

# CHAPTER 11 Statics

## Answers to Understanding the Concepts Questions

1. The forces that act on the mountaineer are the force of friction between the boots and the steep slope, the force of gravity, and the normal force. The normal force is perpendicular to the direction of the slope, while friction acts along the slope. For a stationary climber, the vertical component of friction together with the vertical component of the normal force balances the force of gravity, and the horizontal component of the force of friction must cancel the horizontal component of the normal force. Because the climber is an extended object, we must take into consideration torques. The climber could fall backwards, or he could fall forward in such a way that his feet slide out from under him. If the climber leans towards the slope so that his center of gravity is in front of the vertical, towards the slope, then there is a torque about the point of contact which would tend to make the climber fall toward the slope. He can, of course, put out his hands to keep from falling, but if he still leans into the mountain, the normal force and hence the force of friction is reduced. It is not enough for his hands to keep him from leaning into the mountain; the hands actually have to keep him from sliding down, and this may be very difficult if the rock surface is smooth. Thus standing away from the mountain is the safer procedure.
2. Very unlikely. When the bureau is on the verge of tipping over the (upward) normal force from the floor is exerted on its front legs, about which the bureau is about to rotate. But the force of the baby pulling down is almost aligned with that force (as the handles are flush with front face of the bureau), so he is unlikely to be able to come up with enough counterclockwise torque about the axis of rotation to overcome the clockwise torque due to the weight of the bureau.
3. If a rope with a mass at the end is not vertically aligned, the force of gravity on the mass produces a torque about the point of suspension. This rotates the system in the direction of the vertical. If there is no friction, the rope oscillates about the vertical position. With friction and/or drag, the system will come to rest in a (vertical) equilibrium position.
4. Yes. For an object to be in equilibrium, both the net force and the net *torque* exerted on it must be zero. It is possible to have a nonzero net torque but zero net force. For example, grab the steering wheel of your vehicle with both hands at two points diametrically opposite to each other, and apply a force with one hand and another force of equal magnitude and opposite direction with the other. The net force exerted by your hands is zero, yet the net torque is not. As a result the steering wheel starts to turn.
5. The equilibrium is stable, since a displacement from the rest position leads to torques that tend to rotate the rocking chair and its occupant towards the rest position.
6. When the mass of the ladder is taken into consideration, the weight of the ladder-washer system increases, and that must be balanced by an increased upward supporting force at the base of the ladder. This, in turn, increases the maximum available static friction between the ladder and the floor, making it less likely to slip. So a massive ladder does help.
7. An ideal rigid object has no deformation whatsoever. A real object can be considered approximately rigid if its deformation is negligible compared with the length scales of the problem in consideration. For example, if you hang a weight at the tip of a horizontal wooden rod that is attached to the wall at the other end, the rod can be thought of as approximately rigid if the weight does not cause the tip of the rod to dip significantly, i.e., if the rod remains nearly straight and horizontal.

8. If the rock is to topple over, it must rotate about its base, where it makes contact with the ground. To maximize the effect of the force you apply in terms of causing the rock to rotate about the base, you want to maximize the lever arm of that force (measured from the base), i.e., you want to apply the force horizontally at the top of the rock.
9. All you need is three independent equations to solve for the three (upward) supporting forces. One of them can be from the balance of force (i.e., the sum of the three upward forces equals the weight of the bar). The other two come from the balance of torque, which you may write about any fixed point. In fact, if you write such an equation about the first contact point, for example, then you would get an equation without the first supporting force in it (as the torque it provides is zero). There are certainly enough equations (but only three of them are independent) for us to uniquely determine the three supporting forces.
10. As far as the equilibrium conditions are concerned, yes, the height of the table is irrelevant. The height becomes important, however, when one considers the stability of the table. A really tall table is more vulnerable to a horizontal disturbance applied along its surface, as the lever arm of that applied force is large in comparison with that of the (counterbalancing) weight of the table about its legs.
11. The equations governing the linear motion of the motorcycle are  $\sum F_x = ma_x$  and  $\sum F_y = ma_y$ . Here in the  $x$ -direction the forward force is the friction from the ground exerted on the back wheel of the motorcycle, while backward force is from the curb. To determine whether the motorcycle can climb the curb, consider  $\sum \tau_A$ , the net torque exerted on it, measured about the contact point A between the motorcycle and the curb. The torque provided by the friction of the road at the back wheel is what tends to make it climb over the curb, while that due to the weight of the motorcycle is what opposes it. Whether the motorcycle can climb the curb depends on which one of these torque is greater. And that, in turn, depends on the size of the motorcycle as well as the height of the curb. If the curb extends above the center of the wheels then it would not be possible for the motorcycle to climb over it. Otherwise it is possible.
12. This is a case of stable equilibrium. A small displacement of the pendulum from the equilibrium angle will lead to a swinging motion about that equilibrium angle, and not to a runaway motion.
13. Yes. The gravitational force exerted on an extended object can always be effectively replaced by a single force acting through its center of mass. The fact that the location of the center of mass may not be on the object itself should not change that rule. For the sake of clarity, one might imagine, for example, that the void between the two arms of a U-shaped steel object is completely filled with transparent acrylic, whose weight is negligible in comparison with the steel object itself. This obviously does not change the location of the center of mass of the object by any significant degree. And yet the object is now a whole rigid piece with no voids — and the center of mass is understandably somewhere on the acrylic part of the object, in between the two arms of the U-shape.
14. No external forces act on the station as an extended body. Once the station starts rotating, the conservation of angular momentum ensures that the rotation continues. One angular velocity is as good as any other, provided there are no disruptive forces entering once the angular velocity becomes too large. Thus the equilibrium is neutral.
15. The net force exerted on the shoe is zero just before one of the shoelaces break. Since the weight of the shoe as well as the downward force you exert it are both vertical, the horizontal components of the tensions in the two shoelaces must have the same magnitudes (and opposite signs, so they can cancel). So  $T_1 \cos 60^\circ = T_2 \cos 30^\circ$ , which implies that  $T_1 > T_2$ . Thus the shoelace which makes an angle of  $60^\circ$  with the horizontal will break first.

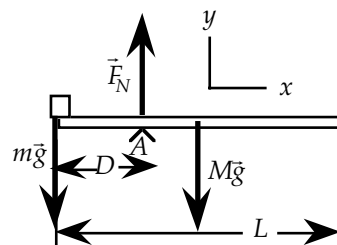
16. Yes it does. If you suspend a flat object at two different points, the center of mass must be directly below each point of suspension, regardless of whether the object is uniform or not. Otherwise the tension and the weight of the object would not be collinear and a net torque would result, upsetting the equilibrium.
17. It is not possible to do this. The vertical component of the tension must balance the weight of the beam if there is no friction. Since the tension acts at the far end of the beam and gravity acts at the midpoint of the beam, there is a net torque about the point of suspension. Equilibrium conditions cannot be satisfied, and the beam rotates by slipping along the frictionless wall.
18. The lever arm (and hence the torque) of the pushing force is twice as much if the force is applied at the top of the wheel rather than at the center.
19. No. As the ladder leans on the wall the wall must exert a force on it, with a horizontal component that points away from the wall. To balance that horizontal force the floor must exert the same amount of horizontal force pointing toward the wall on the bottom of the ladder. This won't be possible if the floor is frictionless. As a result the center of mass of the ladder would move away from the wall, and the ladder would eventually lose contact with the wall.
20. For a straight beam, the net torque about the pivot is zero no matter what the angle of inclination is. A slow change in angle will lead to another equilibrium position. Thus the equilibrium is neutral.
21. For simplicity, assume that the bend is symmetrical about the pivot point. As the equilibrium is upset, say the system has rotated counterclockwise with the person on the right now sitting higher than the other, the lever arm of the weight of the person on the left about the pivot is now reduced, while that of the other person is increased. This causes the system to tend to rotate clockwise back into equilibrium. So the equilibrium is stable.
22. Concrete, while strong in compression, is relatively weak in tension and cannot withstand the large tension produced by the pressure of the boiler.
23. Brick is weak in tension; adding straw increases the tensile strength.
24. Following the hint given in the problem statement, let's consider a (uniform) bar of length  $2L$ , which we can think of as two identical bars of length  $L$  each, joined at the ends. When a tensile force is applied on the bar, each half of the bar is subject to the same force, so each stretches by a certain amount, say  $\Delta L$ . The total amount of stretch of the bar is then  $\Delta L + \Delta L = 2 \Delta L$ . Thus doubling the length of a bar (from  $L$  to  $2L$ ) also doubles the amount of its stretch (from  $\Delta L$  to  $2 \Delta L$ , under the same tensile force). Therefore  $\Delta L$  is linearly proportional to  $L$ .
25. As the technique of making diamonds suggests, diamond is a form of solid carbon in which the carbon atoms are more tightly bound to each other than in graphite. Thus to decompose diamond, it is necessary to add energy to each atom to allow it to dissociate from its neighbor. One way to do this would be to add thermal energy; that is, to cook the diamond. Don't try this with a valuable gemstone!
26. Fluids certainly exist — think of thick molasses — with increasing degrees of resistance to shear. Glass is a fluid with a large viscosity, and this is associated with resistance to shear. Thus the macroscopic resistance to shear is not a clear distinction between fluids and solids.
27. A structure (e.g., graphite) featuring layers of planes is more vulnerable to shear forces applied along the planes. Applying a distortion force in other directions will encounter more resistance. One does have a directional dependence on resistance to stretching.

## Solutions to Problems

1. We choose the coordinate system shown, with positive torques clockwise. We write  $\sum \tau = I\alpha$  about the pivot point  $A$  from the force diagram for the board and mass:

$$\sum \tau_A = Mg(\tfrac{1}{2}L - D) - mgD = 0, \text{ which gives}$$

$$\begin{aligned} m &= M[(\tfrac{1}{2}L/D) - 1] \\ &= (80 \text{ kg})[\tfrac{1}{2}(3.6 \text{ m})/(1.2 \text{ m}) - 1] \\ &= \boxed{40 \text{ kg}}. \end{aligned}$$



2. We choose the coordinate system shown, with positive torques clockwise. We write  $\sum \tau = I\alpha$  about the point  $A$  from the force diagram for the board and worker:

$$\sum \tau_A = mg(\tfrac{1}{2}L) + Mg x - F_{N2}L = 0, \text{ which gives}$$

$$\begin{aligned} F_{N2} &= \tfrac{1}{2}mg + Mg(x/L) \\ &= \tfrac{1}{2}(20 \text{ kg})(9.8 \text{ m/s}^2) + (70 \text{ kg})(9.8 \text{ m/s}^2)(1.2 \text{ m})/(2.5 \text{ m}) \\ &= 4.3 \times 10^2 \text{ N}. \end{aligned}$$

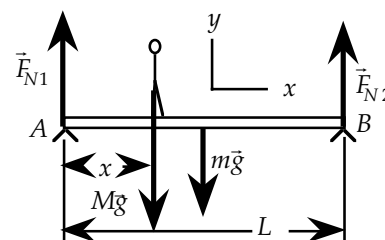
We write  $\sum F_y = ma_y$  from the force diagram for the board and worker:

$$F_{N1} + F_{N2} - mg - Mg = 0, \text{ which gives}$$

$$\begin{aligned} F_{N1} &= mg + Mg - F_{N2} \\ &= (20 \text{ kg})(9.8 \text{ m/s}^2) + (70 \text{ kg})(9.8 \text{ m/s}^2) - (4.3 \times 10^2 \text{ N}) \\ &= 4.5 \times 10^2 \text{ N}. \end{aligned}$$

The forces on the support points are the reactions to these normal forces:

$$\boxed{4.5 \times 10^2 \text{ N down}} \quad \text{and} \quad \boxed{4.3 \times 10^2 \text{ N down}}.$$



3. We choose the coordinate system shown, with positive torques clockwise. We write  $\sum \tau = I\alpha$  about the point  $A$  from the force diagram for the board:

$$\sum \tau_A = MgD - F_{N2}L = 0, \text{ which gives}$$

$$F_{N2} = Mg(D/L) = (24 \text{ kg})(9.8 \text{ m/s}^2)(0.9 \text{ m})/(2.2 \text{ m}) = 96 \text{ N}.$$

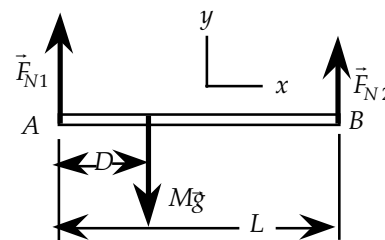
We write  $\sum F_y = ma_y$  from the force diagram for the board and worker:

$$F_{N1} + F_{N2} - Mg = 0, \text{ which gives}$$

$$\begin{aligned} F_{N1} &= Mg - F_{N2} \\ &= (24 \text{ kg})(9.8 \text{ m/s}^2) - 96 \text{ N} \\ &= 139 \text{ N}. \end{aligned}$$

The forces on the workmen are the reactions to these normal forces:

$$\boxed{139 \text{ N down}} \quad \text{and} \quad \boxed{96 \text{ N down}}.$$



4. We choose the coordinate system shown, with positive torques clockwise. The location of the bowling ball is given by

$$x = x_0 + vt = 0 + (0.15 \text{ m/s})t = (0.15 \text{ m/s})t.$$

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the rail and ball:

$$\sum \tau_A = mg(\frac{1}{2}L) + Mg x - F_{N2}L = 0, \text{ which gives}$$

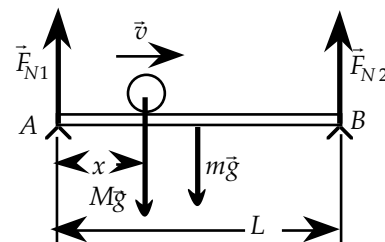
$$\begin{aligned} F_{N2} &= \frac{1}{2}mg + Mg(x/L) \\ &= \frac{1}{2}(8.0 \text{ kg})(9.8 \text{ m/s}^2) + (5.5 \text{ kg})(9.8 \text{ m/s}^2)[(0.15 \text{ m/s})t]/(3.0 \text{ m}) \\ &= \boxed{39.2 \text{ N} + (2.7 \text{ N/s})t} \end{aligned}$$

We write  $\sum F_y = ma_y$  from the force diagram for the rail and ball:

$$F_{N1} + F_{N2} - mg - Mg = 0, \text{ which gives}$$

$$\begin{aligned} F_{N1} &= mg + Mg - F_{N2} \\ &= (8.0 \text{ kg})(9.8 \text{ m/s}^2) + (5.5 \text{ kg})(9.8 \text{ m/s}^2) - (39.2 + 2.7t \text{ N}) \\ &= \boxed{93 \text{ N} - (2.7 \text{ N/s})t}. \end{aligned}$$

Because the scales read the magnitudes of the normal forces, these are the readings.



