{E} - I_b r) = I_b R, \text{ which gives}$$

$$I_b = \frac{\mathcal{E}}{[r + (R/N)]}.$$

In general,  $R$  will be much greater than  $r$ , so  $I_b$  will be greater than  $I_a$ .

24. We assume that the current going to point  $C$  is negligible. If  $I$  is the current through the resistors, which are in series, we have

$$V_{AB} = I(R_1 + R_2), \text{ and } V_{CD} = I R_2.$$

If we eliminate  $I$ , we get

$$V_{CD} = \left[ \frac{R_2}{(R_1 + R_2)} \right] V_{AB}.$$

25. We combine  $R_2$  and  $R_L$ , which are in parallel:

$$1/R = 1/R_2 + 1/R_L, \text{ which gives } R = R_2 R_L / (R_2 + R_L).$$

We now have a single-loop circuit, so the current is

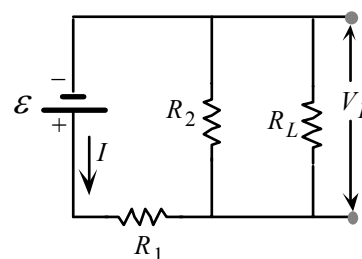
$$I = \mathcal{E} / (R_1 + R).$$

The voltage across the load is the voltage across  $R$ :

$$V_L = I R = \mathcal{E} R / (R_1 + R).$$

When we use the expression for  $R$ , we get

$$\begin{aligned} V_L &= \mathcal{E} R_2 R_L / [(R_1 R_2 + R_1 R_L + R_2 R_L)] \\ &= (10 \text{ V})(3.3 \text{ k}\Omega) R_L / [(3.3 \text{ k}\Omega)(3.3 \text{ k}\Omega) + (3.3 \text{ k}\Omega) R_L + (3.3 \text{ k}\Omega) R_L] \\ &= 33 R_L / (10.9 + 6.6 R_L) \text{ V, with } R_L \text{ in k}\Omega. \end{aligned}$$



For the given loads, we have

$$V_{20\text{ k}\Omega} = (33)(20\text{ k}\Omega)/[10.9 + 6.6(20\text{ k}\Omega)] = 4.62\text{ V}, \Delta V = \boxed{0.38\text{ V}};$$

$$V_{200\text{ k}\Omega} = (33)(200\text{ k}\Omega)/[10.9 + 6.6(200\text{ k}\Omega)] = 4.96\text{ V}, \Delta V = \boxed{0.04\text{ V}};$$

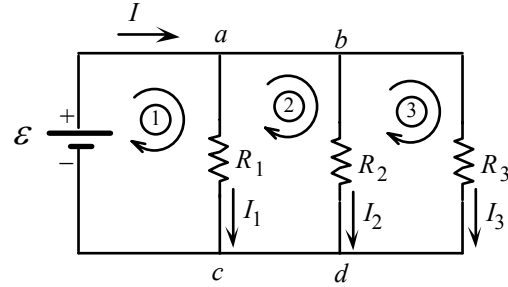
$$V_{2\text{ M}\Omega} = (33)(2 \times 10^3\text{ k}\Omega)/[10.9 + 6.6(2 \times 10^3\text{ k}\Omega)] = 4.996\text{ V}, \Delta V = \boxed{0.004\text{ V}}.$$

26. From the diagram, we have

$$I_1 = V_{ac}/R_1 = \mathcal{E}/R;$$

$$55 \times 10^{-3}\text{ A} = (2.8\text{ V})/R,$$

$$\text{which gives } R = \boxed{51\ \Omega}.$$



27. We can consider point  $a$  to be along the top and point  $b$  to be along the bottom, so the conservation of current gives

$$\sum I_{\text{in}} = 0;$$

$$\text{junction } a: I_1 - I_2 - I_3 - I_4 = 0;$$

$$\text{junction } b: I_2 + I_3 + I_4 - I_1 = 0.$$

Thus there is only one independent junction.

For the three loops indicated on the diagram, we have

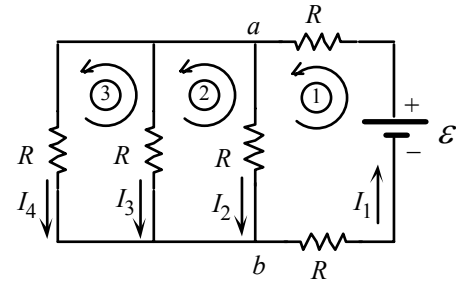
$$\text{loop 1: } -I_1R - I_2R - I_1R + \mathcal{E} = 0;$$

$$\text{loop 2: } +I_2R - I_3R = 0;$$

$$\text{loop 3: } +I_3R - I_4R = 0.$$

The solution of these four equations gives

$$I_1 = \boxed{3\mathcal{E}/7R}, I_2 = I_3 = I_4 = \boxed{\mathcal{E}/7R}.$$



28. For the conservation of current, we have

$$\text{junction } a: I - I_1 - I_2 = 0;$$

$$\text{junction } b: I_1 - I_3 - I_4 = 0;$$

$$\text{junction } d: I_4 + I_5 - I = 0.$$

For the three loops indicated on the diagram, we have

$$\text{loop 1: } -I_1R - I_4R + \mathcal{E} = 0;$$

$$\text{loop 2: } -I_2R + I_3R + I_1R = 0;$$

$$\text{loop 3: } -I_3R - I_5R + I_4R = 0.$$

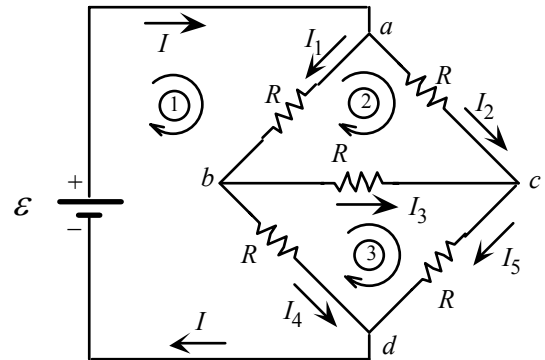
When we solve these equations, we get

$$I_3 = 0, I_1 = I_2 = I_4 = I_5 = \mathcal{E}/2R, I = \mathcal{E}/R.$$

Because the current through the battery must be

$$I = \mathcal{E}/R_{\text{eq}}, \text{ we get } \boxed{R_{\text{eq}} = R}.$$

From the symmetry of the resistance network, we know that there can be no current through the central resistor, which could be removed. Then we would have two pairs of series resistors in parallel, which gives  $R_{\text{eq}} = R$ .





29. (a) For the conservation of current at point  $a$ , we have

$$\sum I_{\text{in}} = 0;$$

$$I_1 + I_2 - I_3 = 0.$$

For the two loops indicated on the diagram, we have

$$\begin{aligned} \text{loop 1: } \mathcal{E}_2 - I_2 R_1 - I_3 R_3 - I_2 R_4 &= 0; \\ + 9 \text{ V} - I_2(100 \, \Omega) - I_3(50 \, \Omega) - I_2(200 \, \Omega) &= 0; \end{aligned}$$

$$\begin{aligned} \text{loop 2: } \mathcal{E}_1 - I_1 R_2 - I_3 R_3 - I_1 R_5 &= 0; \\ + 6 \text{ V} - I_1(150 \, \Omega) - I_3(50 \, \Omega) - I_1(250 \, \Omega) &= 0. \end{aligned}$$

When we solve these equations, we get

$$I_1 = 0.0106 \text{ A}, I_2 = 0.0242 \text{ A}, I_3 = 0.0348 \text{ A}.$$

The power dissipated in the 50- $\Omega$  resistor is

$$P_3 = I_3^2 R_3 = (0.348 \text{ A})^2(50 \, \Omega) = 0.0605 \text{ W} = \boxed{60.5 \text{ mW}}.$$

- (b) If the terminals on the 6-V battery are reversed, we have the same equations, except for the sign of  $\mathcal{E}_1$ :

$$I_1 + I_2 - I_3 = 0.$$

$$\begin{aligned} \text{loop 1: } \mathcal{E}_2 - I_2 R_1 - I_3 R_3 - I_2 R_4 &= 0; \\ + 9 \text{ V} - I_2(100 \, \Omega) - I_3(50 \, \Omega) - I_2(200 \, \Omega) &= 0; \end{aligned}$$

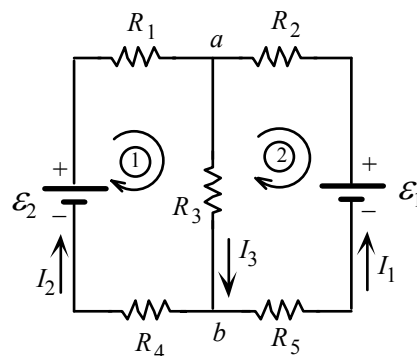
$$\begin{aligned} \text{loop 2: } -\mathcal{E}_1 - I_1 R_2 - I_3 R_3 - I_1 R_5 &= 0; \\ - 6 \text{ V} - I_1(150 \, \Omega) - I_3(50 \, \Omega) - I_1(250 \, \Omega) &= 0. \end{aligned}$$

