

CHAPTER 6 Work and Kinetic Energy

Answers to Understanding the Concepts Questions

1. The real motion of a baseball depends to a large extent on the fact that it rotates and that it is not smooth. But we can proceed here by ignoring these complications and thinking about the drag force as a small effect compared to the gravity-induced projectile motion. In the motion of a thrown baseball (as opposed to that of a pop-up), the change in vertical velocity is small, as is the distance the ball falls. Then we need to worry only about the horizontal motion. Here we can say two things: (a) If we don't know the drag force, we can at least assume that it is constant, because the speed does not change much. We could then find the drag force by dividing the change in kinetic energy by the distance traveled. (b) If we do know the drag force, we can find the work done by that force by integrating it along a straight path and thereby predict the change in the kinetic energy. We can do this in small steps using computers, and this approach allows us to take into account the small changes in the drag force associated with the changes in the velocity of the baseball.
2. You are in a noninertial frame of reference that is decelerating with respect to Earth, so in order to understand the situation based on Newton's second law you must assume that a pseudoforce (fictitious force), opposite to the direction of the acceleration of your car, is exerted on the ball. Since your car accelerates backwards the pseudoforce pushes the ball forward, doing positive work on it and, according to the work-energy theorem, giving it a kinetic energy.
3. Yes, The work done by a force may be positive or negative, depending on the direction of the force relative to that of the velocity of the object. Throw a ball up into the air, for example, and gravity would be doing negative work on it as the ball moves upward, since the gravitational force is opposite in direction to the velocity of the ball. As another example, as you apply the brake to stop your vehicle, the work done on it by the friction of the road is also negative. In general, when a force opposes the motion of an object, it does negative work, causing the object to slow down.
4. We do expend energy when we hold a bag of groceries stationary, but the reason for this is associated with the biochemical processes that allow us to hold our muscles flexed. At the molecular level a tensed muscle is not "locked." Rather, the muscle fibers continuously are released and must be re-flexed. Work is done each time there is a tiny movement of individual muscle fibers.
5. The work done by a centripetal force is zero. According to the work-energy theorem, this force does not result in any change in the kinetic energy, and therefore speed, of the object. Indeed, if the object is subject only to a centripetal force (i.e., no tangential forces), then it would be undergoing *uniform* circular motion, with no change in speed.
6. The net work done on the piano is the same in each case. We are neglecting friction, and the work done by gravity is independent of the path taken by the piano to get from one place to another. "Note that when the crew carries the piano, they must also do work on themselves to raise themselves from ground level to the third floor.
7. No. The downward force that the crew exerts on the rope is not directly exerted on the piano, which is subject to an *upward* lifting force from the rope that does positive work on it (as the lifting force is in the same direction as the velocity of the piano — both upward).

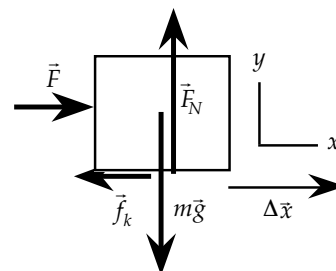
8. If the racket (whose net we assume to be made of perfectly elastic strings) is held fixed while a tennis ball strikes it and bounces off it net, the net is acting as an elastic spring, in which case the force is essentially conservative. In reality, the racket most likely would be pushed by the hand holding it; and it passes part of the (nonconservative) force exerted by the hand on to the tennis ball. In this case the total force on the tennis ball is no longer conservative. Also, the net of a real racket cannot be perfectly elastic, and a small amount of mechanical energy would be lost to friction.
9. Let 's assume for simplicity that the ship is moving uniformly. Then the net force on both the man and the smokestack is zero, and no net work is done on anything. However, the force exerted by the man on the smokestack does indeed do work. This work requires no exertion on his part. The force of friction that the deck exerts on his shoes is in the direction of the ship. Because there is no net force on the man, the smokestack exerts a force equal and opposite to that of deck friction on the man. By Newton's third law, the force the man exerts on the smokestack is equal and opposite to the force the smokestack exerts on him; that is, the force the man exerts on the smokestack is equal to the deck friction on the man. The work done by the man on the smokestack is the force he exerts on the smokestack times the distance moved, equal to the work done on the man by deck friction. The energy for this comes from the engine, and the man only acts as a conduit for the work done. You might think about the case where the ship starts to move and everything accelerates.
10. You need to overcome friction of various origins — from the road, the air, and between your muscles and joints. Also, walking elevates your heart rate and raises the body temperature. All these require energy expenditure.
11. Once the parachutist reaches terminal speed the work done on him or her is zero, because there is no change in his or her kinetic energy. Work is done only during the period over which the parachutist reaches terminal speed. In the measure that the distance over which terminal speed is reached depends only weakly on the starting height, the net work done is also independent of the starting height. Note that the net work is the sum of the work done by gravity and the drag force, which is why the work done by gravity is different from the net work.
12. No net work is done over the entire process. As the object speeds up, a tangential force must be doing positive work; and as it slows down a tangential force must be doing negative work. The net work done equals the change in the kinetic energy of the object. Since there is no net change in speed there is no net change in kinetic energy, either; so the net work done on the object must be zero.
13. Yes. Whenever a force is in the same general direction as the velocity of the object it acts on, the work done by that force is positive. This is true whether or not the force is frictional. For example, as you step on the gas pedal your vehicle accelerates forward, causing its speed (and, therefore, kinetic energy) to increase. The positive work done on the vehicle that results in the increase in its kinetic energy is due to the static friction exerted by the road on the tires. As the vehicle moves forward the tires rotates backwards, so the tendency of motion of the tires where they make contact with the ground is actually backward, causing a forward friction from the road. In this case the friction of the road does positive work.
14. The only influences on the bag are the force of gravity and the contact force from your hands. These cancel if you hold the bag stationary and there is no net work done on the bag. The forces similarly cancel if the bag is moving upward with constant velocity. No net work is done on the bag in that case as well. If you and the bag are *accelerating* upward, then there is indeed net work done on the bag. You must exert an increased force on the bag, one larger in magnitude than its weight. Remember that a floor scale reads an increased amount when you stand on it in an upward-accelerating elevator; in effect your arms must support the bag against what appears to you to be an increased weight.