5. The point that follows a parabolic trajectory is the center of mass. We find its location from

$$\begin{aligned} \vec{R} &= \sum m_i \vec{r}_i / \sum m_i \\ &= [(2m_2)(3, 0, 0) + (m_2)(0, 0, 3)] / (2m_2 + m_2) = \boxed{(2, 0, 1) \text{ m}}. \end{aligned}$$

6. We choose the coordinate system shown, with positive torques clockwise.

- (a) We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the beam:

$$\sum \tau_A = mg(\frac{1}{2}L) - F_{N2}x = 0.$$

If the stronger person supports three-quarters of the weight, we have  $F_{N2} = \frac{3}{4}mg$ . Thus we have

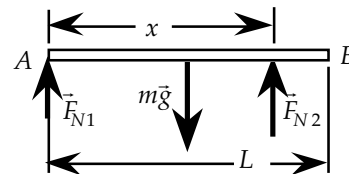
$$x = \frac{1}{2}mgL / F_{N2} = \frac{1}{2}mg / \frac{3}{4}mg = \frac{2}{3}L.$$

The person must hold the beam a distance of  $\boxed{L/3}$  from the end.

- (b) **No.** If  $x = L$ , we have

$$F_{N2} = \frac{1}{2}mg.$$

If the other person moves in,  $F_{N2}$  decreases; so more than half the weight cannot be supported.

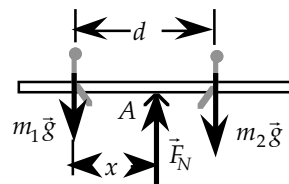


7. We choose the coordinate system shown, with positive torques clockwise.

We write  $\sum \tau = I\alpha$  about the pivot point A from the force diagram for the seesaw and children:

$$\sum \tau_A = m_2g(d - x) - m_1gx = 0, \text{ which gives}$$

$$\begin{aligned} x &= m_2d / (m_1 + m_2) = (40 \text{ kg})(2.8 \text{ m}) / (25 \text{ kg} + 40 \text{ kg}) \\ &= \boxed{1.7 \text{ m}}. \end{aligned}$$



8. We choose the coordinate system shown, with positive torques clockwise.

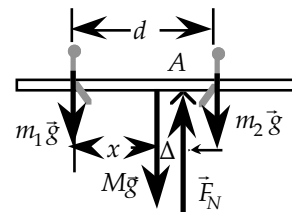
We write  $\sum \tau = I\alpha$  about the pivot point A from the force diagram for the seesaw and children:

$$\sum \tau_A = m_2g(d - x - \Delta) - Mg\Delta - m_1g(x + \Delta) = 0, \text{ which gives}$$

$$\begin{aligned} x &= [m_2d - (m_2 + M + m_1)\Delta] / (m_1 + m_2) \\ &= [(40 \text{ kg})(2.80 \text{ m}) - (40 \text{ kg} + 8 \text{ kg} + 25 \text{ kg})(0.24 \text{ m})] / (25 \text{ kg} + 40 \text{ kg}) \\ &= 1.45 \text{ m}. \end{aligned}$$

The lighter child sits  $\boxed{1.45 \text{ m from the center}}$ .

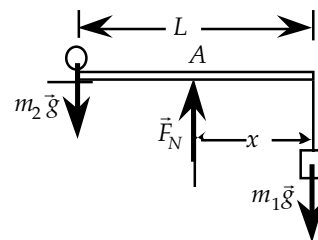
The heavier child sits  $d - x = 2.80 \text{ m} - 1.45 \text{ m} = \boxed{1.35 \text{ m from the center}}$ .



9. We choose the coordinate system shown, with positive torques clockwise. As the plank is moved out from the roof, the effective normal force acts at a point closer to the edge. When the normal force reaches the edge, the plank is on the verge of tipping. We write  $\sum \tau = I\alpha$  about the edge A from the force diagram for the plank, the load and the concrete:

$$\sum \tau_A = M_1gx - M_2g(L - x) = 0, \text{ which gives}$$

$$x = M_2L / (M_1 + M_2) = (15 \text{ kg})(2.4 \text{ m}) / (30 \text{ kg} + 15 \text{ kg}) = \boxed{0.8 \text{ m}}.$$



10. We choose the coordinate system shown, with positive torques clockwise. We write  $\sum \tau = I\alpha$  about the support point B from the force diagram for the beam and mass:

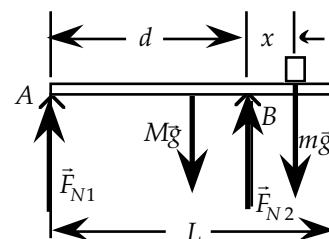
$$\sum \tau_B = mgx + F_{N1}(d) - Mg(d - \frac{1}{2}L) = 0, \text{ which gives}$$

$$x = (M/m)(d - \frac{1}{2}L) - (F_{N1}d/mg).$$

From this we see that  $F_{N1}$  decreases as  $x$  increases. When  $F_{N1}$  becomes zero, the beam will start to tip. This occurs when

$$x = (M/m)(d - \frac{1}{2}L) = (80 \text{ kg}/150 \text{ kg})[2.4 \text{ m} - \frac{1}{2}(3.0 \text{ m})] = \boxed{0.48 \text{ m}}.$$

Note that this is still on the beam.



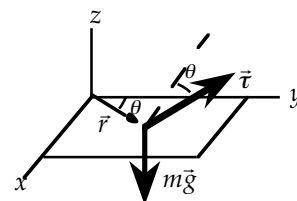
11. We find the torque from

$$\vec{\tau} = \vec{r} \times m\vec{g}$$

$$= [\frac{1}{2}(0.60 \text{ m})\hat{i} + \frac{1}{2}(1.20 \text{ m})\hat{j}] \times (3 \text{ kg/m}^2)(0.60 \text{ m})(1.20 \text{ m})(9.8 \text{ m/s}^2)(-\hat{k})$$

$$= (-12.7\hat{i} + 6.4\hat{j}) \text{ N}\cdot\text{m}$$

$$= \boxed{14.2 \text{ N}\cdot\text{m} \text{ in } xy\text{-plane } 27^\circ \text{ from } x\text{-axis (perpendicular to } \vec{r})}.$$

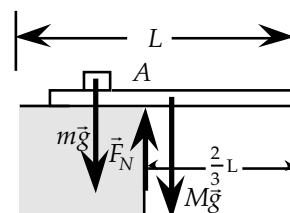


12. We choose the coordinate system shown, with positive torques clockwise. As the mass of the paperweight decreases, the effective normal force acts at a point closer to the edge. The minimum mass to keep the book from falling off the table occurs when the normal force reaches the edge.

We write  $\sum \tau = I\alpha$  about the edge A from the force diagram for the book and paperweight:

$$\sum \tau_A = mg(0.2L) - Mg(0.1L) = 0, \text{ which gives}$$

$$m = \frac{1}{2}M = \boxed{0.50 \text{ kg}}.$$



13. The maximum distance for the top book to remain on the bottom book will be reached when its center of mass is over the edge of the bottom book. The maximum distance for the bottom book to remain on the table will be reached when the center of mass of the combination is over the edge of the table. If we take the edge of the table as the origin, we have

$$R = m(x - L/2) + mx = 0, \text{ which gives } x = L/4.$$

The distance to the extreme edge of the top book is

$$D = L/4 + L/2 = \boxed{3L/4}.$$

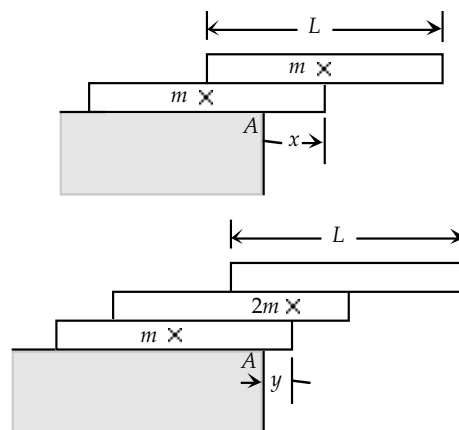
If we add a third book, the maximum distance for the top two books to remain on the bottom book will be reached when their composite center of mass is over the edge of the bottom book.

The maximum distance for the bottom book to remain on the table will be reached when the center of mass of the three books is over the edge of the table. If we take the edge of the table as the origin, we have

$$R = m(y - \frac{1}{2}L) + 2my = 0, \text{ which gives } y = L/6.$$

The distance to the extreme edge of the top book is

$$D = L/6 + 3L/4 = \boxed{11L/12}.$$



14. We choose the coordinate system shown, with positive torques clockwise.

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the door:

$$\sum \tau_A = Mg(\frac{1}{2}w) + F_T H = 0, \text{ which gives}$$

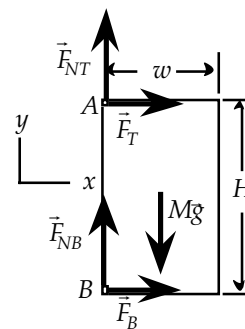
$$F_T = -\frac{1}{2}Mgw/H = -\frac{1}{2}(14 \text{ kg})(9.8 \text{ m/s}^2)(0.90 \text{ m})/(1.95 \text{ m}) \\ = -32 \text{ N}.$$

We write  $\sum F_x = ma_x$  from the force diagram for the door:

$$F_T + F_B = 0, \text{ which gives}$$

$$F_B = -F_T = \boxed{32 \text{ N}}.$$

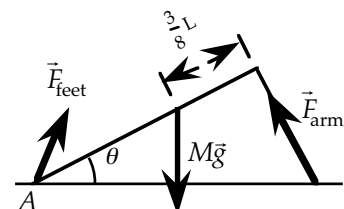
Thus the top hinge pulls away from the door, and the bottom hinge pushes on the door.



15. We take the support force to be along the arms. We write  $\sum \tau = I\alpha$  about the point A from the force diagram:

$$\sum \tau_A = Mg(5L/8) \cos \theta - F_{\text{arms}} L = 0, \text{ which gives}$$

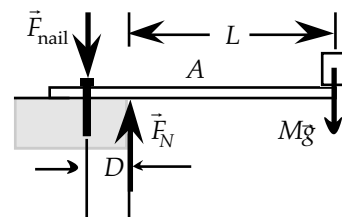
$$F_{\text{arms}} = (5Mg/8) \cos 25^\circ = \boxed{0.57 Mg}.$$



16. We choose positive torques clockwise. The nail prevents the board from tipping, so the effective normal force acts at the edge. We write  $\sum \tau = I\alpha$  about the edge A from the force diagram for the board and flower pot:

$$\sum \tau_A = F_{\text{nail}} D - MgL = 0, \text{ which gives}$$

$$F_{\text{nail}} = MgL/D = (3.5 \text{ kg})(9.8 \text{ m/s}^2)(0.50 \text{ m})/(0.04 \text{ m}) = \boxed{4.3 \times 10^2 \text{ N}}.$$



17. We choose positive torques clockwise.

We write  $\sum \tau = I\alpha$  about the center C from the force diagram for the rod:

$$\sum \tau_C = Mg(\frac{1}{2}L) - F_{NC}(\frac{1}{2}L) = 0, \text{ which gives}$$

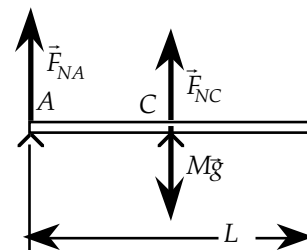
$$F_{NC} = Mg = (12 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{1.2 \times 10^2 \text{ N}}.$$

We write  $\sum \tau = I\alpha$  about the end A from the force diagram for the rod:

$$\sum \tau_A = F_{NA} \frac{1}{2}L = 0, \text{ which gives}$$

$$F_{NA} = \boxed{0}.$$

The rod is balanced on the center point. It is unstable for any slight shift away from the supported end. A shift toward the supported end will cause a normal force there and the rod will be stable.



18. We choose the coordinate system shown, with positive torques counterclockwise. Find the angle between the ladder and the ground from  $\sin \theta = h/L = (8 \text{ m})/(8.5 \text{ m})$ , which gives  $\theta = 70^\circ$ .

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the ladder and man:

$$\begin{aligned}\sum \tau_A &= mg\left(\frac{1}{2}L\right) \cos \theta + Mg d \cos \theta - F_{N\text{top}} L \sin \theta = 0, \text{ which gives} \\ F_{N\text{top}} &= \left(\frac{1}{2}mL + Md\right)g \cos \theta / (L \sin \theta) \\ &= \left[\frac{1}{2}(26 \text{ kg})(8.5 \text{ m}) + (75 \text{ kg})(7 \text{ m})\right](9.8 \text{ m/s}^2)(\cos 70^\circ) / [(8.5 \text{ m})(\sin 70^\circ)] \\ &= 2.7 \times 10^2 \text{ N.}\end{aligned}$$

We write  $\sum F_x = ma_x$  from the force diagram for the ladder and man:

$$F_{N\text{top}} - f_{\text{bottom}} = 0, \text{ which gives } f_{\text{bottom}} = F_{N\text{top}} = 2.7 \times 10^2 \text{ N.}$$

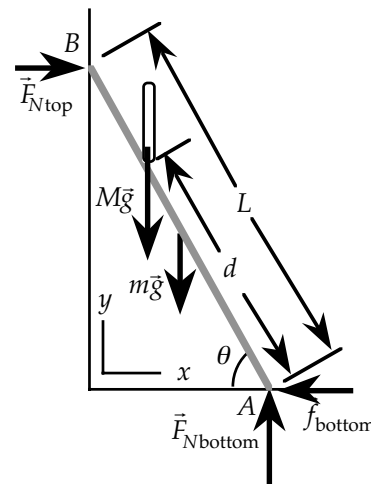
We write  $\sum F_y = ma_y$  from the force diagram for the ladder and man:

$$F_{N\text{bottom}} - (m + M)g = 0, \text{ which gives } F_{N\text{bottom}} = 9.9 \times 10^2 \text{ N.}$$

The force on the ladder from the man is the reaction to the force on the man by the ladder, which must have the magnitude of the man's weight.