When we solve these equations, we get

$$I_1 = -0.0165 \text{ A}, I_2 = 0.0281 \text{ A}, I_3 = 0.0116 \text{ A}.$$

The power dissipated in the 50- $\Omega$  resistor is

$$P_3 = I_3^2 R_3 = (0.0116 \text{ A})^2(50 \, \Omega) = 0.0068 \text{ W} = \boxed{6.8 \text{ mW}}.$$



30. (a) The total resistance of the circuit is  $R = 20(2 \, \Omega) + 80 \, \Omega = 120 \, \Omega$ , while the total emf is

$$\mathcal{E} = 20(12 \text{ V}) = 240 \text{ V}. \text{ The current is then}$$

$$I = \mathcal{E}/R = 240 \text{ V}/120 \, \Omega = \boxed{2.0 \text{ A}}.$$

- (b) Now the total resistance of the circuit is

$$R = (2 \, \Omega)/20 + 20 \, \Omega = 20.1 \, \Omega, \text{ while the total emf is}$$

$$\mathcal{E} = 12 \text{ V}. \text{ The current is then}$$

$$I = \mathcal{E}/R = 12 \text{ V}/20.1 \, \Omega = \boxed{0.60 \text{ A}}.$$

- (c) Now the total resistance of the circuit is

$$R = 5(2 \, \Omega)/4 + 40 \, \Omega = 42.5 \, \Omega, \text{ while the total emf is}$$

$$\mathcal{E} = 5(12 \text{ V}) = 60 \text{ V}. \text{ The current is then}$$

$$I = \mathcal{E}/R = 60 \text{ V}/42.5 \, \Omega = \boxed{1.4 \text{ A}}.$$

31. (a) We can reduce the circuit to a single loop by successively combining parallel and series combinations.

We combine  $R_3$  and  $R_4$ , which are in parallel:

$$\begin{aligned} 1/R_5 &= 1/R_3 + 1/R_4 \\ &= 1/(100\ \Omega) + 1/(50\ \Omega), \end{aligned}$$

which gives  $R_5 = 33.3\ \Omega$ .

We combine  $R_1$  and  $R_2 + R_5$ , which are in parallel:

$$\begin{aligned} 1/R_6 &= 1/R_1 + 1/(R_2 + R_5) \\ &= 1/(100\ \Omega) + 1/(20\ \Omega + 33.3\ \Omega), \end{aligned}$$

which gives  $R_6 = 34.8\ \Omega$ .

Because  $V_{ab} = 6\text{ V}$ , we find  $I_2$  from

$$I_2 = V_{ab}/(R_5 + R_2) = (6\text{ V})/(33.3\ \Omega + 0\ \Omega) = 0.113\text{ A}.$$

We can now find  $V_{cb}$  from

$$V_{cb} = I_2 R_5 = (0.113\text{ A})(33.3\ \Omega) = 3.75\text{ V}.$$

The current through the  $50\text{-}\Omega$  resistor is

$$I_4 = V_{cb}/R_4 = (3.75\text{ V})/(50\ \Omega) = \boxed{0.075\text{ A}}.$$

- (b) For the conservation of current, we have

$$\text{junction } a: I - I_1 - I_2 = 0;$$

$$\text{junction } c: I_2 - I_3 - I_4 = 0.$$

For the three loops indicated on the diagram, we have

$$\text{loop 1: } \mathcal{E} - I_1 R_1 = 0;$$

$$6\text{ V} - I_1(100\ \Omega) = 0;$$

$$\text{loop 2: } I_1 R_1 - I_2 R_2 - I_3 R_3 = 0;$$

$$I_1(100\ \Omega) - I_2(20\ \Omega) - I_3(100\ \Omega) = 0;$$

$$\text{loop 3: } -I_3 R_3 + I_4 R_4 = 0;$$

$$-I_3(100\ \Omega) + I_4(50\ \Omega) = 0.$$

When we solve these equations, we get

$$I_1 = 0.060\text{ A}, I_2 = 0.113\text{ A}, I_3 = 0.038\text{ A}, I_4 = 0.075\text{ A}.$$

Thus, the current through the  $50\text{-}\Omega$  resistor is  $\boxed{0.075\text{ A}}$ .

32. For the conservation of current at point  $a$ , we have

$$\sum I_{\text{in}} = 0;$$

$$I_1 - I_2 - I_3 = 0;$$

$$I_1 - I_2 - 0.1\text{ A} = 0.$$

For the two loops indicated on the diagram, we have

$$\text{loop 1: } \mathcal{E}_1 - I_1 R_1 - I_3 R_3 = 0;$$

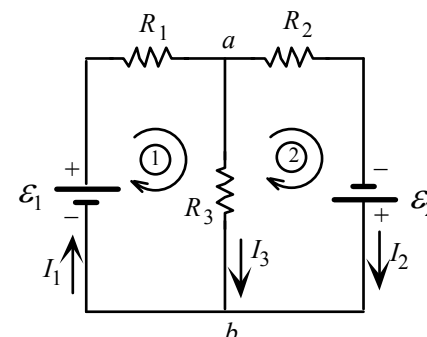
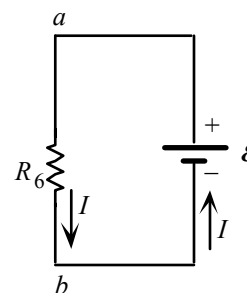
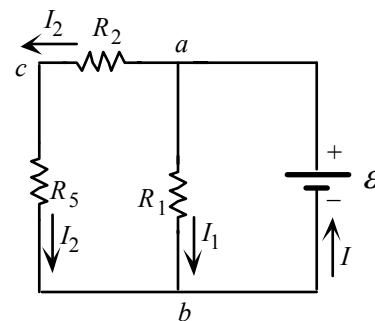
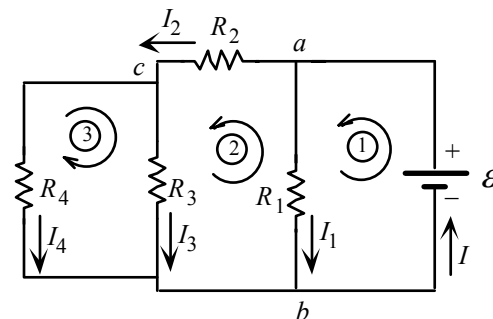
$$+3\text{ V} - I_1(5\ \Omega) - (0.1\text{ A})R_3 = 0;$$

$$\text{loop 2: } \mathcal{E}_2 + I_3 R_3 - I_2 R_2 = 0;$$

$$+6\text{ V} + (0.1\text{ A})R_3 - I_2(20\ \Omega) = 0.$$

When we solve these equations, we get

$$I_1 = 0.44\text{ A}, I_2 = 0.34\text{ A}, \text{ and } R_3 = \boxed{8\ \Omega}.$$



If  $I_3 = -0.1$  A, the equations become

$$I_1 = I_2 - 0.1 \text{ A};$$

$$+3 \text{ V} - I_1(5 \Omega) - (-0.1 \text{ A})R_3 = 0;$$

$$+6 \text{ V} + (-0.1 \text{ A})R_3 - I_2(20 \Omega) = 0.$$

When we solve these equations, we get  $I_1 = 0.28$  A,  $I_2 = 0.38$  A, and  $R_3 = -16 \Omega$ .

Because we cannot have a negative resistance, it is **not possible** to have  $I_3 = -0.1$  A.

33. For the conservation of current at point  $b$ , we have

$$\sum I_{\text{in}} = 0;$$

$$I - I_1 - I_2 = 0;$$

For the two loops indicated on the diagram, we have

$$\text{loop 1: } \mathcal{E}_1 - I_1 r_1 - IR = 0;$$

$$+12 \text{ V} - I_1(0.1 \Omega) - I(5 \Omega) = 0;$$

$$\text{loop 2: } \mathcal{E}_2 + I_2 r_2 - IR = 0;$$

$$+10 \text{ V} - I_2(10 \Omega) - I(5 \Omega) = 0.$$

When we solve these equations, we get

$$I_1 = 2.52 \text{ A}, I_2 = -0.17 \text{ A}, \text{ and } I = 2.35 \text{ A}.$$

The current through the load resistor is **2.35 A**.

$\mathcal{E}_1$  supplies **2.52 A**;  $\mathcal{E}_2$  supplies **no current**; it is being charged by  $\mathcal{E}_1$ .