Thus the forces are

$$\begin{aligned}F_{N\text{bottom}} &= 9.9 \times 10^2 \text{ N up,} \\ f_{\text{bottom}} &= 2.7 \times 10^2 \text{ N toward the wall,} \\ F_{N\text{top}} &= 2.7 \times 10^2 \text{ N away from the wall,} \\ F_{\text{man}} &= 7.4 \times 10^2 \text{ N down,} \\ W_{\text{ladder}} &= 2.5 \times 10^2 \text{ N down.}\end{aligned}$$



19. We choose the coordinate system shown, with positive torques counterclockwise.

- (a) We write  $\sum \tau = I\alpha$  about the point B from the force diagram for the ladder and man:

$$\sum \tau_B = mg\left(\frac{1}{2}L\right) \sin \theta + Mg d \sin \theta - F_{N\text{top}} L \cos \theta = 0.$$

We write  $\sum F_x = ma_x$  from the force diagram for the ladder and man:

$$F_{N\text{top}} - f_{\text{bottom}} = 0.$$

We write  $\sum F_y = ma_y$  from the force diagram for the ladder and man:

$$F_{N\text{bottom}} - (m + M)g = 0.$$

As the man climbs the ladder, the friction force at the bottom increases. At the point where slipping begins,

$$f_{\text{bottom}} = f_{\text{bottom,max}} = \mu_s F_{N\text{bottom}} = \mu_s (m + M)g. \text{ Then we have}$$

$$F_{N\text{top}} = \mu_s (m + M)g.$$

Using these in the torque equation, we get

$$\begin{aligned}d &= [\mu_s (m + M)L \cos \theta - m\left(\frac{1}{2}L\right) \sin \theta] / (M \sin \theta) \\ &= [0.40(10 \text{ kg} + 80 \text{ kg})(4 \text{ m}) \cos 30^\circ - 10(4 \text{ m}/2) \sin 30^\circ] / [(80 \text{ kg}) \sin 30^\circ] = 2.9 \text{ m.}\end{aligned}$$

- (b) We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the ladder and man:

$$\sum \tau_A = F_{N\text{bottom}} (L \sin \theta) - Mg(L - d) \sin \theta - mg\left(\frac{1}{2}L\right) \sin \theta - f_{\text{bottom}} L \cos \theta = 0.$$

We write  $\sum \tau = I\alpha$  about the point O from the force diagram for the ladder and man:

$$\sum \tau_O = F_{N\text{bottom}} (L \sin \theta) - Mg(L - d) \sin \theta - mg\left(\frac{1}{2}L\right) \sin \theta - F_{N\text{top}} L \cos \theta = 0.$$

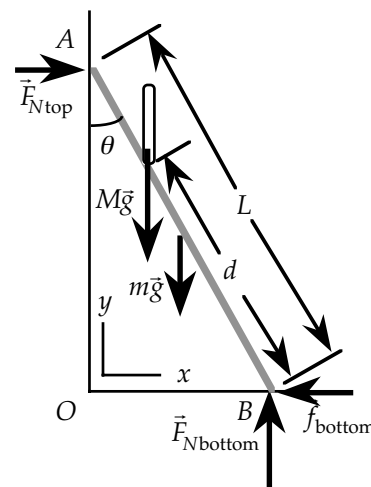
We can combine these three torque equations to obtain the force equations.

If we add the equations for points A and O, we get

$$F_{N\text{top}} = f_{\text{bottom}}, \text{ which is the } x\text{-equation.}$$

If we subtract the equations for points B and O, we get

$$F_{N\text{bottom}} = \mu_s (m + M)g, \text{ which is the } y\text{-equation.}$$





20. We choose the coordinate system shown, with positive torques counterclockwise. Write  $\sum F_y = ma_y$  from the force diagram for the ladder and man:

$$F_{N\text{bottom}} - Mg = 0, \text{ which gives}$$

$$F_{N\text{bottom}} = Mg.$$

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the ladder and man:

$$\sum \tau_A = F_{N\text{bottom}}(L \cos \theta) - Mg(L - d) \cos \theta - f_{\text{bottom}}L \sin \theta = 0.$$

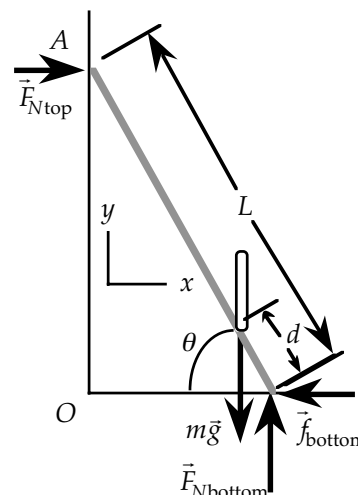
When we use  $F_{N\text{bottom}} = Mg$ , we get

$$f_{\text{bottom}} = (Mgd \cos \theta) / L \sin \theta.$$

At the instant that the ladder slips, the friction force is maximum,

$$f_{\text{bottom}} = \mu_s F_{N\text{bottom}}, \text{ so we have}$$

$$\begin{aligned} \mu_s &= (d \cos \theta) / L \sin \theta \\ &= 2(0.33 \text{ m}) \cos 58^\circ / [(3.0 \text{ m}) \sin 58^\circ] \\ &= \boxed{0.14}. \end{aligned}$$



21. (a) We choose the coordinate system shown, with positive torques clockwise. The force exerted by the person on the rope is equal to the tension in the rope. Because the rope is smooth, the tension does not change around the pulley. From the equilibrium of the engine, we have

$$T = Mg = (30 \text{ kg})(9.8 \text{ m/s}^2) = 2.9 \times 10^2 \text{ N}.$$

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the strut and engine:

$$\sum \tau_A = -T \cos \theta L \sin 45^\circ - T \sin \theta L \cos 45^\circ + mg(\frac{1}{2}L) \cos 45^\circ + MgL \cos 45^\circ = 0.$$

Note that we found the torque from the tension by finding the torque from each of its components.

Because  $\sin 45^\circ = \cos 45^\circ$  and the factor  $L$  is in each term, when we replace the tension we get

$$\begin{aligned} -M \cos \theta - M \sin \theta + \frac{1}{2}m + Mg &= 0, \text{ or} \\ \cos \theta + \sin \theta &= (M + \frac{1}{2}m) / M \\ &= [30 \text{ kg} + \frac{1}{2}(12.5 \text{ kg})] / (30 \text{ kg}) = 1.21. \end{aligned}$$

If we use  $\sin^2 \theta + \cos^2 \theta = 1$ , we get a quadratic equation for  $\cos \theta$ :

$$\cos^2 \theta - 1.21 \cos \theta + 0.46, \text{ which has two solutions: } \theta = 13.5^\circ \text{ and } 76^\circ.$$

We choose the smaller angle as more practical.

The person exerts a force of  $\boxed{2.9 \times 10^2 \text{ N } 13.5^\circ \text{ above the horizontal}}.$

- (b) We write  $\sum F_x = ma_x$  from the force diagram for the strut and engine:

$$F_H - T \cos \theta = 0, \text{ which gives}$$

$$F_H = (2.9 \times 10^2 \text{ N}) \cos 13.5^\circ = 2.8 \times 10^2 \text{ N}.$$

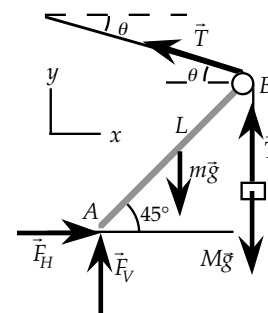
We write  $\sum F_y = ma_y$  from the force diagram for the strut and engine:

$$F_V + T \sin \theta - (m + M)g = 0, \text{ which gives}$$

$$\begin{aligned} F_V &= -(2.9 \times 10^2 \text{ N}) \sin 13.5^\circ + (12.5 \text{ kg} + 30 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 3.5 \times 10^2 \text{ N}. \end{aligned}$$

The forces exerted by the strut on the ground are the reactions to these:

$$F_V = \boxed{3.5 \times 10^2 \text{ N down}} \quad \text{and} \quad F_H = \boxed{2.8 \times 10^2 \text{ N}}.$$



22. We choose the coordinate system shown, with positive torques counterclockwise.

(a) We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the board:

$$\sum \tau_A = mg(\frac{1}{2}L) \cos \theta - F_{N2}L \sin \theta - f_2L \cos \theta = 0.$$

We write  $\sum F_x = ma_x$  from the force diagram for the ladder and man:

$$f_1 - F_{N2} = 0.$$

We write  $\sum F_y = ma_y$  from the force diagram for the ladder and man:

$$F_{N1} + f_2 - mg = 0.$$

We assume that just before slipping, the friction force at the wall is maximum:

$$f_2 = \mu_{s2}F_{N2}.$$

From the torque equation, we get

$$F_{N2} = mg \cos \theta / [2(\sin \theta + \mu_{s2} \cos \theta)] = mg / [2(\tan \theta + \mu_{s2})], \text{ which is also } f_1.$$

From the y-equation, we get

$$\begin{aligned} F_{N1} &= mg - f_2 = mg - \mu_{s2} mg / [2(\tan \theta + \mu_{s2})] \\ &= mg(2 \tan \theta + \mu_{s2}) / [2(\tan \theta + \mu_{s2})]. \end{aligned}$$

For the bottom not to slip, we must have

$$f_1 \leq \mu_{s1}F_{N1}, \text{ or}$$

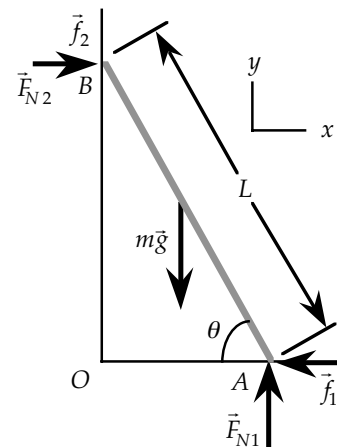
$$mg / [2(\tan \theta + \mu_{s2})] \leq \mu_{s1} mg(2 \tan \theta + \mu_{s2}) / [2(\tan \theta + \mu_{s2})], \text{ from which we get}$$

$$\tan \theta \geq \frac{1}{2}L(1 - \mu_{s1}\mu_{s2}) / \mu_{s2}.$$

We find the minimum angle from

$$\tan \theta_{\min} = \frac{1}{2}[1 - (0.35)(0.28)] / 0.28 = 1.61, \text{ which gives } \theta_{\min} = \boxed{58^\circ}.$$

- (b) If there is no friction at the floor, the only horizontal force is the normal force at the wall. This will push the center of mass to the right, and the board will fall.



23. (a) The component of the weight of the lawn mower along the slope is  $mg \sin \theta$ , down; which should balance the static friction force  $f_s$  pointing up the slope if it is not to slide. When the lawn mower is on the verge of sliding  $f_s$  is at its maximum value, of  $\mu_s mg \cos \theta$ ; so

$$mg \sin \theta - \mu_s mg \cos \theta = 0, \text{ which gives}$$

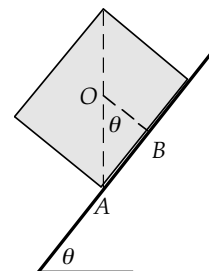
$$\tan \theta = \mu_s, \text{ or } \theta = \boxed{\tan^{-1} \mu_s}.$$

This is the largest angle for which the mower will not slip.

- (b) When the lawn mower is on the verge of tipping over, its center of mass (point O) must be along the same vertical line as the axis of its rotation (A). Thus in the triangle OAB

$$AB/OB = (\frac{1}{2}\Delta)/L = \tan \theta, \text{ or}$$

$$\theta = \boxed{\tan^{-1} (\Delta/2L)}, \text{ the largest angle for which the mower will not tip over.}$$



24. The balance of force equations remain the same as those in Example 11-6, namely,  $F_{Nx} - T \cos \theta_T = 0$  for the x-component and  $F_{Ny} - T \sin \theta_T - mg = 0$  for the y-component. Relative to point A, the lever arm of the weight of the mass being lifted is  $l \cos \theta_{arm}$ , that of  $F_{Nx}$  is  $L \sin \theta_{arm}$ , and that of  $F_{Ny}$  is  $L \cos \theta_{arm}$ . The torque to the tension  $T$  in the cable is zero since its lever arm about point A is zero. The torque equation now reads

$$mg l \cos \theta_{arm} - F_{Nx} L \sin \theta_{arm} + F_{Ny} L \cos \theta_{arm} = 0.$$

Solve the three equations above for  $T$ :

$$\begin{aligned} T &= \frac{mg(l \cos \theta_{arm} - L \sin \theta_{arm})}{\cos \theta_T L (\sin \theta_{arm} \tan \theta_T - \cos \theta_{arm})} \\ &= \frac{(5300 \text{ kg})(9.8 \text{ m/s}^2)(0.52 \cos 45^\circ - 10.0 \sin 45^\circ)}{\cos 32^\circ (10.0 \text{ m})[(\sin 45^\circ)(\tan 32^\circ) - \cos 45^\circ]} \\ &= 1.5 \times 10^5 \text{ N}. \end{aligned}$$

25. We choose the coordinate system shown, with positive torques clockwise. We find the angle  $\theta$  from  $\sin \theta = \frac{1}{4}L / \frac{1}{2}L = 0.5$ , which gives  $\theta = 30^\circ$ .

Using Newton's third law, we know that the forces between the ladders at the top will have equal magnitudes and opposite directions. From symmetry, they and the forces from the crossbar must be horizontal.