34. On the diagram, we show the potential difference applied between points  $A$  and  $B$ . Because all of the resistors are the same, symmetry means that the three currents leaving point  $A$  must be the same three currents entering point  $B$ . This means that there is no current in the resistor between points  $C$  and  $D$ , which can be removed without changing the currents. When we redraw the circuit, we see that we have three parallel branches between points  $A$  and  $B$ . The currents are

$$I_1 = V_{AB}/R = (4 \text{ V})/(1 \Omega) = 4 \text{ A};$$

$$I_2 = I_3 = V_{AB}/(R + R) = (4 \text{ V})/(1 \Omega + 1 \Omega) = 2 \text{ A}.$$

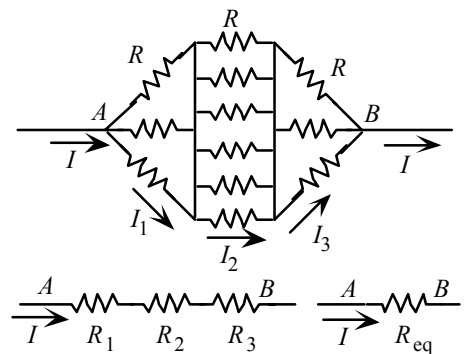
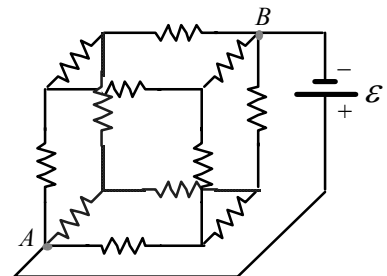
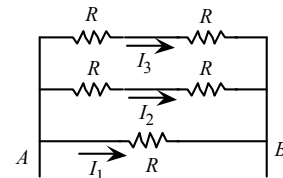
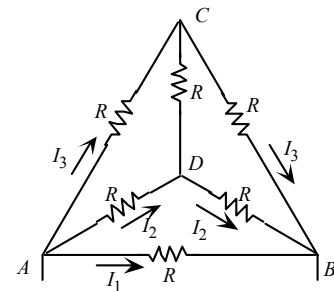
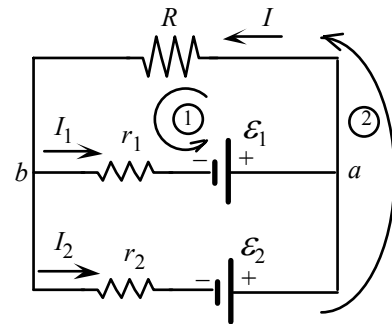
The power dissipated in each of the resistors is

$$P_{AB} = I_1^2 R = (4 \text{ A})^2(1 \Omega) = \mathbf{16 \text{ W}};$$

$$P_{CD} = \mathbf{0};$$

$$P_{\text{all others}} = I_2^2 R = (2 \text{ A})^2(1 \Omega) = \mathbf{4 \text{ W}}.$$

35. In the original configuration, there are no series or parallel combinations; however, from the symmetry of the resistors, we know that the current that goes into point  $A$  must split equally to go through the cube. The three points on the other side of the three resistors must be at the same potential, so we can connect them with a wire without changing the currents. In the same way, the other three corners of the cube must be at equal potentials, so we can connect them with a wire. From the redrawn circuit, we see that we have three parallel combinations, two of which are the same:



$$1/R_1 = 1/R_3 = (1/R) + (1/R) + (1/R),$$

which gives  $R_1 = R_3 = R/3$ ;

$$1/R_2 = (1/R) + (1/R) + (1/R) + (1/R) + (1/R) + (1/R),$$

which gives  $R_2 = R/6$ ;

We combine these three in series to get

$$R_{eq} = (R/3) + (R/6) + (R/3) = \boxed{5R/6}.$$

The current in the equivalent resistor is

$$I = V/(5R/6) = 6V/5R.$$

The current in a resistor connected to point A or point B is

$$I_1 = I_3 = I/3 = \boxed{2V/5R}.$$

The current in each of the other resistors is

$$I_2 = I/6 = \boxed{V/5R}.$$

36. (a) For  $n = 1$ , we have two resistors in series:

$$R_1 = R + R = \boxed{2R}.$$

- (b) For  $n = 2$ , we have a resistor in series with a parallel combination of a resistor and resistance  $R_1$ :

$$\begin{aligned} R_2 &= R + R_1 R / (R_1 + R) \\ &= R + 2RR / (2R + R) = \boxed{5R/3}. \end{aligned}$$

- (c) For  $n = 3$ , we have a resistor in series with a parallel combination of a resistor and resistance  $R_2$ :

$$\begin{aligned} R_3 &= R + R_2 R / (R_2 + R) \\ &= R + (5RR/3) / (5R/3 + R) = \boxed{13R/8}. \end{aligned}$$

- (d) For  $n$  rungs, we have a resistor in series with a parallel combination of a resistor and resistance  $R_{n-1}$ :

$$R_n = R + R_{n-1} R / (R_{n-1} + R).$$

In the limit of  $n \rightarrow \infty$ , we have

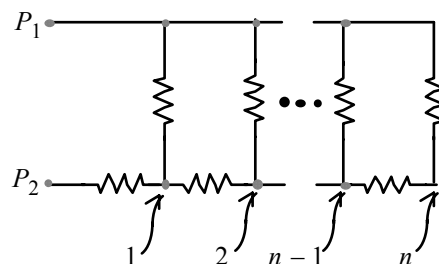
$$R_n = R_{n-1} = R_{eq}, \text{ which gives}$$

$$R_{eq} = R + R_{eq} R / (R_{eq} + R), \text{ which reduces to } R_{eq}^2 - RR_{eq} - R^2 = 0.$$

The solutions to this quadratic equation are

$$R_{eq} = \frac{1}{2}(1 \pm \sqrt{5})R.$$

Because the resistance cannot be negative, we have  $R_{eq} = \frac{1}{2}(1 + \sqrt{5})R = \boxed{1.618R}$ .



37. The equivalent resistance of the circuit is  $R = R_1 R_V / (R_V + R_1) + R_2$ , where  $R_V$  is the internal resistance of the voltmeter. The current in the circuit is then

$$I = \mathcal{E} / R = \mathcal{E} / [R_1 R_V / (R_V + R_1) + R_2].$$

The voltage across  $R_2$  is  $V_2 = IR_2$ , so the voltage across  $R_1$  is

$$V_1 = \mathcal{E} - V_2 = \mathcal{E} - IR_2 = \mathcal{E} - \mathcal{E} R_2 / [R_1 R_V / (R_V + R_1) + R_2].$$

Plug in  $\mathcal{E} = 6 \text{ V}$ ,  $R_1 = 1400 \Omega$ ,  $R_2 = 10 \text{ k}\Omega$ , and  $R_V = 200 \text{ k}\Omega$  to obtain

$$V_1 = \boxed{0.732 \text{ V}} \quad (\text{for } R_V = 200 \text{ k}\Omega).$$

If  $R_V$  is changed to  $10 \text{ M}\Omega$ , then from the formula above we get

$$V_1 = \boxed{0.737 \text{ V}} \quad (\text{for } R_V = 10 \text{ M}\Omega).$$

38. We have a single-loop circuit of the battery and the voltmeter. Because the terminal voltage of the battery is the voltage across the voltmeter, we have

$$V_{\text{terminal}} = IR_V;$$

$$1.45\text{ V} = I(60 \times 10^3 \Omega), \text{ which gives } I = 2.4 \times 10^{-5} \text{ A}.$$

For the battery, we have

$$V_{\text{terminal}} = \mathcal{E} - Ir;$$

$$1.45\text{ V} = 1.5\text{ V} - (2.4 \times 10^{-5} \text{ A})r, \text{ which gives } r = \boxed{2.1 \text{ k}\Omega}.$$

39. Without the ammeter in the circuit, we have

$$I = \mathcal{E}/R.$$

With the ammeter in the circuit, we have

$$I' = \mathcal{E}/(R_A + R).$$

When we combine these two equations, we get

$$I'/I = R/(R + R_A) = 1/(1 + R_A/R) \approx 1 - (R_A/R), \text{ if } R_A/R \ll 1.$$

To get the desired accuracy, we want  $I'/I \geq 0.999$ , or  $R_A/R \leq 0.001$ .

The maximum allowable value of  $R_A$  is determined by the smallest value of  $R$ :

$$R_A = 0.001(10 \Omega) = \boxed{0.010 \Omega}.$$

40. The voltmeter is placed in parallel with the resistor, so the equivalent resistance is

$$1/R_{\text{eq}} = 1/R + 1/R_V, \text{ which becomes}$$

$$R_{\text{eq}}/R = R_V/(R + R_V) = 1/(1 + R/R_V) \approx 1 - R/R_V, \text{ if } R/R_V \ll 1.$$

To get the desired accuracy, we want  $R_{\text{eq}}/R \geq 0.999$ , or  $R/R_V \leq 0.001$ .