We write  $\sum \tau = I\alpha$  about the point B from the force diagram for the left ladder:

$$\sum \tau_B = F_{NA}(L \sin \theta) - Mg(\frac{1}{2}L) \sin \theta - F(\frac{1}{2}L) \cos \theta = 0.$$

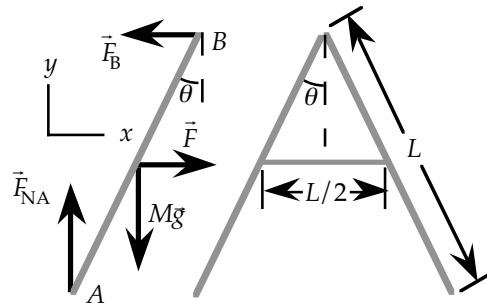
We write  $\sum F_y = ma_y$  from the force diagram for the left ladder:

$$F_{NA} - Mg = 0, \text{ which gives } F_{NA} = Mg.$$

If we use this in the torque equation, we get

$$F = [Mg(\frac{1}{2}L) \sin \theta] / [(\frac{1}{2}L) \cos \theta] \\ = Mg \tan 30^\circ = 0.58 Mg.$$

The force on the crossbar is the reaction to this:  $0.58 Mg$  outward.



26. We choose the coordinate system shown, with positive torques clockwise. We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the flagpole:

$$\sum \tau_A = Mg(\frac{1}{2}L) - (T \sin \theta)L = 0, \text{ which gives}$$

$$T = \frac{1}{2}Mg / (\sin \theta) = \frac{1}{2}(6 \text{ kg})(9.8 \text{ m/s}^2) / \sin 25^\circ = \text{70 N}.$$

Note that we found the torque of the tension by finding the torque from each of its components. Write  $\sum F_x = ma_x$  from the force diagram for the flagpole:

$$F_{\text{wall}H} - T \cos \theta = 0, \text{ which gives}$$

$$F_{\text{wall}H} = (70 \text{ N}) \cos 25^\circ = 63 \text{ N}.$$

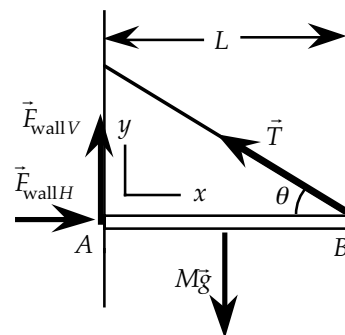
We write  $\sum F_y = ma_y$  from the force diagram for the flagpole:

$$F_{\text{wall}V} + T \sin \theta - Mg = 0, \text{ which gives}$$

$$F_{\text{wall}V} = (6 \text{ kg})(9.8 \text{ m/s}^2) - (70 \text{ N}) \sin 25^\circ = 29 \text{ N}.$$

Combining these two components, we get

$$\vec{F}_{\text{wall}} = (63\hat{i} + 29\hat{j}) \text{ N}.$$



27. The force  $F$  supplies a counterclockwise torque about the axis of the pulleys,  $\tau_1 = FR_1$ , where  $R_1 = 20 \text{ cm}$ ; while the weight of the engine (of mass  $m$ ) exerts a clockwise torque about the axis,  $\tau_2 = -mgR_2$ , where  $R_2 = 8.0 \text{ cm}$ . To balance the torques, set

$$\sum \tau_A = FR_1 - mgR_2 = 0, \text{ which gives}$$

$$F = mgR_2 / R_1 = (300 \text{ kg})(9.8 \text{ m/s}^2)(8.0 \text{ cm}) / 20 \text{ cm} = \text{1.2 kN}.$$

28. Consider the left side of the ladder. The net force exerted on it is zero, so

$$f - F_{NA} = 0 \text{ in the horizontal direction and}$$

$$mg - F_{NB} = 0 \text{ in the vertical direction. Also, the net torque about point B must vanish:}$$

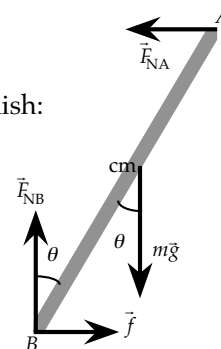
$$\sum \tau_B = F_{NA} L \cos \theta - mg(\frac{1}{2}L) \sin \theta = 0.$$

When  $\theta$  is at its largest possible value, the ladder is just about to slide outward at point B, so

$$f = f_{\text{max}} = \mu F_{NB}. \text{ Combine these equation to obtain}$$

$$\tan \theta = 2\mu, \text{ or}$$

$$\theta = \text{tan}^{-1}(2\mu).$$



29. We choose the coordinate system shown, with positive torques clockwise. On the force diagram we have added the two forces on each front leg and on each rear leg.

We write  $\sum F_x = ma_x$  from the force diagram for the desk:

$$F - 2f_{\text{rear}} - 2f_{\text{front}} = 0.$$

We write  $\sum F_y = ma_y$  from the force diagram for the desk:

$$2F_{N\text{rear}} + 2F_{N\text{front}} - Mg = 0, \text{ which gives}$$

$$F_{N\text{rear}} + F_{N\text{front}} = \frac{1}{2}Mg = \frac{1}{2}(43 \text{ kg})(9.8 \text{ m/s}^2) = 2.1 \times 10^2 \text{ N}.$$

When this is used in the  $x$ -equation, we get

$$F = 2(f_{\text{rear}} + f_{\text{front}}) = 2\mu_k(F_{N\text{rear}} + F_{N\text{front}}) = 2(0.45)(2.1 \times 10^2 \text{ N}) = \boxed{1.9 \times 10^2 \text{ N right}}.$$

Because three of the forces will have no torque about the point A,

we choose it as the axis and write  $\sum \tau = I\alpha$  from the force diagram for the desk:

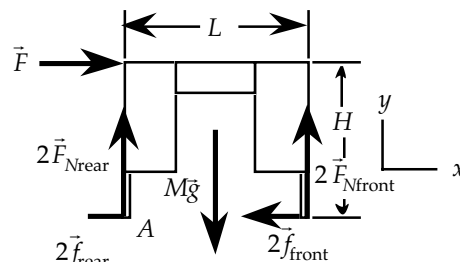
$$\sum \tau_A = Mg\left(\frac{1}{2}L\right) - (2F_{N\text{front}})L + FH = 0, \text{ which gives}$$

$$F_{N\text{front}} = \left(\frac{1}{2}MgL + FH\right)/2L = \left[\frac{1}{2}(43 \text{ kg})(9.8 \text{ m/s}^2)(1.54 \text{ m}) + (1.9 \times 10^2 \text{ N})(0.82 \text{ m})\right]/[2(1.54 \text{ m})] = 1.6 \times 10^2 \text{ N}.$$

Thus  $F_{N\text{rear}} = 2.1 \times 10^2 \text{ N} - 1.6 \times 10^2 \text{ N} = 50 \text{ N}$ , so we get

$$f_{\text{front}} = \mu_k F_{N\text{front}} = (0.45)(1.6 \times 10^2 \text{ N}) = \boxed{73 \text{ N left}} \text{ and}$$

$$f_{\text{rear}} = \mu_k F_{N\text{rear}} = (0.45)(50 \text{ N}) = \boxed{23 \text{ N left}}.$$



30. We choose the coordinate system shown, with positive torques clockwise. On the force diagram we have added the two forces on each front leg and on each rear leg.

We write  $\sum F_x = ma_x$  from the force diagram for the desk:

$$F - 2f_{\text{rear}} - 2f_{\text{front}} + Mg \sin \theta = 0.$$

We write  $\sum F_y = ma_y$  from the force diagram for the desk:

$$2F_{N\text{rear}} + 2F_{N\text{front}} - Mg \cos \theta = 0, \text{ which gives}$$

$$F_{N\text{rear}} + F_{N\text{front}} = \frac{1}{2}Mg \cos \theta = \frac{1}{2}(43 \text{ kg})(9.8 \text{ m/s}^2) \cos 3.5^\circ = 2.1 \times 10^2 \text{ N}.$$

When this is used in the  $x$ -equation, we get

$$F = 2\mu_k(F_{N\text{rear}} + F_{N\text{front}}) - Mg \sin \theta = 2(0.45)(2.1 \times 10^2 \text{ N}) - (43 \text{ kg})(9.8 \text{ m/s}^2) \sin 3.5^\circ = \boxed{1.6 \times 10^2 \text{ N down slope}}.$$

Because three of the forces will have no torque about point A, we choose it as the axis and write  $\sum \tau = I\alpha$  from the force diagram for the desk:

$$\sum \tau_A = Mg \cos \theta \left(\frac{1}{2}L\right) + Mg \sin \theta (H - d) - (2F_{N\text{front}})L + FH = 0, \text{ which gives}$$

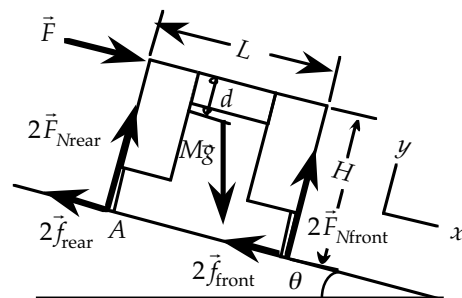
$$F_{N\text{front}} = \{Mg[\frac{1}{2}L \cos \theta + (H - d) \sin \theta] + FH\}/2L = \{(43 \text{ kg})(9.8 \text{ m/s}^2)[\frac{1}{2}(1.54 \text{ m} \cos 3.5^\circ) + (0.44 \text{ m}) \sin 3.5^\circ] + (1.6 \times 10^2 \text{ N})(0.82 \text{ m})\}/[2(1.54 \text{ m})] = \boxed{1.5 \times 10^2 \text{ N}}.$$

Note that we found the torque of  $Mg$  by finding the torque from each of its components.

Thus  $F_{N\text{rear}} = 2.1 \times 10^2 \text{ N} - 1.5 \times 10^2 \text{ N} = \boxed{0.6 \times 10^2 \text{ N}}$ , so we get

$$f_{\text{front}} = \mu_k F_{N\text{front}} = (0.45)(1.5 \times 10^2 \text{ N}) = \boxed{68 \text{ N/leg up slope}} \text{ and}$$

$$f_{\text{rear}} = \mu_k F_{N\text{rear}} = (0.45)(0.6 \times 10^2 \text{ N}) = \boxed{27 \text{ N/leg up slope}}.$$



31. We choose the coordinate system shown, with positive torques clockwise. On the force diagram we have added the two forces on each leg at A and on each leg at B.

(a) We write  $\sum \tau = I\alpha$  about the point B from the force diagram for the seat:

$$\sum \tau_B = -mg\ell_2 + 2F_{NA}\ell_1 = 0, \text{ which gives}$$

$$F_{NA} = \frac{1}{2}mg\ell_2/\ell_1.$$

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the seat:

$$\sum \tau_A = -mg(\ell_2 - \ell_1) + 2F_{NB}\ell_1 = 0, \text{ which gives}$$

$$F_{NB} = \frac{1}{2}mg(\ell_2 - \ell_1)/\ell_1.$$

The forces on the table will be the reactions to these forces:

$$F_{NA} = \boxed{\frac{1}{2}mg\ell_2/\ell_1 \text{ down}} \quad \text{and} \quad F_{NB} = \boxed{\frac{1}{2}mg(\ell_2 - \ell_1)/\ell_1 \text{ up}}.$$

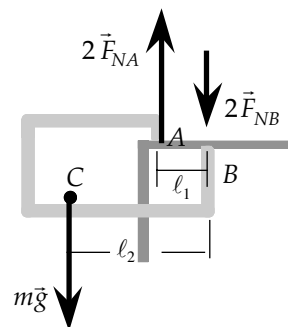
(b) As  $\ell_2 \rightarrow 0$ ;  $F_{NA} \rightarrow 0$ ,  $F_{NB} \rightarrow -\frac{1}{2}mg$ . Because the normal forces can not become negative, the seat will lose contact at B and turn clockwise.

As  $\ell_1 \rightarrow 0$ ;  $F_{NA} \rightarrow \infty$ ,  $F_{NB} \rightarrow \infty$ . The normal forces will have no torque about the contact point, and the seat will turn counterclockwise.

(c) From the above expressions, we get

$$F_{NA} = \frac{1}{2}mg\ell_2/\ell_1 = \frac{1}{2}(10 \text{ kg})(9.8 \text{ m/s}^2)(0.30 \text{ m})/0.20 \text{ m} = \boxed{74 \text{ N}} \quad \text{and}$$

$$F_{NB} = \frac{1}{2}mg(\ell_2 - \ell_1)/\ell_1 = \frac{1}{2}(10 \text{ kg})(9.8 \text{ m/s}^2)(0.30 \text{ m} - 0.20 \text{ m})/0.20 \text{ m} = \boxed{25 \text{ N}}.$$



32. We choose the coordinate system shown, with positive torques clockwise. On the force diagram we have added the two forces on each front leg and on each rear leg of the table. We write  $\sum \tau = I\alpha$  about the point P from the force diagram for the seat and table:

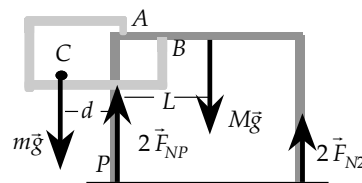
$$\sum \tau_P = -mgd + MgL - 2F_{N2}(2L) = 0.$$

When the table is about to tip,  $F_{N2} = 0$ , so we get

$$m = MgL/gd = (18 \text{ kg})(70 \text{ cm})/(10 \text{ cm}) = \boxed{126 \text{ kg}}.$$

Assuming that the mass of baby and seat is not near this limit, we see from the results of Problem 31 that small

changes in  $\ell_2$  will cause the normal forces on the seat to change slightly but not destabilize the equilibrium. From the results of this problem, we see that small changes in  $d$  will be compensated by small changes in the normal forces on the legs of the table but will not destabilize the situation.



33. We choose the coordinate system shown, with positive torques clockwise.

(a) We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the trap door:

$$\sum \tau_A = Mg(\frac{1}{2}L \cos \theta) - TL = 0, \text{ which gives}$$

$$T = \frac{1}{2}Mg \cos \theta$$

$$= \frac{1}{2}(20 \text{ kg})(9.8 \text{ m/s}^2) \cos 55^\circ = \boxed{56 \text{ N}}.$$

(b) We write  $\sum F_x = ma_x$  from the force diagram for the trap door:

$$F_{\text{hinge}H} - T \sin \theta = 0, \text{ which gives}$$

$$F_{\text{hinge}H} = (56 \text{ N}) \sin 55^\circ = 46 \text{ N}.$$

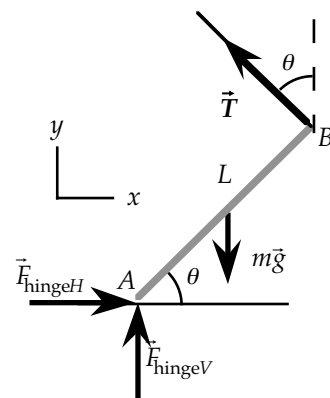
We write  $\sum F_y = ma_y$  from the force diagram for the trap door:

$$F_{\text{hinge}V} + T \cos \theta - Mg = 0, \text{ which gives}$$

$$F_{\text{hinge}V} = (20 \text{ kg})(9.8 \text{ m/s}^2) - (56 \text{ N}) \cos 55^\circ = 164 \text{ N}.$$

Combining these two components, we get

$$\vec{F}_{\text{hinge}} = \boxed{(164\hat{i} + 46\hat{j}) \text{ N}}.$$



34. We choose the coordinate system shown, with positive torques clockwise.

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the beam:

$$\sum \tau_A = \frac{1}{2}Mg(L \sin \theta) - T(L \cos \theta) = 0, \text{ which gives}$$

$$\begin{aligned} T &= \frac{1}{2}Mg \tan \theta \\ &= \frac{1}{2}(12 \text{ kg})(9.8 \text{ m/s}^2) \tan 15^\circ \\ &= \boxed{16 \text{ N}}. \end{aligned}$$

We write  $\sum F_x = ma_x$  from the force diagram for the beam:

$$F_H + T = 0, \text{ which gives}$$

$$F_H = \boxed{-16 \text{ N}}.$$

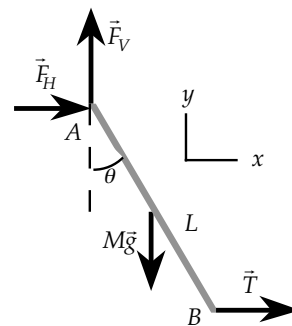
We write  $\sum F_y = ma_y$  from the force diagram for the beam:

$$F_V - Mg = 0, \text{ which gives}$$

$$F_V = (12 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{118 \text{ N}}.$$

Combining these two components, we get

$$\vec{F} = \boxed{(-16\hat{i} + 118\hat{j}) \text{ N}}.$$



35. We choose the coordinate system shown, with positive torques clockwise.

We write  $\sum \tau = I\alpha$  about the point B from the force diagram for the beam:

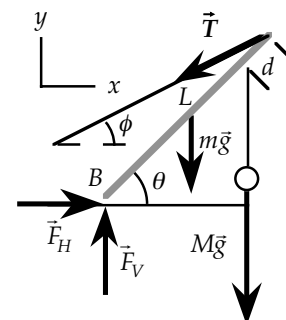
$$\begin{aligned} \sum \tau_B &= mg\left(\frac{1}{2}L \cos \theta\right) + Mg[(L - d) \sin \theta] - T \cos \phi (L \sin \theta) - T \sin \phi (L \cos \theta) \\ &= 0, \end{aligned}$$

which gives

$$M = \{[TL(\cos \phi \sin \theta - \sin \phi \cos \theta)/g] - \frac{1}{2}mL \cos \theta\} / [(L - d) \cos \theta].$$

We see that maximum M corresponds to maximum T, so we have

$$\begin{aligned} M_{\max} &= \{[(10 \times 10^3 \text{ N})(3.00 \text{ m})(\cos 30^\circ \sin 45^\circ - \sin 30^\circ \cos 45^\circ)/(9.8 \text{ m/s}^2)] - \\ &\quad \frac{1}{2}(100 \text{ kg})(3.00 \text{ m}) \cos 45^\circ\} / [(3.00 \text{ m} - 0.25 \text{ m}) \cos 45^\circ] \\ &= \boxed{353 \text{ kg}}. \end{aligned}$$



36. For static equilibrium we can write

$$\sum F_x = 0, \sum F_y = 0, \sum \tau = 0.$$

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the door:

$$\sum \tau_A = F_{1x}d + \left(\frac{1}{2}Mgw\right) = 0, \text{ which gives}$$

$$\boxed{F_{1x} = -Mgw/2d}.$$

We write  $\sum F_x = ma_x$  from the force diagram for the door:

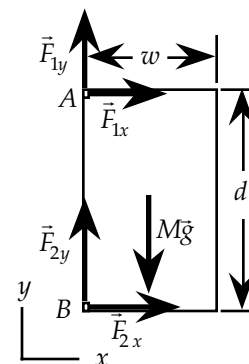
$$F_{1x} + F_{2x} = 0, \text{ which gives}$$

$$\boxed{F_{2x} = Mgw/2d}.$$

We write  $\sum F_y = ma_y$  from the force diagram for the door:

$$F_{1y} + F_{2y} - Mg = 0.$$

The system is underdetermined. We cannot determine  $F_{1y}$  and  $F_{2y}$  separately.



37. We simplify the three-dimensional system by showing a top view of the table, with the  $z$ -axis out of the page. The normal forces at the legs come out of the page and the forces of gravity are into the page. With the teacup placed on the diagonal, we know from symmetry that  $F_{N2} = F_{N3}$ .

- (a) When the teacup is placed so that  $x > 0$ , there will be a normal force at leg 4, and thus  $F_{N1} = 0$ . We write  $\sum \tau = I\alpha$  about the  $x$ -axis from the force diagram:

$$\sum \tau_{x\text{-axis}} = F_{N3}L + F_{N4}L - Mg(\frac{1}{2}L) - mg(\frac{1}{2}L + x) = 0, \text{ which gives}$$

$$F_{N4} = \frac{1}{2}Mg - \frac{1}{2}mg(1 + x/L) - F_{N3}.$$

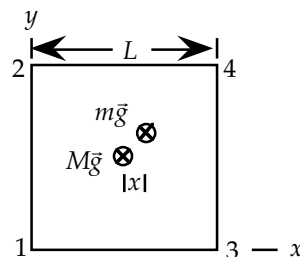
We write  $\sum F_z = ma_z$  from the force diagram:

$$2F_{N3} + F_{N4} - Mg - mg = 0.$$

By combining these two equations, we get

$$F_{N3} = F_{N2} = \frac{1}{5}(M + m)g - mgx/L;$$

$$F_{N4} = \frac{2mgx}{L}.$$



- (b) When the teacup is placed so that  $x < 0$ , there will be a normal force at leg 1 and thus  $F_{N4} = 0$ . We could repeat the analysis of part (a), but instead we write  $\sum \tau = I\alpha$  about the diagonal axis from leg 2 to leg 3:

$$\sum \tau_{\text{diagonal axis}} = F_{N1}(\frac{1}{2}\sqrt{2}L) + mg(\sqrt{2}x) = 0, \text{ which gives}$$

$$F_{N1} = \frac{2mgx}{L}, \quad x < 0.$$

From  $\sum F_z = ma_z$  we get

$$2F_{N3} + F_{N1} - Mg - mg = 0, \text{ which gives}$$

$$F_{N3} = F_{N2} = \frac{1}{5}(M + m)g + mgx/L, \quad x < 0. \quad [\text{The same as part (a).}]$$

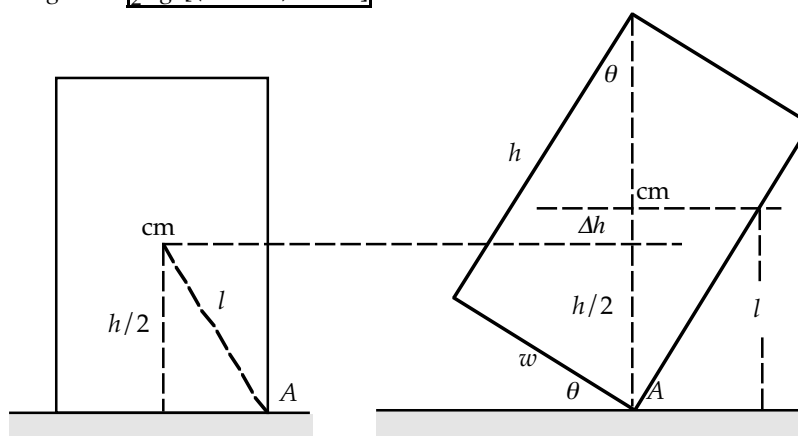
38. See the diagram below. The block must be brought to such a position that its center of mass (point O) is directly above the axis of rotation (through point A). The critical angle in question is

$$\theta = \tan^{-1}(w/h).$$

The work  $W$  done must be at least equal to the increase in gravitational potential energy of the block (assuming that the block virtually stops at the critical position). Since the center of mass rises by

$$\Delta h = l - \frac{1}{2}h = [(\frac{1}{2}h)^2 + (\frac{1}{2}w)^2]^{1/2} - \frac{1}{2}h,$$

$$W_{\min} = mg \Delta h = \frac{1}{2}mg [(h^2 + w^2)^{1/2} - h].$$



39. Assuming that the additional torque about the center of mass is introduced without adding a net force. This can be achieved with the use of a force couple (a pair of forces of equal magnitude and opposite direction, that are not co-linear). Neither the normal force nor the gravitational force exerts a torque about the center of mass, as they both pass through the cm. The force of friction has a magnitude of  $f = mg \sin \theta$  (which balances that of the component of the weight of the cylinder along the incline) and its lever arm about the cm of the cylinder is  $R$ , so it provides a torque of  $\tau_f = (mg \sin \theta)R$ . Thus to prevent the cylinder from rolling the magnitude of the extra torque  $\tau$  applied about the cm must be

$$\tau = \tau_f = \boxed{mgR \sin \theta}.$$

40. (a) Because all forces have a line of action through one point, we can not use the torque equations. Thus we have  

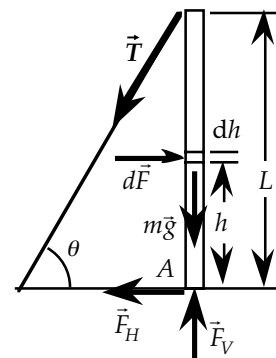
$$\boxed{\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0}.$$
- (b) For each set of opposite ropes, the above equations tell us that the two tensions must be equal. Thus after using the three equations, we still have no way of determining any of the three different tensions. The system is underdetermined.
- (c) If all of the ropes are under identical tension, we still have no way of determining the magnitude of the tension. The system is underdetermined.

41. We choose the coordinate system shown, with positive torques clockwise. We use the point A at the ground as the axis so the force exerted by the ground will produce no torque. Because the wind force is variable, we determine its torque by selecting a small element  $dh$  at the height  $h$ . We write  $\Sigma \tau = 0$  about the point A from the force diagram for the beam:

$$\Sigma \tau_A = \int_0^L h \, dF - T \cos \theta L = 0, \text{ which we can write as}$$

$$T \cos \theta L = \int_0^L h \alpha h \, dh = \frac{1}{3} \alpha h^3 \Big|_0^L = \frac{1}{3} \alpha L^3, \text{ which gives}$$

$$T = \alpha L^2 / (3 \cos \theta) = (50 \, \text{N/m}^2)(20 \, \text{m}) / (3 \cos 60^\circ) \\ = 1.33 \times 10^4 \, \text{N}.$$



42. We choose the coordinate system shown, with positive torques clockwise.  
 (a) We write  $\Sigma \tau = I\alpha$  about the point A from the force diagram for the rock:

$$\Sigma \tau_A = F_N R \sin \theta - f_s (R + R \cos \theta) = 0, \text{ which gives}$$

$$f_s = [\sin \theta / (1 + \cos \theta)] F_N = \tan(\frac{1}{2} \theta) F_N.$$

The larger the angle becomes the greater the friction force required. The maximum friction force is  $f_{s\max} = \mu_s F_N$ , so we find the maximum angle from

$$\tan(\frac{1}{2} \theta_{\max}) = f_{s\max} / F_N = \mu_s = 0.6, \text{ which gives}$$

$$\theta_{\max} = \boxed{62^\circ}.$$

- (b) We write  $\Sigma \tau = I\alpha$  about the point B from the force diagram for the rock:

$$\Sigma \tau_B = T(R + R \cos \theta) - mgR \sin \theta = 0, \text{ which gives}$$

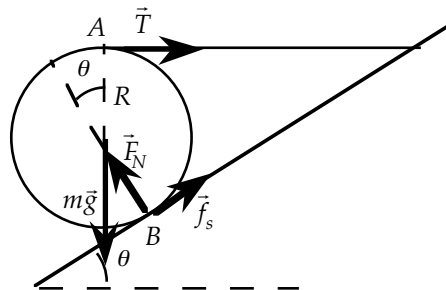
$$T_{\max} = [\sin \theta_{\max} / (1 + \cos \theta_{\max})] mg$$

$$= \tan(\frac{1}{2} \theta_{\max}) mg = \mu_s mg$$

$$= (0.6)(1088 \, \text{kg})(9.8 \, \text{m/s}^2) = \boxed{6 \times 10^3 \, \text{N}}.$$

- (c) When the angle is less than the maximum angle, from part (b) we get

$$T = \boxed{mg \tan(\frac{1}{2} \theta)}.$$



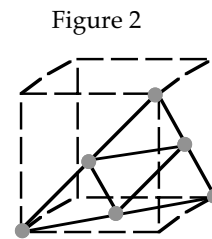
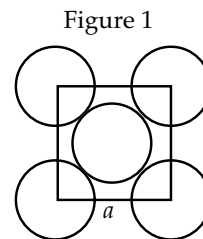


43. (a) Figure 1 shows one face of the cube with sides  $a$  and the atoms touching. The side diagonal has length  $a\sqrt{2}$ . We see that this corresponds to 2 atomic diameters:

$$2D = a\sqrt{2}, \text{ which gives}$$

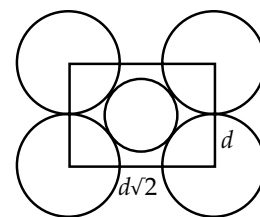
$$D = a/\sqrt{2} = (0.361 \text{ nm})/\sqrt{2} = \boxed{0.255 \text{ nm}}.$$

- (b) Figure 2 shows the atoms that lie in a plane perpendicular to the maximum diagonal, showing the triangular arrangement.