The minimum allowable value of  $R_V$  is determined by the largest value of  $R$ :

$$R_V = R/0.001 = (5 \times 10^3 \Omega)/0.001 = \boxed{5 \times 10^6 \Omega}.$$

41. We find the equivalent resistance for  $R$  and  $R_V$ , which are in parallel, from

$$1/R_{\text{eq}} = 1/R + 1/R_V, \text{ which gives } R_{\text{eq}} = RR_V/(R + R_V).$$

$$(a) \quad R_{\text{eq}} = (10 \Omega)(10^5 \Omega)/(10 \Omega + 10^5 \Omega) \approx \boxed{10 \Omega}.$$

$$(b) \quad R_{\text{eq}} = (10^5 \Omega)(10^5 \Omega)/(10^5 \Omega + 10^5 \Omega) = \boxed{5 \times 10^4 \Omega}.$$

$$(c) \quad R_{\text{eq}} = (100 \times 10^6 \Omega)(10^5 \Omega)/(100 \times 10^6 \Omega + 10^5 \Omega) \approx \boxed{10^5 \Omega}.$$

The equivalent resistance has the value of the resistor when  $R_V \gg R$ .

42. The resistance of the voltmeter is  $R_V = R + R_A$ . The maximum current through the voltmeter is the maximum current through the ammeter, so we have

$$V_{\text{max}} = I_{\text{max}}(R + R_A);$$

$$3\text{ V} = (5 \times 10^{-3} \text{ A})(R + 1.8 \times 10^{-4} \Omega), \text{ which gives}$$

$$R = \boxed{0.6 \text{ k}\Omega}.$$

43. The equivalent resistance of the circuit is  $R + r$ , where we assumed that the internal resistance of the voltmeter is nearly infinity. The current in the circuit is then

$$I = \mathcal{E}/(R + r).$$

The voltage across  $R$ , i.e., the reading of the voltmeter, is then

$$V = \mathcal{E} - Ir = \mathcal{E} - \mathcal{E}r/(R + r) = \mathcal{E}R/(R + r).$$

For  $R = 20 \Omega$  we have  $V = 23\text{ V}$ , and for  $R = 5 \Omega$  we have  $V = 16\text{ V}$ ; so

$$23\text{ V} = \mathcal{E}(20 \Omega)/(20 \Omega + r);$$

$$16\text{ V} = \mathcal{E}(5 \Omega)/(5 \Omega + r).$$

Solve these two equations to obtain  $\mathcal{E} = 26.9\text{ V}$  and  $r = 3.41 \Omega$ . Thus for  $R = 50 \Omega$

$$V = \mathcal{E}R/(R + r) = (26.9\text{ V})(50 \Omega)/(50 \Omega + 3.41 \Omega) = \boxed{25\text{ V}}.$$



44. To be able to accommodate  $2 \times 10^{-3}$  A, which is 10 times the current that causes the galvanometer to fully deflect, we need to add a resistor of resistance  $R$  in parallel to it to route 90% of the current, leaving only 10% of  $2 \times 10^{-3}$  A, or  $2 \times 10^{-4}$  A, to flow through the galvanometer. If the full-deflection voltage is  $V$  then

$$V = I_G R_G = I_R R;$$

$$(2 \times 10^{-4} \text{ A})(20 \Omega) = [(90\%)(2 \times 10^{-3} \text{ A})]R; \text{ which yields}$$

$$R = 2.2 \Omega.$$

So one needs to connect a 2.2- $\Omega$  resistor in parallel with the galvanometer.

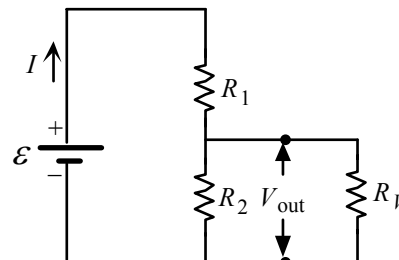
To change the full-deflection potential to  $V' = 0.2$  V, place a resistor of resistance  $R'$  in series with the galvanometer. Upon full deflection the current in both of them is  $I_G = 2 \times 10^{-4}$  A, so

$$V' = I_G (R_G + R');$$

$$0.2 \text{ V} = (2 \times 10^{-4} \text{ A})(20 \Omega + R'); \text{ which yields}$$

$$R' = 0.98 \text{ k}\Omega.$$

So one needs to connect a 0.98-k $\Omega$  resistor in series with the galvanometer.



45. Because the shunt resistor is in parallel with the galvanometer, we have

$$V_{\text{meter}} = I_G R_G = I_s R_s, \text{ which gives } R_G/R_s = I_s/I_G.$$

We use the junction at one side of the meter to find the total current through the meter:

$$I = I_G + I_s = I_G(1 + I_s/I_G) = I_G(1 + R_G/R_s).$$

46. We combine  $R_2$  and  $R_V$ , which are in parallel:

$$1/R_3 = 1/R_2 + 1/R_V, \text{ which gives } R_3 = R_2 R_V / (R_2 + R_V).$$

For the resulting single-loop circuit, we have

$$I = \mathcal{E} / (R_1 + R_3) = \mathcal{E} / [R_1 + R_2 R_V / (R_2 + R_V)].$$

The output voltage is

$$\begin{aligned} V_{\text{out}} &= IR_3 = \left\{ \mathcal{E} / [R_1 + R_2 R_V / (R_2 + R_V)] \right\} [R_2 R_V / (R_2 + R_V)] \\ &= \mathcal{E} R_2 R_V / (R_1 R_2 + R_1 R_V + R_2 R_V) \\ &= \mathcal{E} / (1 + R_1/R_2 + R_1/R_V). \end{aligned}$$

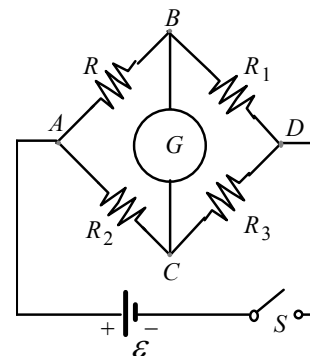
For the two voltmeters, we have

$$R_V = 500 \text{ k}\Omega:$$

$$V_{\text{out}} = (1200 \text{ V}) / [1 + (30 \text{ k}\Omega) / (50 \text{ k}\Omega) + (30 \text{ k}\Omega) / (500 \text{ k}\Omega)] = \boxed{723 \text{ V}}.$$

$$R_V = 100 \text{ M}\Omega:$$

$$V_{\text{out}} = (1200 \text{ V}) / [1 + (30 \text{ k}\Omega) / (50 \text{ k}\Omega) + (30 \text{ k}\Omega) / (100 \times 10^3 \text{ k}\Omega)] = \boxed{750 \text{ V}}.$$



47. When there is no current through the galvanometer, we have  $V_{BC} = 0$ , a current  $I_1$  through  $R$  and  $R_1$ , and a current  $I_2$  through  $R_2$  and  $R_3$ . Thus we have

$$V_{AD} = I_1(R + R_1) = I_2(R_2 + R_3), \text{ and}$$

$$V_{AB} = V_{AC} \text{ or } I_1 R = I_2 R_2.$$

When we divide these two equations, we get

$$(R + R_1)/R = (R_2 + R_3)/R_2, \text{ or}$$

$$1 + (R_1/R) = 1 + (R_3/R_2), \text{ which gives } R_1/R = R_3/R_2.$$

The unknown resistance is

$$R = R_1 R_2 / R_3.$$

48. The voltage read by the voltmeter is also the voltage across  $R_x$  and the ammeter:

$$V = I(R_A + R_x), \text{ which gives}$$

$$R_x = \boxed{V/I - R_A}.$$

We will have  $R_x = V/I$  when  $R_A \ll V/I$  (when  $R_A \ll R_x$ ).

49. Because the voltmeter and  $R_x$  are in parallel, their equivalent resistance is

$$R_{eq} = R_V R_x / (R_V + R_x).$$

The voltage read by the voltmeter is also the voltage across  $R_{eq}$ :

$$V = IR_{eq} = IR_V R_x / (R_V + R_x), \text{ which gives}$$

$$R_x = \boxed{(V/I) / (1 - V/IR_V)}.$$

We will have  $R_x = V/I$  when  $V/IR_V \ll 1$ , or

$$R_V \gg V/I \text{ (when } R_V \gg R_x \text{)}.$$

50. We find the resistance from

$$\text{time constant} = RC;$$

$$5 \times 10^{-4} \text{ s} = R(16 \times 10^{-6} \text{ F}), \text{ which gives } R = \boxed{31 \Omega}.$$

51. We find the capacitance from

$$\text{time constant} = RC;$$

$$2.0 \text{ s} = (10^5 \Omega)C, \text{ which gives } C = 2.0 \times 10^{-5} \text{ F} = \boxed{20 \mu\text{F}}.$$

52. We use the definitions of  $R$  and  $C$  to find the units of  $RC$ :

$$RC = (V/I)(Q/V) = Q/I = \text{coulomb}/(\text{coulomb}/\text{second}) = \text{second}.$$

For the given data, we have

$$R_1 C_1 = (5 \times 10^6 \Omega)(30 \times 10^{-6} \text{ F}) = \boxed{150 \text{ s}};$$