44. The figure shows the section through the cube with the shorter side an edge of the cube with length equal to the atomic diameter  $d$ . The longer side is a face diagonal of length  $d\sqrt{2}$ . The diagonal of the section is the diagonal through the cube with length  $d\sqrt{3}$ . In this length we see that there is one atomic diameter and one impurity diameter, so we have

$$d\sqrt{3} = d + d_{\text{impurity}}, \text{ which gives } d_{\text{impurity}} = \boxed{0.732d}.$$

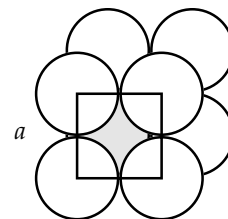


45. To determine the filling ratio we find the length of the basic cube edge  $a$  in terms of the atomic diameter  $d$  and the number of atoms in the basic cube.

Simple cubic cell: There is one atom at each corner. From the figure we see that  $a = d$ .

The atom at each of the 8 corners has  $1/8$  of its volume in the cell, so there is 1 atom/basic cube.

$$\text{Filling ratio} = (1)\frac{4}{3}\pi(\frac{1}{2}d)^3/d^3 = \boxed{0.524}.$$

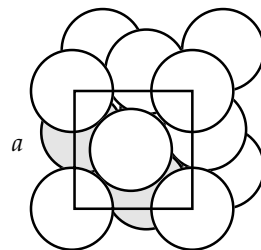


Face centered cubic cell: There is one atom at each corner and one atom at the center of each of the six faces. From the figure we see that the face diagonal contains 2 atomic diameters:

$$a\sqrt{2} = 2d, \text{ or } a = d\sqrt{2}.$$

The atom at each of the 8 corners has  $1/8$  of its volume in the cell and the atom at each of the 6 faces has  $1/2$  of its volume in the cell, so there are 4 atoms/basic cube.

$$\text{Filling ratio} = (4)\frac{4}{3}\pi(\frac{1}{2}d)^3/(d\sqrt{2})^3 = \boxed{0.740}.$$



46. From the relation between stress and strain, we have

$$\text{stress} = Y(\text{strain});$$

$$mg/A = Y(\Delta h/h);$$

$$(0.91 \times 10^5 \text{ kg})(9.8 \text{ m/s}^2)/(0.85 \text{ m}^2) = (1.7 \times 10^{10} \text{ N/m}^2)[\Delta h/(3.6 \text{ m})], \text{ which gives}$$

$$\Delta h = 2.2 \times 10^{-4} \text{ m} = \boxed{0.22 \text{ mm}}.$$

47. From the relation between stress and strain, we have

$$\text{stress} = Y(\text{strain});$$

$$mg/\pi r^2 = Y(\Delta L/L);$$

$$(800 \text{ kg})(9.8 \text{ m/s}^2)/[\pi(1.0 \times 10^{-2} \text{ m})^2] = (21 \times 10^{10} \text{ N/m}^2)[\Delta L/(250 \text{ m})], \text{ which gives}$$

$$\Delta L = 2.97 \times 10^{-2} \text{ m} = \boxed{2.97 \text{ cm}}.$$

48. If we use the lowest value of tensile strength from Table 11-2, we have

$$\text{tensile strength} = F/A;$$

$$400 \times 10^6 \text{ N/m}^2 = 3(1000 \text{ N})/(\frac{1}{4}\pi d^2), \text{ which gives } d = 3.1 \times 10^{-3} \text{ m} = \boxed{3.1 \text{ mm}}.$$

49. From the definition of critical strain, we have

$$\text{tensile strength} = Y(\Delta L/L)_c;$$

$$3000 \times 10^6 \text{ N/m}^2 = (21 \times 10^{10} \text{ N/m}^2)(\Delta L/L)_c, \text{ which gives } (\Delta L/L)_c = \boxed{0.014}.$$

$$\text{For a wire 1 m long, we have } \Delta L = (\Delta L/L)_c L = (0.014)(1 \text{ m}) = 0.014 \text{ m} = \boxed{14 \text{ mm}}.$$

50. We find the maximum weight from the tensile strength:

$$\text{tensile strength} = F/A;$$

$$3000 \times 10^6 \text{ N/m}^2 = mg / [\frac{1}{4}\pi(0.85 \times 10^{-3} \text{ m})^2], \text{ which gives } mg = \boxed{1.7 \text{ kN}}.$$

51. The shear stress is created by the horizontal friction force, so we have

$$\text{shear stress} = f/A = \mu mg/A = (0.3)(30 \text{ kg})(9.8 \text{ m/s}^2)/(0.35 \text{ m}^2) = \boxed{2.5 \times 10^2 \text{ N/m}^2}.$$

52. We find Young's modulus from

$$Y = \text{stress}/\text{strain} = (mg/A)/(\Delta L/L)$$

$$= [(60.0 \text{ kg})(9.8 \text{ m/s}^2)/\pi(2.5 \times 10^{-3} \text{ m})^2]/[(0.30 \times 10^{-3} \text{ m})/(2.0 \text{ m})] = \boxed{2.0 \times 10^{11} \text{ N/m}^2}.$$

We find the maximum mass from the tensile strength:

$$\text{tensile strength} = m_{\text{max}}g/A$$

$$1000 \times 10^6 \text{ N/m}^2 = m_{\text{max}}(9.8 \text{ m/s}^2)/[\pi(2.5 \times 10^{-3} \text{ m})^2], \text{ which gives } m_{\text{max}} = \boxed{2.0 \times 10^3 \text{ kg}}.$$

53. If we take the separation of the atoms to be  $y = 10^{-10} \text{ m}$ , we have

$$e = \Delta L/L = \Delta y/y;$$

$$(2 \text{ m})/(3 \times 10^3 \text{ m}) \approx \Delta y/(10^{-10} \text{ m}), \text{ which gives } \boxed{\Delta y \approx 10^{-13} \text{ m}}.$$

54. We find the stress that causes the maximum allowable strain:

$$\text{stress} = Y(\text{strain}_{\text{max}}) = (21 \times 10^{10} \text{ N/m}^2)[(3 \times 10^{-3} \text{ m})/(3.5 \text{ m})] = 17 \times 10^7 \text{ N/m}^2 = 180 \text{ MN/m}^2.$$

The bar does not break, because the maximum stress is less than the tensile strength.

- 55.** We call the length of the beam  $L_{\text{beam}}$ . The initial length of the cable is  $L_0 = L_{\text{beam}}/\cos 30^\circ = 2L_{\text{beam}}/\sqrt{3}$  and the distance along the wall from the beam to the cable is  $D = L_{\text{beam}} \tan 30^\circ = L_{\text{beam}}/\sqrt{3}$ . When the load is hung at the end of the beam, there must be an additional tension in the cable to maintain equilibrium:

$$\sum \tau_A = (\Delta T \sin 30^\circ)L_{\text{beam}} - Mg L_{\text{beam}} = 0, \text{ or}$$

$$\Delta T = Mg/\sin 30^\circ = (30 \text{ kg})(9.8 \text{ m/s}^2)/\sin 30^\circ = 588 \text{ N},$$

which is an additional stress:

$$\Delta T/\pi r^2 = 588 \text{ N}/[\pi(1 \times 10^{-3} \text{ m})^2] = 1.87 \times 10^8 \text{ N/m}^2.$$

(Note that even when the initial tension is added, this is less than the tensile strength, so the cable does not break.)

The additional stress produces an elongation of the cable:

$$\Delta L/L_0 = \Delta T/AY.$$

This elongation causes the beam to drop below the horizontal (exaggerated in the diagram). We find the angle  $\theta$  from a geometrical formula for a triangle:

$$D^2 + L_{\text{beam}}^2 - 2DL_{\text{beam}} \cos \theta = (L_0 + \Delta L)^2 = L_0^2(1 + \Delta L/L_0)^2 \approx L_0^2(1 + 2\Delta L/L_0),$$

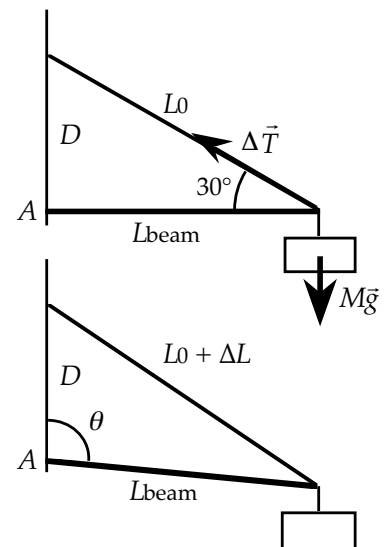
where we have used the fact that  $\Delta L \ll L_0$ . Because  $D^2 + L_{\text{beam}}^2 = L_0^2$ , this becomes

$$-2(L_{\text{beam}}/\sqrt{3})L_{\text{beam}} \cos \theta = (2L_{\text{beam}}/\sqrt{3})^2 2\Delta L/L_0, \text{ which reduces to}$$

$$\cos \theta = -(4/\sqrt{3})(\Delta L/L_0) = -(4/\sqrt{3})(\Delta T/AY)$$

$$= (4/\sqrt{3})(588 \text{ N})/[\pi(1 \times 10^{-3} \text{ m})^2(2.1 \times 10^5 \text{ MN/m}^2)] = 0.00206, \text{ which gives } \theta = 90.12^\circ.$$

Thus the beam is 0.12° below the horizontal.



56. The compressional strain is

$$e = \Delta L / L = (-0.1 \times 10^{-3} \text{ m}) / (0.5 \text{ m}) = -2.0 \times 10^{-4}.$$

We find the lateral strains from Poisson's ratio:

$$\Delta h / h = \Delta w / w = -\sigma e = -(0.32)(-2.0 \times 10^{-4}) = 0.64 \times 10^{-4}.$$

Because the volume is  $V = h w L$ , when there are small changes in the dimensions, we have

$$\Delta V = w L \Delta h + h L \Delta w + h w \Delta L.$$

When we divide this by  $V = h w L$  to get the volume strain, we have

$$\Delta V / V = \Delta h / h + \Delta w / w + \Delta L / L = 2(0.64 \times 10^{-4}) - (2.0 \times 10^{-4}) = \boxed{-7.2 \times 10^{-5}}.$$

57. From Example 11-9, the compressional strain is

$$e = \Delta L / L = 1.2 \times 10^{-3}.$$

We find the fractional change in the diameter from Poisson's ratio:

$$\Delta d / d = -\sigma e = -0.3(1.2 \times 10^{-3}) = -3.6 \times 10^{-4}.$$

The change in the diameter is

$$\Delta d = (-3.6 \times 10^{-4})(2.0 \text{ cm}) = -7.2 \times 10^{-4} \text{ cm, so the new diameter is}$$

$$d' = d + \Delta d = 2.0 \text{ cm} - (7.2 \times 10^{-4} \text{ cm}) = \boxed{1.99928 \text{ cm}}.$$

58. We choose the
- $x$
- axis along the longitudinal direction. From symmetry, the
- $y$
- and
- $z$
- axes will have equal stresses and strains. The direct elongation from a stress is

$$e = \Delta L / L = (1 / Y)(F / A).$$

The induced transverse elongation is determined by Poisson's ratio:

$$e_{\text{tr}} = -\sigma e = -\sigma(1 / Y)(F / A).$$

We will ignore the strain induced in the  $y$ -direction from the stress in the  $z$ -direction. (This is equivalent to keeping terms up to order  $\sigma^2$  only.) The total strains in the  $x$ - and  $y$ -directions are

$$e_x = (1 / Y)(F / A)_x - \sigma(1 / Y)(F / A)_y - \sigma(1 / Y)(F / A)_z = (1 / Y)(F / A)_x - 2\sigma(1 / Y)(F / A)_y;$$

$$e_y = (1 / Y)(F / A)_y - \sigma(1 / Y)(F / A)_x.$$

To have zero strain in the  $y$ -direction, we must have a pressure on the sides:

$$\boxed{(F / A)_y = \sigma(F / A)_x}.$$

The total compression in the longitudinal direction is

$$e_x = (1 / Y)(F / A)_x - 2\sigma(1 / Y)(F / A)_y = \boxed{(1 - 2\sigma^2)(1 / Y)(F / A)_x}.$$

We find the effective Young's modulus from

$$Y' = (F / A)_x / e_x = Y / (1 - 2\sigma^2).$$

We see that  $Y' > Y$ , and the bar is stiffer, i. e., there is smaller strain for the same stress.

59. (a) We have an induced transverse strain, given by
- $e_{\text{tr}} = -\sigma e_1$
- . The total strain on a side of the cube will be due to the direct strain and the induced transverse strain from the other two sides:

$$e = \Delta L / L = e_1 + 2e_{\text{tr}} = e_1(1 - 2\sigma).$$

The volume of the cube is  $V = L^3$ , so a small change in  $L$  means a small change in  $V$ , given by

$$\Delta V = 3L^2 \Delta L, \text{ which gives a volume strain:}$$

$$\Delta V / V = 3 \Delta L / L = 3e_1(1 - 2\sigma).$$

If we use the relation between stress and strain, we have

$$\Delta V / V = 3[(F / A) / Y](1 - 2\sigma).$$

All three sides are under pressure, which is the stress, so we have for the magnitude

$$\Delta V / V = 3p(1 - 2\sigma) / Y.$$

- (b) The bulk modulus is defined as

$$B = -p / (\Delta V / V).$$

If we substitute the given volume change, which must be negative for a positive pressure, we get

$$B = -p / [-3p(1 - 2\sigma) / Y] = Y / [3(1 - 2\sigma)].$$

60. We rearrange the expression for the bulk modulus:

$$\Delta V/V = -p/B = -3p(1-2\sigma)/Y.$$

If  $p > 0$  and  $2\sigma > 1$ , we have  $\Delta V/V > 0$ , and a positive pressure would cause an increase in volume; thus  $\sigma < 0.5$ .