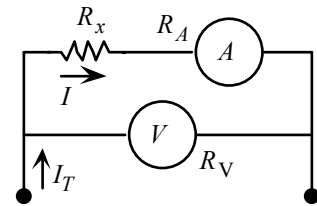
$$R_2 C_2 = (8 \times 10^3 \Omega)(3 \times 10^{-6} \text{ F}) = 24 \times 10^{-3} \text{ s} = \boxed{24 \text{ ms}};$$

$$R_3 C_3 = (20 \Omega)(50 \times 10^{-12} \text{ F}) = 1 \times 10^{-9} \text{ s} = \boxed{1 \text{ ns}}.$$

53. With the time constant as the flash time, we have

$$\text{time constant} = RC;$$

$$(1/500) \text{ s} = R(600 \times 10^{-6} \text{ F}), \text{ which gives } R = \boxed{3.3 \Omega}.$$



54. From  $Q = C\mathcal{E}(1 - e^{-t/RC})$ , we obtain

$$dQ/dt = C\mathcal{E}[-(-1/RC)e^{-t/RC}] = (\mathcal{E}/R)e^{-t/RC}.$$

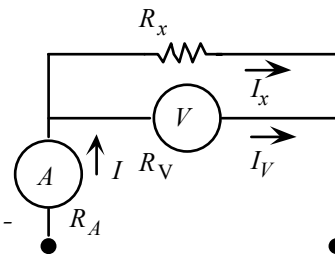
We substitute these equations into

$$\mathcal{E} - R(dQ/dt) - Q/C;$$

$$\mathcal{E} - R(\mathcal{E}/R)e^{-t/RC} - (C\mathcal{E}/C)(1 - e^{-t/RC}) = \mathcal{E} - \mathcal{E}e^{-t/RC} - \mathcal{E} + \mathcal{E}e^{-t/RC} = 0,$$

$$t/RC = 0,$$

so Eq. (27-21) is satisfied.



55. From  $Q = Q_0 e^{-t/RC}$ , we obtain

$$dQ/dt = (-Q_0/RC)e^{-t/RC}.$$

We substitute these equations into

$$R(dQ/dt) + Q/C;$$

$$(-RQ_0/RC)e^{-t/RC} + (Q_0/C)e^{-t/RC} = 0, \text{ so Eq. (27-25) is satisfied.}$$

56. (a) The time constant is

$$RC = (3 \times 10^6 \Omega)(350 \times 10^{-6} \text{ F}) = 1050 \text{ s.}$$

(b) For the charge on the capacitor, we have

$$Q = Q_0(1 - e^{-t/RC}) = 0.90Q_0, \text{ which gives } e^{-t/RC} = 0.10.$$

For the charging current, we have

$$I = (\mathcal{E}/R)e^{-t/RC} = [(50 \text{ V})/(3 \times 10^6 \Omega)](0.10) = \boxed{1.7 \times 10^{-6} \text{ A}}.$$

57. Because there is no internal resistance in the battery, the potential difference across  $R_2$  and across the capacitor branch is  $\mathcal{E}$ . The current in  $R_2$  is constant:

$$I_2 = \mathcal{E}/R_2.$$

The charging current in the capacitor branch is

$$I_1 = (\mathcal{E}/R_1)e^{-t/R_1C}.$$

From the junction, the current in the battery is

$$I_{\text{battery}} = I_1 + I_2 = (\mathcal{E}/R_1)e^{-t/R_1C} + (\mathcal{E}/R_2).$$

58. The time constant of the circuit is

$$RC = (350 \times 10^3 \Omega)(20 \times 10^{-6} \text{ F}) = 7.0 \text{ s.}$$

The charging current in the resistor is

$$I = (\mathcal{E}/R)e^{-t/RC}, \text{ so the voltage across the resistor is}$$

$$V = IR = \mathcal{E}e^{-t/RC},$$

$$= (200 \text{ V})e^{-(4.0 \text{ s})/(7.0 \text{ s})} = \boxed{113 \text{ V}}.$$

The charge on the capacitor is

$$Q = C\mathcal{E}(1 - e^{-t/RC})$$

$$= [(20 \times 10^{-6} \text{ F})(200 \text{ V})][1 - e^{-(4.0 \text{ s})/(7.0 \text{ s})}] = 1.7 \times 10^{-3} \text{ C} = \boxed{1.7 \text{ mC}}.$$

59. The possible capacitance values that we have are

$$C_1 = C_2 = 5 \mu\text{F};$$

$$C_{\text{parallel}} = C_1 + C_2 = 5 \mu\text{F} + 5 \mu\text{F} = 10 \mu\text{F};$$

$$C_{\text{series}} = C_1 C_2 / (C_1 + C_2) = (5 \mu\text{F})(5 \mu\text{F}) / [(5 \mu\text{F}) + (5 \mu\text{F})] = 2.5 \mu\text{F}.$$

We need to combine the resistors to produce one of the following resistance values:

$$R_a = RC/C_1 = (1 \times 10^{-3} \text{ s}) / (5 \times 10^{-6} \text{ F}) = 200 \Omega;$$

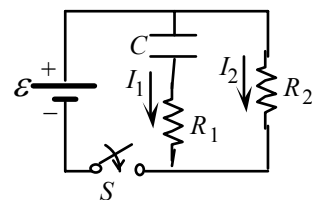
$$R_b = RC/C_{\text{parallel}} = (1 \times 10^{-3} \text{ s}) / (10 \times 10^{-6} \text{ F}) = 100 \Omega;$$

$$R_c = RC/C_{\text{series}} = (1 \times 10^{-3} \text{ s}) / (2.5 \times 10^{-6} \text{ F}) = 400 \Omega;$$

If we connect the 300- $\Omega$  resistors in parallel, we get

$$R_3 = R_2 R_2 / (R_2 + R_2)$$

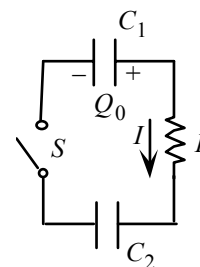
$$= (300 \Omega)(300 \Omega) / [(300 \Omega) + (300 \Omega)] = 150 \Omega.$$



We see that we can produce  $R_c$  by putting this combination in series with the 250- $\Omega$  resistor:

$$R_c = R_1 + R_3 = 250\ \Omega + 150\ \Omega = 400\ \Omega.$$

Thus we connect the 300- $\Omega$  resistors in parallel with each other and in series with the 250- $\Omega$  resistor and the two capacitors.



60. For a parallel-plate capacitor with separation  $d$ , we have

$$C = \kappa \epsilon_0 A / d.$$

For the resistance of the dielectric, we have

$$R = \rho d / A.$$

The time constant is

$$RC = (\rho d / A)(\kappa \epsilon_0 A / d) = \kappa \epsilon_0 \rho, \text{ which is independent of the area and separation.}$$

61. We use the result of Problem 62 to find the time constant:

$$RC = \kappa \epsilon_0 \rho$$

$$= (3.2)(8.85 \times 10^{-12} \text{ F/m})(2 \times 10^{14} \ \Omega \cdot \text{m}) = 5.66 \times 10^3 \text{ s.}$$

As the capacitor discharges, when 70% of the charge on the plates has leaked away, we have

$$Q = Q_0 e^{-t/RC} = 0.30 Q_0, \text{ which gives}$$

$$e^{-t/RC} = 0.30; \quad t/RC = 1.2.$$

The time for 70% of the charge to leak away is

$$t = 1.2RC = (1.2)(5.66 \times 10^3 \text{ s}) = \boxed{6.8 \times 10^3 \text{ s (1.9 h)}}.$$

62. When the switch is closed, charge will move from  $C_1$  to  $C_2$ ; with a variable current  $I$  in the single loop or series circuit:

$$I = -dQ_1/dt = dQ_2/dt.$$

With the same current in the two capacitors, they are connected in series.

We find the equivalent capacitance of the circuit from

$$1/C = 1/C_1 + 1/C_2, \text{ which gives}$$

$$C = C_2 C_1 / (C_1 + C_2).$$

The time constant of the circuit is  $RC$ .

$Q_1$ , the charge on  $C_1$ , will decrease and  $Q_2$ , the charge on  $C_2$ ,

will increase until the potential difference will be the same across both capacitors, at which point the current becomes 0.

The charges will have reached their final value, which we find from

$$V_1 = V_2; \quad \text{or}$$

$$Q_{1f}/C_1 = Q_{2f}/C_2.$$

Because there has been no loss of charge, we have  $Q_0 = Q_{1f} + Q_{2f}$ .

When we combine these two equations, we get

$$Q_{1f} = Q_0 C_1 / (C_1 + C_2), \text{ and}$$

$$Q_{2f} = Q_0 C_2 / (C_1 + C_2).$$

Capacitor  $C_2$  will charge just like a single capacitor from 0 to its final value  $Q_{2f}$ , so we have

$$Q_2 = [Q_0 C_2 / (C_1 + C_2)](1 - e^{-t/RC}).$$

At any time the total charge is conserved and equal to the initial charge  $Q_0$ . We find the charge on  $C_1$  as a function of time from

$$Q_1 = Q_0 - Q_2, \text{ which gives}$$

$$Q_1 = [Q_0 C_1 / (C_1 + C_2)] + [Q_0 C_2 / (C_1 + C_2)]e^{-t/RC}.$$

Note that at  $t = 0$ , this gives  $Q_0$  and after a long time it gives  $Q_{1f}$ .