61. For the compressibility, we have

$$\begin{aligned}\text{compressibility} &= -\Delta V/pV = +3(1-2\sigma)/Y \\ &= 3[1-2(0.38)]/(7.9 \times 10^{10} \text{ N/m}^2) = \boxed{9.1 \times 10^{-12} \text{ m}^2/\text{N}}.\end{aligned}$$

62. For the simple spring, we have a potential energy

$$U = \frac{1}{2}kx^2 = \frac{1}{2}Fx, \text{ where } F \text{ is the magnitude of the restoring force.}$$

The relation between stress and strain is

$$F/A = Y\epsilon = Yx/L, \text{ where } x \text{ is the elongation, which gives}$$

$$F = YAx/L, \text{ which has the same form as the spring force, with } k = YA/L.$$

The potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(YA/L)x^2, \text{ which can be rearranged:}$$

$$U = \frac{1}{2}Y(x/L)(x/L)(AL) = \frac{1}{2}Y(x/L)(x/L)V.$$

The potential energy per unit volume is

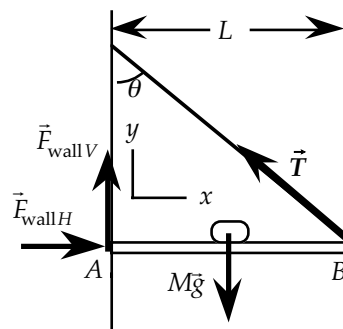
$$u = U/V = \frac{1}{2}(\text{stress})(\text{strain}).$$

63. We choose the coordinate system shown, with positive torques clockwise. From the symmetry of the placement, the tensions in the two cables will be equal.

We write  $\sum \tau = I\alpha$  about the axis  $A$  from the force diagram for the shelf and sack:

$$\sum \tau_A = Mg(\frac{1}{2}L) - (2T \sin \theta)L = 0, \text{ which gives}$$

$$\begin{aligned}T &= \frac{1}{4}Mg/\sin \theta \\ &= \frac{1}{4}(20 \text{ kg})(9.8 \text{ m/s}^2)/\sin 45^\circ = \boxed{69 \text{ N}}.\end{aligned}$$



64. We choose the coordinate system shown, with positive torques clockwise. We find the angle  $\theta$  from

$$\begin{aligned}\tan \theta &= \frac{1}{3}H/\frac{1}{2}D = [\frac{1}{3}(5 \text{ m})]/[\frac{1}{2}(1.5 \text{ m})] \\ &= 2.22, \text{ which gives } \theta = 65.8^\circ.\end{aligned}$$

We assume there is no horizontal force at the ground.

Using Newton's third law, we know that the forces at the top between the beams will have equal magnitudes and opposite directions. From symmetry, they and the forces from the crossbeam must be horizontal.

We write  $\sum \tau = I\alpha$  about the point  $B$  from the force diagram for the left roof beam:

$$\sum \tau_B = F_{NA}(L \cos \theta) - Mg(\frac{1}{2}L) \cos \theta - F(\frac{1}{3}L) \sin \theta = 0.$$

We write  $\sum F_y = ma_y$  from the force diagram for the left roof beam:

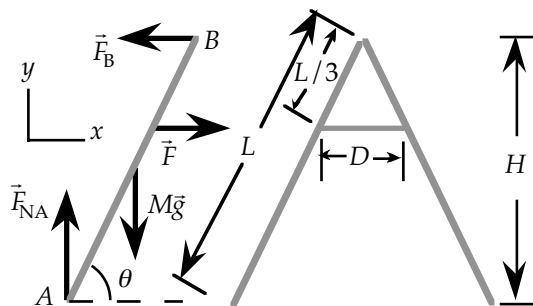
$$F_{NA} - Mg = 0, \text{ which gives } F_{NA} = Mg.$$

If we use this in the torque equation, we get

$$\begin{aligned}F &= Mg(\frac{1}{2}L) \cos \theta / [(\frac{1}{3}L) \sin \theta] = 3Mg/(2 \tan \theta) \\ &= 3(3000 \text{ kg})(9.8 \text{ m/s}^2)/(2 \tan 65.8^\circ) = 1.98 \times 10^4 \text{ N}.\end{aligned}$$

- (a) The positive result means that our assumed direction for  $F$  is correct, so the crossbeam **pulls in**.

- (b) The force on the crossbeam is the reaction to  $F$ :  **$1.98 \times 10^4 \text{ N outward}$** .



65. We choose the coordinate system shown, with positive torques counterclockwise.

(a) We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the beam:

$$\sum \tau_A = Mg(\frac{1}{2}L) \cos \theta_0 - TL \sin \theta_0 = 0, \text{ which gives } T = \frac{1}{2}Mg \cot \theta_0.$$

(b) When the cable snaps, the tension is zero, so we have

$$\sum \tau_A = I\alpha;$$

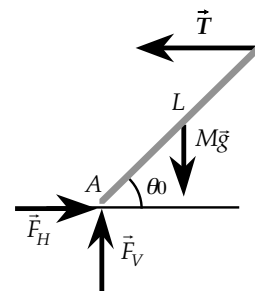
$$Mg(\frac{1}{2}L) \cos \theta_0 = \frac{1}{3}ML^2\alpha, \text{ which gives } \alpha = (3g \cos \theta_0)/2L.$$

(c) The angular acceleration is not constant. Rather than integrate, we use the work-energy theorem, with the reference level for potential energy at the horizontal position. No work is done by the force at the pivot, so we have

$$W = \Delta K + \Delta U$$

$$0 = (\frac{1}{2}I_A\omega^2 - 0) + Mg(0 - \frac{1}{2}L \sin \theta_0).$$

$$\text{When we use } I_A = \frac{1}{3}ML^2, \text{ we get } \omega = \sqrt{(3g \sin \theta_0)/L}.$$



66. We choose the coordinate system shown, with positive torques clockwise.

We write  $\sum F_y = ma_y$  from the force diagram for the board:

$$F_N - mg = 0, \text{ which gives } F_N = mg.$$

We write  $\sum \tau = I\alpha$  about the point B from the force diagram for the board:

$$\sum \tau_B = mg(\frac{1}{2}L) \sin \theta - F_N L \sin \theta + f_s L \cos \theta = 0, \text{ which gives}$$

$$f_s = (-\frac{1}{2}mg + F_N) \tan \theta = \frac{1}{2}mg \tan \theta.$$

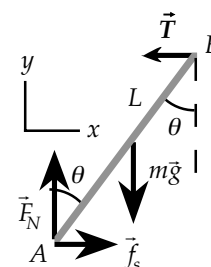
Because static friction does only what is needed, up to its limit, we have

$$f_{s\min} = 0, \text{ which gives } \theta_{\min} = 0 \text{ and}$$

$$f_{s\max} = \mu_s F_N = \mu_s mg, \text{ which gives } \tan \theta_{\max} = 2\mu_s = 2(0.32) = 0.64, \theta_{\max} = 33^\circ.$$

We write  $\sum F_x = ma_x$  from the force diagram for the board:

$$T - f_s = 0, \text{ which gives } T = \frac{1}{2}mg \tan \theta = \frac{1}{2}(47 \text{ N}) \tan \theta = (23 \text{ N}) \tan \theta.$$



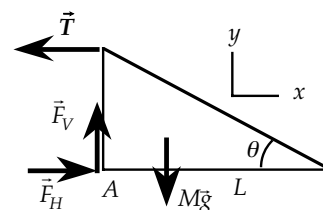
67. We choose the coordinate system shown, with positive torques clockwise.

The center of mass of a triangle is  $\frac{1}{3}$  the distance from its base.

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the sign:

$$\sum \tau_A = Mg(\frac{1}{3}L) - T(L \tan \theta) = 0, \text{ which gives}$$

$$T = \frac{1}{3}Mg / \tan \theta = \frac{1}{3}(15 \text{ kg})(9.8 \text{ m/s}^2) / \tan 30^\circ = 85 \text{ N}.$$



68. We choose the coordinate system shown for this 3-dimensional system. The y-axis lies along the axle of the wheels. We write  $\sum \tau = I\alpha$  about the y-axis from the force diagram for the car:

$$\sum \tau_{y\text{-axis}} = MgL_1 - F_{N\text{jack}}(L_1 + L_2) = 0, \text{ which gives}$$

$$F_{N\text{jack}} = MgL_1 / (L_1 + L_2)$$

$$= Mg(0.80 \text{ m}) / (0.80 \text{ m} + 2.10 \text{ m}) = 0.28Mg.$$

We write  $\sum \tau = I\alpha$  about the x-axis from the force diagram for the car:

$$\sum \tau_{x\text{-axis}} = +F_{N\text{jack}}(\frac{1}{2}H + d) + F_{NR}H - Mg(\frac{1}{2}H) = 0, \text{ which}$$

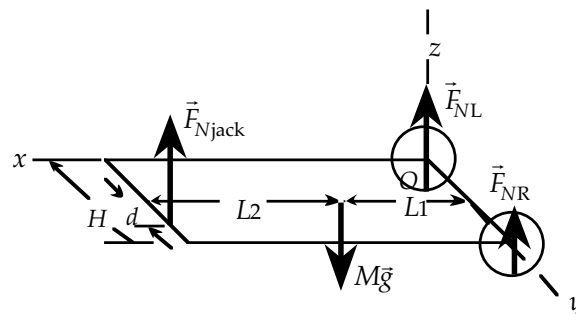
gives

$$F_{NR} = \frac{1}{2}(Mg - F_{N\text{jack}}) - (F_{N\text{jack}}d/H) = \frac{1}{2}(Mg - 0.28Mg) - 0.28Mg(0.40 \text{ m}/1.60 \text{ m}) = 0.29Mg.$$

We write  $\sum F_z = ma_z$  from the force diagram for the car:

$$F_{NL} + F_{NR} + F_{N\text{jack}} - Mg = 0, \text{ which gives}$$

$$F_{NL} = Mg - F_{NR} - F_{N\text{jack}} = Mg - 0.29Mg - 0.28Mg = 0.43Mg.$$



69. To climb the step, the roller will roll about the contact point A.

We choose the coordinate system shown, with positive torques clockwise. We find the angle  $\theta$  from

$$\sin \theta = (R - h)/R \\ = (30 \text{ cm} - 15 \text{ cm})/(30 \text{ cm}) = 0.50, \text{ which gives } \theta = 30^\circ.$$

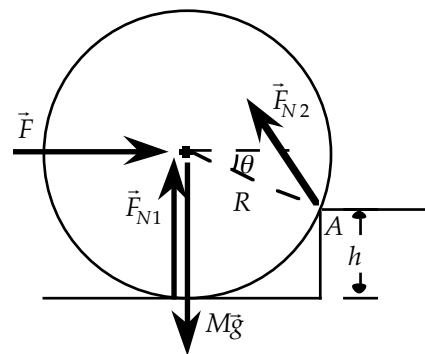
When the roller goes over the curb, contact with the ground is lost and  $F_{N1} = 0$ .

We write  $\Sigma \tau = I \alpha$  about the point A:

$$F(R - h) - MgR \cos \theta = I_A \alpha.$$

The minimum force occurs when  $\alpha = 0$ :

$$F_{\min} = (MgR \cos \theta)/(R - h). \\ = (80 \text{ kg})(9.8 \text{ m/s}^2)(0.30 \text{ m})(\cos 30^\circ)/(0.30 \text{ m} - 0.15 \text{ m}) = \boxed{1.36 \times 10^3 \text{ N}}.$$



70. We can specify the direction of  $F$  by the angle  $\phi$  from the horizontal or the angle  $\beta$  from the line from the center to the step. Our equation for  $\Sigma \tau = I \alpha$  about the point A, with  $F_{N1} = 0$ , becomes

$$FR \sin \beta - MgR \cos \theta = 0, \text{ which gives}$$

$$F = (MgR \cos \theta)/(R \sin \beta).$$

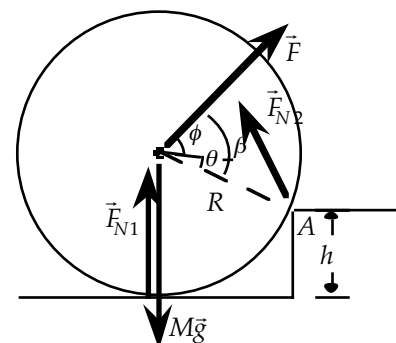
To find the angle  $\beta$  which minimizes  $F$ , we set  $dF/d\beta = 0$ :

$$dF/d\beta = Mg \cos \theta (-\cos \beta / \sin^2 \beta) = 0, \text{ which gives}$$

$$\cos \beta = 0, \text{ or } \beta = 90^\circ.$$

Thus  $\phi = \beta - \theta = 90^\circ - 30^\circ = \boxed{60^\circ \text{ above the horizontal}}.$

Note that this means  $\vec{F}$  is perpendicular to the line from the center to the step.



71. We choose the coordinate system shown, with positive torques clockwise. On the force diagram we have added the two forces on each near leg and on each far leg.