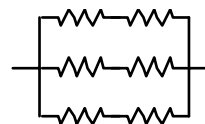
[It is also possible to solve the circuit equation, obtained by adding the potential drops around the loop:  $Q_1/C_1 - IR - Q_2/C_2 = 0$ . When the current is put in terms of the rate of change of the charge and the conservation of charge is used, a differential equation is obtained. The solution is the given equations.]

63. The power used by each appliance is  $P_i = I_i V$ , so the currents draw from the main supply are

$$I_1 = P_1 / V = (50 \text{ W}) / (120 \text{ V}) = \boxed{0.417 \text{ A}};$$

$$I_2 = P_2 / V = (60 \text{ W}) / (120 \text{ V}) = \boxed{0.500 \text{ A}};$$

$$I_3 = P_3 / V = (20 \text{ W}) / (120 \text{ V}) = \boxed{0.167 \text{ A}}.$$



64. (a) The current in the circuit will be clockwise.  
For the single loop, we have

$$I = (\mathcal{E}_1 - \mathcal{E}_2) / R$$

$$= (12 \text{ V} - 6 \text{ V}) / (20 \Omega) = \boxed{0.30 \text{ A}}.$$

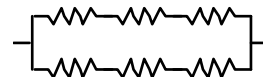
- (b) The rate at which energy is being stored in the smaller battery is

$$P = I \mathcal{E}_2 = (0.30 \text{ A})(6 \text{ V}) = \boxed{1.8 \text{ W}}.$$

- (c) The rate of energy dissipation in the resistor is

$$P = I^2 R = (0.30 \text{ A})^2 (20 \Omega) = \boxed{1.8 \text{ W}}.$$

Note that the sum of the answers to parts (b) and (c) is the rate at which energy is being provided by the larger battery, 3.6 W.



65. Because the power dissipated in the effective resistance is the sum of the powers dissipated in the individual resistors, we have

$$P_{\text{eq}} = \sum P_i = n P_i;$$

$$30 \text{ W} = n(5 \text{ W}), \text{ which gives } n = 6 \text{ resistors.}$$

If we connect the six resistors in parallel, we have

$$1/R_{\text{eq}} = \sum (1/R_i) = n/R_i;$$

$$1/(100 \Omega) = 6/R_i, \text{ which gives } R_i = 600 \Omega. \quad \boxed{\text{We can}}$$

connect six 600-Ω resistors in parallel.

If we connect the six resistors in series, we have

$$R_{\text{eq}} = \sum R_i = n R_i;$$

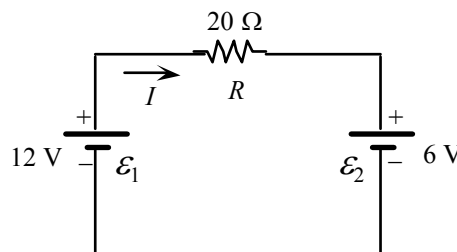
$$100 \Omega = 6 R_i, \text{ which gives } R_i = 16.7 \Omega. \quad \boxed{\text{We can connect six}}$$

16.7-Ω resistors in series.

To have the same power dissipated in each resistor requires that the currents be the same, which means there must be symmetry in the arrangement of the resistors.

If we connect two series resistors in parallel with two more sets of two series resistors, we have

$$1/R_{\text{eq}} = 1/2R_i + 1/2R_i + 1/2R_i = 3/2R_i;$$



$$1/(100\ \Omega) = 3/2R_i, \text{ which gives } R_i = 150\ \Omega.$$

We can connect two series 150- $\Omega$  resistors in parallel with two more sets of two series 150- $\Omega$  resistors.

If we connect three series resistors in parallel with a set of three series resistors, we have

$$1/R_{\text{eq}} = 1/3R_i + 1/3R_i = 2/3R_i;$$

$$1/(100\ \Omega) = 2/3R_i, \text{ which gives } R_i = 66.7\ \Omega.$$

We can connect three series 66.7- $\Omega$  resistors in parallel with three series 66.7- $\Omega$  resistors.

66. The current for this single loop is

$$I = \mathcal{E}/(R + r).$$

The power delivered to the external resistor is the power dissipated in the resistor:

$$P_{\text{ext}} = I^2 R = \mathcal{E}^2 R / (r + R)^2.$$

To find the value of  $R$  that maximizes the power, we set  $dP_{\text{ext}}/dR = 0$ :

$$dP/dR = [\mathcal{E}^2/(r + R)^2] - [2\mathcal{E}^2 R/(r + R)^3] = \mathcal{E}^2(r - R)/(r + R)^3 = 0, \text{ which gives } r = R.$$

67. (a) The short-circuit current (when there is no load) provided by the battery is

$$I = \mathcal{E}/r = 12.6\ \text{V}/0.05\ \Omega = \boxed{0.25\ \text{kA}}.$$

- (b) The voltage  $V$  across the battery terminal during recharging must overcome both  $\mathcal{E}$  and the internal resistance of the battery:

$$V = \mathcal{E} + Ir = 12.6\ \text{V} + (2.5\ \text{A})(0.05\ \Omega) = \boxed{12.7\ \text{V}}.$$

- (c) Energy stored =  $\mathcal{E}It = (12.6\ \text{V})(2.5\ \text{A})(10\ \text{h})(3600\ \text{s/h}) = \boxed{1.1 \times 10^6\ \text{J}}.$

68. If the batteries are in parallel then the equivalent resistance of the circuit is  $R + r/n$ , and the emf is  $\mathcal{E}$ . The current is

$$I_{\text{parallel}} = \mathcal{E}/(R + r/n) = n\mathcal{E}/(nR + r).$$

If the batteries are in series then the equivalent resistance of the circuit is  $R + nr$ , and the emf is  $n\mathcal{E}$ . The current is

$$I_{\text{series}} = n\mathcal{E}/(R + nr).$$

If we compare  $I_{\text{parallel}}$  with  $I_{\text{series}}$ , it is clear that  $I_{\text{parallel}} > I_{\text{series}}$  if

$$nR + r < R + nr, \text{ or } R < r.$$

Similarly, if  $R > r$  then  $I_{\text{series}} > I_{\text{parallel}}.$

Therefore, to maximize the current, put the batteries in series if  $R > r$  and in parallel if  $R < r$ .

69. For this single-loop circuit, we have

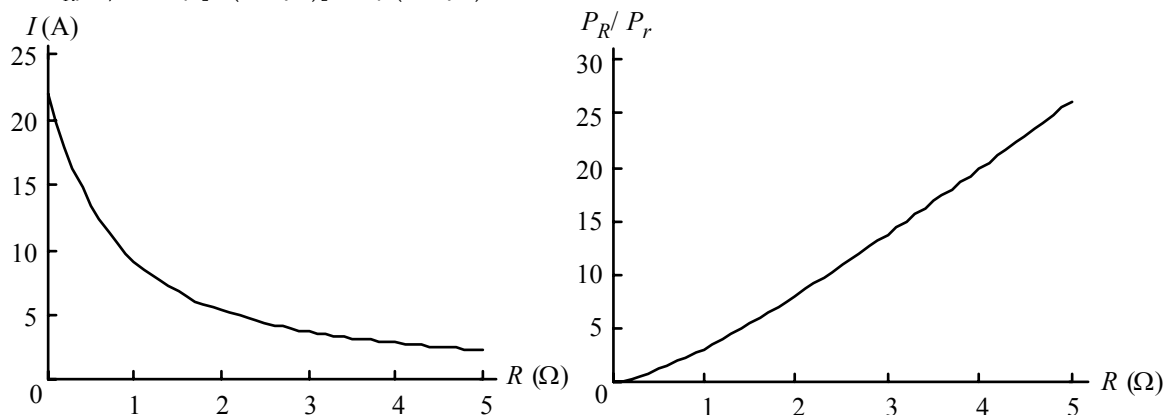
$$I = \mathcal{E}/(R + r) = \mathcal{E}/[R + (\alpha + \beta I)], \text{ which is a quadratic equation for } I: \beta I^2 + (R + \alpha)I - \mathcal{E} = 0.$$

The positive solution is

$$I = \left\{ -(R + \alpha) + [(R + \alpha)^2 + 4\beta\mathcal{E}]^{1/2} \right\} / 2\beta.$$

The ratio of power delivered to the load to the power dissipated in the battery is

$$P_R/P_r = I^2 R / [I^2(\alpha + \beta I)] = R/(\alpha + \beta I).$$



70. Using  $P = V^2/R$ , we find the two resistances:

$$R_1 = V^2/P_1 \quad \text{and} \quad R_2 = V^2/P_2.$$

The equivalent resistance for the series connection is

$$R_s = R_1 + R_2 = (V^2/P_1) + (V^2/P_2) = V^2(P_1 + P_2)/P_1P_2.$$

The power generated is

$$P_s = V^2/R_s = \boxed{P_1P_2/(P_1 + P_2)}.$$

The equivalent resistance for the parallel connection is

$$R_p = R_1R_2/(R_1 + R_2) = (V^4/P_1P_2)/(V^2/P_1 + V^2/P_2) = V^2/(P_1 + P_2).$$

The power generated is

$$P_p = V^2/R_p = V^2/[V^2/(P_1 + P_2)] = \boxed{P_1 + P_2}.$$

71. The two heating elements in the furnace are initially connected in parallel. The power dissipated in a resistor is  $P = I^2R = V^2/R$ . The resistances are

$$R_1 = V^2/P_1 = (110 \text{ V})^2/(1000 \text{ W}) = 12.1 \, \Omega.$$

$$R_2 = V^2/P_2 = (110 \text{ V})^2/(2000 \text{ W}) = 6.05 \, \Omega.$$

The power can be reduced by increasing the resistance, which means connecting them in series:

$$P = V^2/(R_1 + R_2) = (110 \text{ V})^2/(12.1 \, \Omega + 6.05 \, \Omega) = \boxed{667 \text{ W}}.$$

72. (a) For the series arrangement, we have

$$I_s = (\mathcal{E} + \mathcal{E})/(R + 2r), \text{ which gives}$$

$$I_s = \boxed{\mathcal{E}/(R + r)}.$$

(b) For the parallel arrangement, we use symmetry to see that the current in each battery is the same.

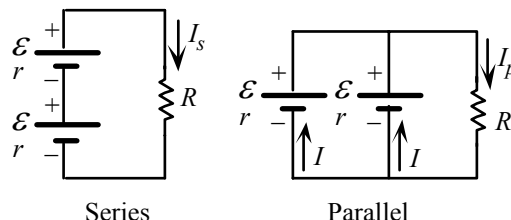
For the junction, we have

$$I_p - I - I = 0, \text{ or } I_p = 2I.$$

For one loop, we have

$$\mathcal{E} - I_p R - Ir = 0 = \mathcal{E} - I_p R - \frac{1}{2} I_p r, \text{ which gives}$$

$$I_p = \boxed{\mathcal{E}/(R + \frac{1}{2}r)}.$$



For large  $R$ :  $I_s \approx 2\mathcal{E}/R$ ,  $I_p \approx \mathcal{E}/R$ ; so  $I_s$  is larger. For small  $R$ :  $I_s \approx \mathcal{E}/r$ ,  $I_p \approx 2\mathcal{E}/r$ ; so  $I_p$  is larger.



73. The devices are connected in parallel, so we have

$$I = PV;$$

$$I_{\text{mixer}} = (800 \text{ W})/(120 \text{ V}) = \boxed{6.67 \text{ A}}.$$

$$I_{\text{vacuum}} = (600 \text{ W})/(120 \text{ V}) = \boxed{5.00 \text{ A}}.$$

$$I_{\text{chandelier}} = 10(60 \text{ W})/(120 \text{ V}) = \boxed{5.00 \text{ A}}.$$

If all three devices are used at the same time, the fuse will blow. Each bulb draws 0.50 A. To find the number of bulbs that can be used without blowing the fuse, we have

$$I_{\text{max}} = I_{\text{mixer}} + I_{\text{vacuum}} + NI_{\text{bulb}};$$

$$15 \text{ A} = 6.67 \text{ A} + 5.00 \text{ A} + N(0.50 \text{ A}), \text{ which gives } N = 6.7 \text{ bulbs. Thus } \boxed{7 \text{ bulbs}} \text{ will blow the fuse.}$$

74. We find the maximum current through a resistor from

$$P_{1\text{max}} = I_{\text{max}}^2 R;$$

$$2 \text{ W} = I_{\text{max}}^2 (30 \Omega), \text{ which gives } I_{\text{max}} = 0.26 \text{ A}.$$

When the three are connected in series, circuit A, the maximum current goes through each resistor, so we have

$$P_{A\text{max}} = 3P_{1\text{max}} = 3(2 \text{ W}) = 6 \text{ W}.$$

When the three are connected in parallel, circuit B, the maximum current goes through each resistor, so we have

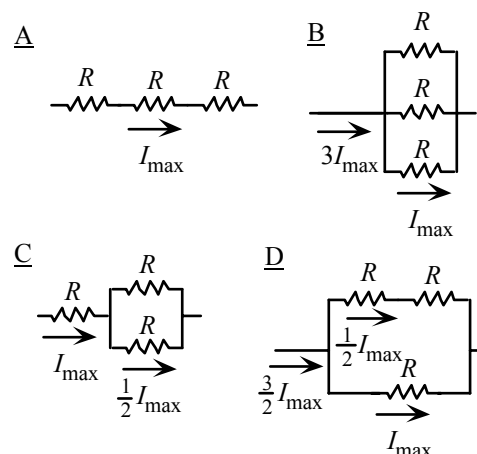
$$P_{B\text{max}} = 3P_{1\text{max}} = 3(2 \text{ W}) = \boxed{6 \text{ W}}.$$

In circuit C, the resistor in series has the maximum current. From symmetry the current in the other two resistors will be  $\frac{1}{2}I_{\text{max}}$ . The maximum power is

$$\begin{aligned} P_{C\text{max}} &= P_{1\text{max}} + 2\left(\frac{1}{2}I_{\text{max}}\right)^2 R \\ &= P_{1\text{max}} + \frac{1}{2}P_{1\text{max}} = \frac{3}{2}(2 \text{ W}) = \boxed{3 \text{ W}}. \end{aligned}$$

In circuit D, the branch with the single resistor has the maximum current. Because the total resistance in the other branch is  $2R$ , the current in the other branch will be  $\frac{1}{2}I_{\text{max}}$ . The maximum power is

$$\begin{aligned} P_{D\text{max}} &= P_{1\text{max}} + 2\left(\frac{1}{2}I_{\text{max}}\right)^2 R \\ &= P_{1\text{max}} + \frac{1}{2}P_{1\text{max}} = \frac{3}{2}(2 \text{ W}) = \boxed{3 \text{ W}}. \end{aligned}$$



75. Normally, this circuit would have six currents, one for each branch. We have used the symmetry of the circuit to reduce the number of currents to four, as shown in the diagram.

For the junction equations, we have

$$\text{junction A (or D): } I - I_1 - I_2 = 0; \quad (1)$$

$$\text{junction B (or C): } I_1 + I_3 - I_2 = 0. \quad (2)$$

For the loop equations, we have

$$\text{loop ACDA: } -I_2 R - I_1 R + \mathcal{E} = 0; \quad (3)$$

$$\text{loop BDCB: } -I_2 R + I_1 R - I_3 R = 0; \quad (4)$$

When we combine Eq. (2) and Eq. (4), we find

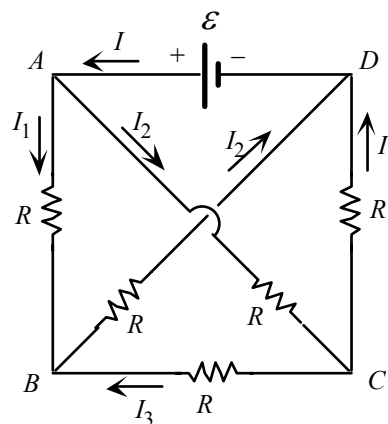
$$\boxed{I_3 = 0} \quad (\text{as suggested by symmetry}) \quad \text{and} \quad I_1 = I_2.$$

From Eq. (3), we get

$$\boxed{I_1 = I_2 = \mathcal{E}/2R}.$$

From Eq. (1), we get

$$\boxed{I = 2I_1 = \mathcal{E}/R}.$$





76. For a cylindrical wire, we have

$$R = \rho L/A, \quad I = V/R = VA/\rho L, \quad J = I/A = V/\rho L = E/\rho = V/\rho L, \quad \text{and} \quad P = I^2 R = I^2 \rho L/A.$$

The two wires have the same length and area.