(a) We write  $\Sigma F_x = ma_x$  from the force diagram for the chest:

$$F - 2f_{\text{far}} - 2f_{\text{near}} = 0.$$

We write  $\Sigma F_y = ma_y$  from the force diagram for the chest:

$$2F_{N\text{far}} + 2F_{N\text{near}} - Mg = 0, \text{ which gives}$$

$$F_{N\text{far}} + F_{N\text{near}} = \frac{1}{2}Mg.$$

When this is used in the  $x$ -equation, we get

$$F = 2(f_{\text{far}} + f_{\text{near}}) = 2\mu_k(F_{N\text{far}} + F_{N\text{near}}) = \mu_k Mg.$$

Because three of the forces will have no torque about the point A,

we choose it as the axis and write  $\Sigma \tau = I \alpha$  from the force diagram for the chest:

$$\Sigma \tau_A = Fd - Mg(\frac{1}{2}w) + (2F_{N\text{far}})w = 0, \text{ which gives}$$

$$F_{N\text{far}} = (\frac{1}{2}Mgw - Fd)/2w = \frac{1}{4}Mg - \frac{1}{2}\mu_k Mg(d/w) \\ = Mg[\frac{1}{4} - \frac{1}{2}\mu_k(58 \text{ cm}/58 \text{ cm})] = Mg(\frac{1}{4} - \frac{1}{2}\mu_k).$$

From the  $y$ -equation, we get

$$F_{N\text{near}} = \frac{1}{2}Mg - Mg(\frac{1}{4} - \frac{1}{2}\mu_k) = Mg(\frac{1}{4} + \frac{1}{2}\mu_k).$$

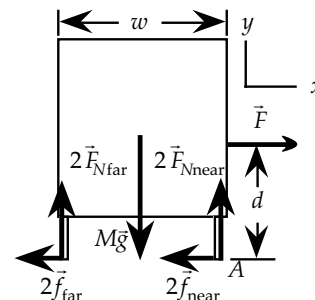
When we include the friction forces, we have

$$\text{near legs: } \boxed{Mg(\frac{1}{4} + \frac{1}{2}\mu_k) \text{ up and } \mu_k Mg(\frac{1}{4} + \frac{1}{2}\mu_k) \text{ to the left;}}$$

$$\text{far legs: } \boxed{Mg(\frac{1}{4} - \frac{1}{2}\mu_k) \text{ up and } \mu_k Mg(\frac{1}{4} - \frac{1}{2}\mu_k) \text{ to the left.}}$$

(b) If the chest topples, the forces on the far legs become zero, so we have

$$F_{N\text{far}} = Mg(\frac{1}{4} - \frac{1}{2}\mu_k) = 0, \text{ which gives } \mu_k = \boxed{0.5}.$$



72. Suppose a horizontal external force of magnitude  $F$  is applied to one side of the box at a height  $x$  above the ground, and that, as a result, the box is on the verge of toppling over but has not started sliding. Then the box is making contact with the ground at axis  $A$ , about which the torque applied by  $F$  is  $-Fx$ , while that due to the weight of the box is  $Mg(\frac{1}{2}w)$ , with  $w$  the width of the base of the box. The net torque is

$$\sum \tau_A = -Fx + Mg(\frac{1}{2}w) = 0, \text{ or } F = Mgw/2x.$$

Meanwhile, the net force exerted on the box is

$$\sum F = F - f = 0, \text{ or } F = f.$$

Since sliding has not started,

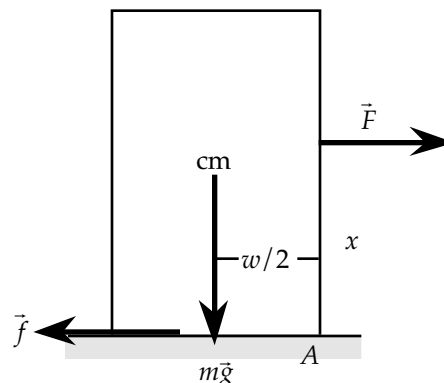
$$F = f < f_{\max} = \mu Mg, \text{ or } F < \mu Mg. \text{ Thus}$$

$$F = Mgw/2x < \mu Mg, \text{ which gives}$$

$$x > w/2\mu.$$

This is the condition for the box to topple over *before* it slides. The condition for the box to slide first must then be

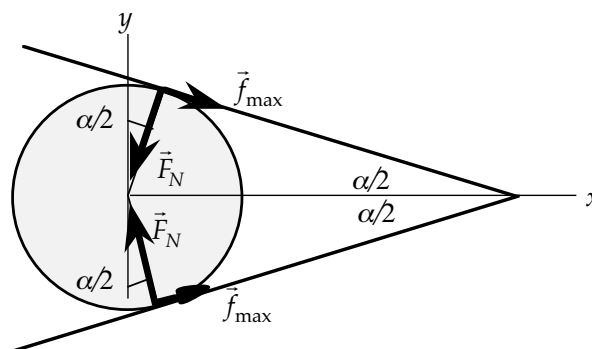
$$x < w/2\mu = 0.3 \text{ m} / 0.70 = \boxed{0.4 \text{ m}}.$$



73. The free-body diagram for the nut is as shown. Since  $\alpha$  is the largest angle that enables the nut to stop, the static friction  $f$  is at its maximum value:  $f_{\max} = \mu F_N$ . For mechanical equilibrium set the net force on the nut to be zero. Due to the symmetry of the setup this is always the case in the  $y$ -direction. The net force in the  $x$ -direction is

$$\begin{aligned} \sum F_x &= 2f_{\max} \cos(\frac{1}{2}\alpha) - 2F_N \sin(\frac{1}{2}\alpha) \\ &= 2\mu F_N \cos(\frac{1}{2}\alpha) - 2F_N \sin(\frac{1}{2}\alpha) = 0, \text{ or} \\ \mu &= [2F_N \sin(\frac{1}{2}\alpha)] / [2F_N \cos(\frac{1}{2}\alpha)] \\ &= \tan(\frac{1}{2}\alpha), \end{aligned}$$

regardless of the value of  $F_N$ .



74. We choose the inertial ground coordinate system shown, with positive torques clockwise. We assume the effective normal force acts at point  $A$ , a distance  $x$  from the centerline. The friction force on the bed of the truck provides the acceleration of the box.

We write  $\sum F_x = ma_x$  from the force diagram for the box:

$$f = ma.$$

We write  $\sum F_y = ma_y$  from the force diagram for the box:

$$F_N - mg = 0, \text{ which gives}$$

$$F_N = mg.$$

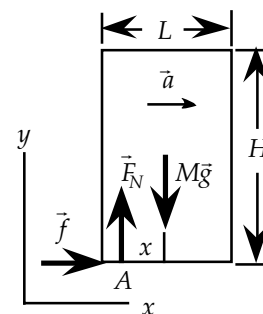
If we assume that the box does not topple, there will be no angular motion about the center of mass. To find  $x$ , we write  $\sum \tau = I\alpha$  about the center of mass from the force diagram for the box:

$$\sum \tau_A = f(\frac{1}{2}H) + F_N x = 0, \text{ which gives}$$

$$\begin{aligned} x &= \frac{1}{2}fH / F_N = \frac{1}{2}maH / mg = \frac{1}{2}aH / g \\ &= \frac{1}{2}(1.2 \text{ m/s}^2)(1.5 \text{ m}) / (9.8 \text{ m/s}^2) \\ &= 0.092 \text{ m}. \end{aligned}$$

Because this is less than  $\frac{1}{2}L = 0.3 \text{ m}$ , **the box will not topple**.

[Note that if the non-inertial reference frame on the truck is used, a non-inertial force  $-ma$  must be added at the center of mass. The axis for torques could then be the corner of the box.]



75. We choose the coordinate system shown, with positive torques clockwise. We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the cylinder:

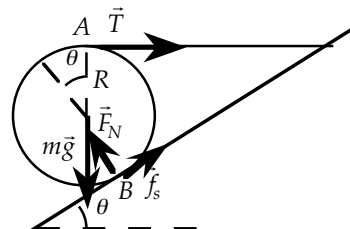
$$\sum \tau_A = F_N R \sin \theta - f_s (R + R \cos \theta) = 0, \text{ which gives}$$

$$f_s = [\sin \theta / (1 + \cos \theta)] F_N = \tan(\frac{1}{2} \theta) F_N.$$

Because  $f_s \leq \mu_s F_N$ , the minimum value of  $\mu_s$  is when  $f_s = \mu_s F_N$ . So

$$\mu_{s\min} F_N = \tan(\frac{1}{2} \theta) F_N, \text{ which gives}$$

$$\mu_{s\min} = \boxed{\tan(\frac{1}{2} \theta)}$$



76. We choose the coordinate system shown, with positive torques clockwise. From the symmetry of the system, the tensions in the interior rods must be equal. We write  $\sum F_y = ma_y$  from the force diagram for the middle mass:

$$2T \cos \theta - Mg = 0, \text{ which gives}$$

$$T = \frac{1}{2} Mg \cos \theta.$$

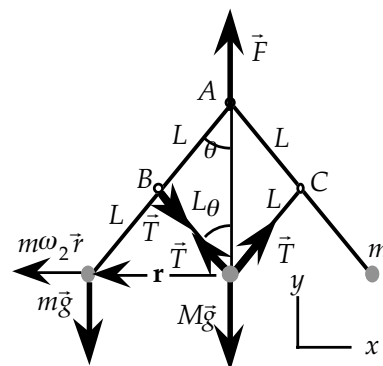
The other masses are rotating in a circle of radius  $2L \sin \theta$ , with a centripetal acceleration  $r\omega^2$ . In the rotating non inertial frame, we add an inertial (centrifugal) force  $m r \omega^2$  away from the center, so we can treat the mass  $m$  as being in equilibrium.

We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the left-hand rod:

$$\sum \tau_A = m(2L \sin \theta) \omega^2 (2L \cos \theta) - mg(2L \sin \theta) - T \cos \theta (L \sin \theta) - T \sin \theta (L \cos \theta) = 0.$$

When we substitute for  $T$ , we get

$$\boxed{\omega^2 = (2m + M)g / (4mL \cos \theta)}.$$



77. The net force exerted on the ball is zero, so

$$F_{NA} - T \sin \theta = 0 \text{ in the horizontal direction and}$$

$$f + T \cos \theta - mg = 0 \text{ in the vertical direction.}$$

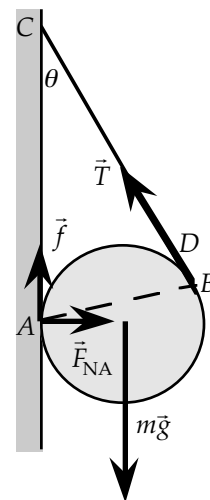
Also, the net torque about point A must vanish. The lever arm of  $T$  about point A is  $AD = CA \sin \theta = CB \sin \theta$ , and that of the weight of the ball is  $R$ , its radius. Here  $\theta = \sin^{-1}(11 \text{ cm} / 90 \text{ cm}) = 7.02^\circ$ . Thus

$$\sum \tau_A = T(CB \sin \theta) - mgR = 0.$$

And when  $\mu$  is at its smallest possible value

$$f = f_{\max} = \mu F_{NA}. \text{ Combine these equations to obtain}$$

$$\mu = CB/R - \cot \theta = 90 \text{ cm} / 6.0 \text{ cm} - \cot 7.02^\circ = \boxed{6.88}.$$





78. According to the force diagram, we know that all forces must be in a plane. The analysis for the top ball will be

$$F_{N2} \cos \theta = mg \text{ and } F_{N2} \sin \theta = F_{N1}.$$

Since  $\sin \theta = (4.00 - 2.66)/2.66 = 0.504$  and  $\cos \theta = 0.864$  we have

$$F_{N2} = 1.16mg, \quad F_{N1} = 0.585mg.$$

$\sum \tau = I\alpha$  about the point  $B$  from the force diagram for the middle ball:

$$(F_{N2} \sin \theta)(2R \cos \theta) + mgR \sin \theta - F_{N4}R \cos \theta = 0,$$

which gives

$$F_{N4} = mg \tan \theta + 2 F_{N2} \sin \theta = 1.75mg.$$

We write  $\sum F_z = ma_z$  from the force diagram for the middle ball:

$$F_{N3} \cos \theta - mg - F_{N2} \cos \theta = 0, \text{ so}$$

$$F_{N3} = F_{N2} + mg / \cos \theta = 2.31mg.$$

To summarize:

Middle ball:

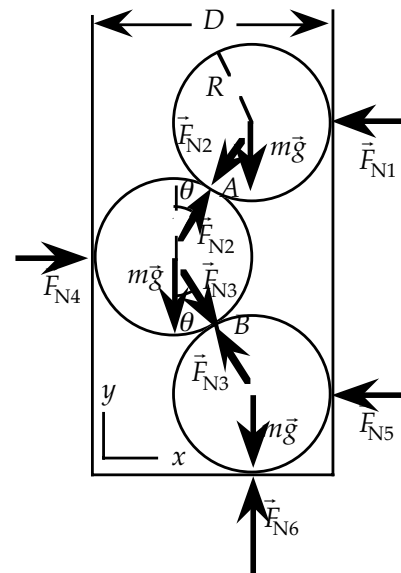
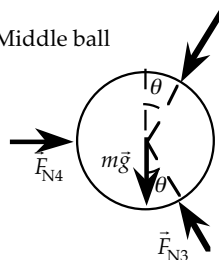
$$F_{N1} = 0.585mg$$

$$F_{N2} = 1.16mg$$

$$F_{N3} = 2.31mg$$

$$F_{N4} = 1.75mg$$

Middle ball



79. From the symmetry, we know that the contact between the balls is at the center of the cylinder. This allows us to find the angle  $\theta$  by adding the horizontal segments:

$$R + R \sin \theta + R \sin \theta + R = D, \text{ which gives}$$

$$\sin \theta = D/2R - 1 = (6 \text{ cm})/[2(2 \text{ cm})] - 1 = 0.5; \quad \theta = 30^\circ.$$

To have  $\sum \vec{\tau} = 0$ , the three forces on the top ball lie in a plane and intersect at the center of the ball. As expected, all normal forces are perpendicular to the contact surface. Thus for the bottom ball, we must also have the four forces in the same plane.

We choose the coordinate system shown, with positive torques clockwise. We write  $\sum \tau = I\alpha$  about the point  $A$  from the force diagram for the top ball:

$$\sum \tau_A = mgR \sin \theta - F_{N1}R \cos \theta = 0, \text{ which gives}$$

$$F_{N1} = mg \tan \theta = mg \tan 30^\circ = 0.577mg.$$

We write  $\sum F_x = ma_x$  from the force diagram for the top ball:

$$F_{N2} \sin \theta - F_{N1} = 0, \text{ which gives}$$

$$F_{N2} = F_{N1} / \sin \theta = 1.155mg.$$

We write  $\sum \tau = I\alpha$  about the point  $B$  from the force diagram for the bottom ball:

$$\sum \tau_B = F_{N4}R - F_{N2}R \sin \theta = 0, \text{ which gives}$$

$$F_{N4} = F_{N2} \sin \theta = F_{N1} = 0.577mg.$$

(This is expected. Consider the two balls as a system and use  $\sum F_x = ma_x$ .)

We write  $\sum F_z = ma_z$  from the force diagram for the bottom ball:

$$F_{N3} - mg - F_{N2} \cos \theta = 0, \text{ which gives}$$

$$F_{N3} = mg + F_{N2} \cos \theta = 2.000mg.$$

To summarize:

$$\text{Top ball: } mg, \quad F_{N1} = 0.577mg, \quad F_{N2} = 1.155mg;$$

$$\text{Bottom ball: } mg, \quad F_{N2} = 1.155mg, \quad F_{N3} = 2.000mg, \quad F_{N4} = 0.577mg.$$