(a) When the wires are in series, the currents must be the same, so we have

$$J_{\text{Al}}/J_{\text{Cu}} = I_{\text{Al}}/I_{\text{Cu}} = \boxed{1}.$$

$$E_{\text{Al}}/E_{\text{Cu}} = J_{\text{Al}}\rho_{\text{Al}}/J_{\text{Cu}}\rho_{\text{Cu}} = \rho_{\text{Al}}/\rho_{\text{Cu}} = (2.82 \times 10^{-8} \Omega \cdot \text{m})/(1.72 \times 10^{-8} \Omega \cdot \text{m}) = \boxed{1.64}.$$

$$P_{\text{Al}}/P_{\text{Cu}} = (I^2 \rho_{\text{Al}} L/A)/(I^2 \rho_{\text{Cu}} L/A) = \rho_{\text{Al}}/\rho_{\text{Cu}} = \boxed{1.64}.$$

(b) When the wires are in parallel, the potential difference is the same, so we have

$$J_{\text{Al}}/J_{\text{Cu}} = I_{\text{Al}}/I_{\text{Cu}} = (VA/\rho_{\text{Al}}L)/(VA/\rho_{\text{Cu}}L) = \rho_{\text{Cu}}/\rho_{\text{Al}} = \boxed{0.61}.$$

$$E_{\text{Al}}/E_{\text{Cu}} = J_{\text{Al}}\rho_{\text{Al}}/J_{\text{Cu}}\rho_{\text{Cu}} = (J_{\text{Al}}/J_{\text{Cu}})(\rho_{\text{Al}}/\rho_{\text{Cu}}) = \boxed{1}.$$

$$P_{\text{Al}}/P_{\text{Cu}} = (I_{\text{Al}}^2 \rho_{\text{Al}} L/A)/(I_{\text{Cu}}^2 \rho_{\text{Cu}} L/A) \\ = (I_{\text{Al}}/I_{\text{Cu}})^2 (\rho_{\text{Al}}/\rho_{\text{Cu}}) = (\rho_{\text{Cu}}/\rho_{\text{Al}})^2 (\rho_{\text{Al}}/\rho_{\text{Cu}}) = \rho_{\text{Cu}}/\rho_{\text{Al}} = \boxed{0.61}.$$

(c) Because the currents and areas are the same, all wires have the same current density.

For constant current, the electric field and the power loss is proportional to the resistivity.

Thus silver, with the smallest resistivity, has the weakest field and the least power loss.

77. If there is no current through the ammeter, we find the current in the source loop from

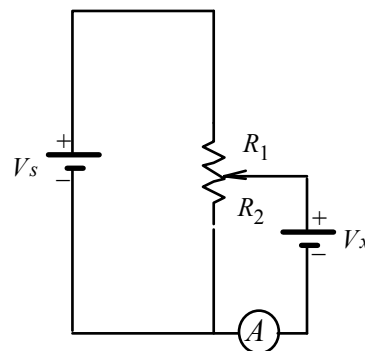
$$V_S - I_S(R_1 + R_2) = 0, \quad \text{which gives } I_S = V_S/(R_1 + R_2).$$

For the loop with the unknown, we have

$$V_x - I_S R_2 = 0.$$

When we use the expression for  $I_S$ , we get

$$V_x = V_S R_2/(R_1 + R_2).$$



78. (a) Immediately after the switch is closed, there is no charge on the capacitors and thus no potential difference across them. For the loop we have

$$\mathcal{E} - I_i R_1 - I_i R_2 = 0;$$

$$9 \text{ V} - I_i(300 \Omega) - I_i(1000 \Omega) = 0, \quad \text{which gives } I_i = 0.0069 \text{ A}.$$

Between  $b$  and  $a$  we have

$$(V_B - V_A)_i = + I_i R_2 = (0.0069 \text{ A})(1000 \Omega) = \boxed{6.9 \text{ V}}.$$

(b) After a long time, the current will be 0. The two capacitors are in series, with an equivalent capacitance:

$$1/C_{\text{eq}} = 1/C_1 + 1/C_2 = 1/(5 \mu\text{F}) + 1/(2 \mu\text{F}), \quad \text{which gives } C_{\text{eq}} = 1.43 \mu\text{F}.$$

The final charge on either capacitor is

$$Q = C_{\text{eq}} \mathcal{E} = (1.43 \mu\text{F})(9 \text{ V}) = 12.9 \mu\text{C}.$$

Between  $B$  and  $A$  we have

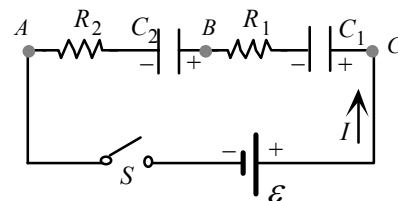
$$(V_B - V_A)_f = + Q/C_2 = (12.9 \mu\text{C})/(1.43 \mu\text{F}) = \boxed{6.4 \text{ V}}.$$

(c) The time constant of the circuit is

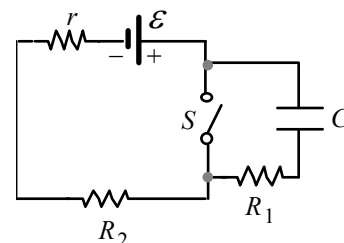
$$\text{time constant} = R_{\text{eq}} C_{\text{eq}} = (1 \text{ k}\Omega + 0.3 \text{ k}\Omega)(1.43 \mu\text{F}) = 1.86 \text{ ms}.$$

As a measure of how fast the circuit reaches a steady state, we use 10 time constants:

$$t = 10 R_{\text{eq}} C_{\text{eq}} = 10(1.86 \text{ ms}) = \boxed{19 \text{ ms}}.$$



79. (a) With the switch open, we have a series circuit of the three resistors and the capacitor. For the time constant we have  
 $\text{time constant} = R_{\text{eq}}C = (r + R_1 + R_2)C$   
 $= [(0.04 \Omega) + (0.1 \Omega) + (2 \Omega)](10 \mu\text{F}) = \boxed{21.4 \mu\text{s}}$



- (b) After a long time, there will be no current in the circuit. The battery emf will be across the capacitor:

$$Q = C\mathcal{E} = (10 \mu\text{F})(240 \text{ V}) = 2400 \mu\text{C} = \boxed{2.4 \text{ mC}}$$

80. (a) After a long time there will be a steady state; there will be no current in the capacitor branch:

$$I_5 = 0; \quad I_1 = I_3, \quad \text{and} \quad I_2 = I_4.$$

For the two resistor branches we have

$$V_f - V_d = \mathcal{E} = I_2(R_2 + R_4);$$

$$6 \text{ V} = I_2(180 \Omega + 60 \Omega), \text{ which gives } I_2 = 0.025 \text{ A};$$

$$V_c - V_a = \mathcal{E} = I_1(R_1 + R_3);$$

$$6 \text{ V} = I_1(35 \Omega + 25 \Omega), \text{ which gives } I_1 = 0.10 \text{ A}.$$

Because  $V_a = V_d$ , we can find the relative potentials of  $b$  and  $e$ :

$$V_b - V_e = (V_b - V_a) - (V_e - V_d) = I_1 R_3 - I_2 R_4$$

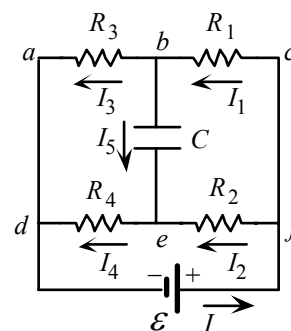
$$= (0.10 \text{ A})(25 \Omega) - (0.025 \text{ A})(60 \Omega) = +1.0 \text{ V}.$$

The charge on the capacitor is

$$Q = C(V_b - V_e) = (5 \mu\text{F})(1.0 \text{ V}) = \boxed{5 \mu\text{C}}.$$

Because  $V_b > V_e$ , the top plate is positive.

- (b) The current through the  $35\text{-}\Omega$  resistor is  $I_1 = \boxed{0.10 \text{ A}}$ .



81. Because the emf has negligible resistance, the terminal voltage, which is the voltage across the capacitor, is the emf  $V_0$ . The resistance of the material in the capacitor is

$$R = \rho L/A = L/\sigma A = d/\sigma \pi r^2.$$

- (a) We find the electric field between the capacitor plates from the potential gradient:

$$E = \boxed{V_0/d}.$$

- (b) The current density depends on the electric field:

$$J = \sigma E = \sigma V_0/d.$$

The current is

$$I = JA = (\sigma V_0/d)\pi r^2 = \boxed{\sigma \pi r^2 V_0/d} = V_0/R.$$

82. When we add another rung, as shown in the diagram, we have the resistance  $R^*$  in parallel with a resistor, which has an equivalent resistance of

$$R_{\text{eq}} = RR^*/(R + R^*).$$

This equivalent resistance is in series with two resistors.

Because the resistance does not change, we have

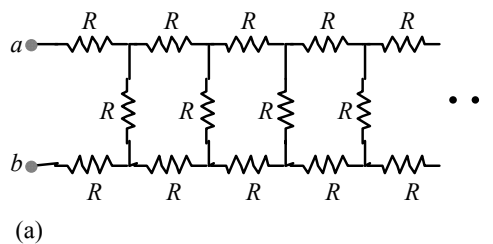
$$R^* = 2R + [RR^*/(R + R^*)], \text{ which gives}$$

$$R^{*2} - 2RR^* - 2R^2 = 0.$$

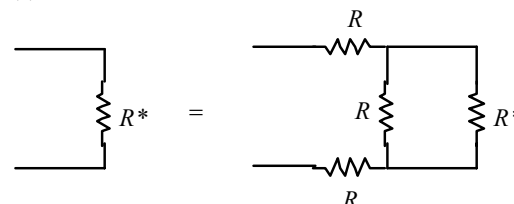
The solutions to this quadratic equation are

$$R^* = (1 \pm \sqrt{3})R.$$

Because the resistance cannot be negative,



(a)



(b)

we have

$$R^* = (1 + \sqrt{3})R = \boxed{2.732R}.$$