15. As the car moves forward it encounters resistance (friction), both from the air and from the road. Part of the friction of the road is due to your weight (as it is proportional to the total normal force between the car and the road), and work as to be done in overcoming that portion of the resistance.
16. The textbook defines a force to be conservative if the work W done by it is independent of the path taken. This means that it is possible to define a potential energy U , such that $W = U_i - U_f$. Combine this with the work-energy theorem, $W = K_f - K_i$, to obtain $E_f = E_i$, with $E = K + U$ the total mechanical energy of the system. So the definition in the textbook leads to the conservation of mechanical energy, which means that there is no dissipation of mechanical energy into other forms (such as thermal energy, which can be caused by friction, a nonconservative force). In this sense these two definitions are equivalent.
17. There are two stages to consider here. The first stage is the one where the speed of the participants increases. At this stage there must be a tangential acceleration; the net force has a component along the displacement, and work is done on the participants. The appropriate tangential force is friction between floor and walls and the participants. The work-energy theorem is realized because the kinetic energy of the participants increases. In the second stage, the participants have reached their final speed and the floor has fallen away. The contact force between the walls and the participants keeps the participants moving along their circular path. This force does no work, as it is perpendicular to the displacement. Without net work there should be no change in kinetic energy, and indeed the speed is constant even if the velocity is not.
18. Three forces are exerted on him: his weight, and the tension in the vine, and air friction. Since the tension is always perpendicular to his velocity, it does no work. The force of gravity does positive work as he swings from the top to the bottom of the swing, speeding him up; and it does negative work as he swings back up from the bottom. The net work done by gravity is zero as he returns to the same horizontal level after completing one swing. Air friction, which is small compared with the other two forces, always opposes his motion and therefore does negative work.
19. You might measure the maximum height reached by the first acrobat; that tells you speed and hence the kinetic energy he or she was given when he or she was flipped. The work done on the first acrobat is, by the work-energy theorem, that kinetic energy.
20. Yes. As the egg moves downward while making contact with the ground, the ground exerts an upward force on the egg, doing negative work on it (as the force is opposite to the direction of motion of the egg). This negative work causes the kinetic energy of the egg to decrease and eventually to vanish.
21. The amount of work done depends on two factors: the magnitude of the force applied, and the displacement. Since the displacements are identical in this case, it is the magnitude of the force that makes a difference here. To slash through the large bone would require a greater force, so the person who chops bones instead of tender meat does more work per swing.
22. The forces acting on the sled are gravity (its weight), the tension from the rope, the contact force from the ground, and friction from the ground. If the motion is horizontal, then only the forces with horizontal components do work. Friction is purely horizontal, while tension has a horizontal component given by its magnitude times the cosine of the angle θ . As long as the sled moves with a constant velocity, the net work is zero.
23. Suppose we choose the direction of the force as positive. In the original reference frame both the force and the velocity are positive, so W is also positive. In the reference frame of the moving observer, the force is still positive (as its orientation did not change), but the direction of motion of the object is now negative; so W becomes negative.

24. The work-energy theorem offers a simple way to determine the average drag force. By the work-energy theorem, the work done by the drag force is equal to the loss of kinetic energy, which is the entire amount of kinetic energy the diver has when she enters the water. This work can be expressed as the net displacement in the water — the depth reached by the diver — times the average force. Measure the depth reached and the speed with which the diver enters the water, and you can calculate the average drag force. Did you understand the signs of the quantities that enter here?
25. You need to measure the mass m of the parachutist and the height h of his or her jump. This gives the work done by gravity as mgh . Also, we know from the parachutist's final speed v just before landing the kinetic energy gained: $\Delta K = \frac{1}{2}mv^2$. The net work done on the parachutist during the entire jump is $W_{\text{net}} = mgh + W_{\text{drag}}$, which equals ΔK according to the work-energy theorem: $W_{\text{net}} = \Delta K$, or $mgh + W_{\text{drag}} = \frac{1}{2}mv^2$. This gives the work done by the drag force as $W_{\text{drag}} = \frac{1}{2}mv^2 - mgh$.
26. (a) Drag forces are like friction, always opposed to the direction of motion and hence nonconservative. (b) Again a drag force, again nonconservative. (c) The force here is the pressure of the expanding gases behind the bullet. These are somewhat more complicated and not entirely conservative, as we shall see when we study hot gases in chapters 17-20. (d) The trampoline is like a spring. The elastic forces it is responsible for are conservative.
27. The net work done by a conservative force over any enclosed path is zero. This is not the case for the drag forces you encounter when you swim; otherwise as you complete one lap to return to your starting point you would need to have done no net work paddling the water (assuming that your initial and final speeds are both the same, say, zero), as there is no net negative work from the drag force that you need to overcome. That means that swimming would be effortless --- no matter how long the lap is, as long as your return to your initial position. This is certainly not true. In reality, you need to keep paddling to overcome the drag forces, which always do negative work on you.
28. No. While we can say that an object possesses a certain amount of kinetic energy, we cannot say that it has a certain amount of work. Rather, work is the transfer of energy. The work done on an object equals the *change* in its kinetic energy.

Solutions to Problems

1. (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(10^3 \text{ kg})(0.28 \text{ m/s})^2 = \boxed{39 \text{ J}}$.
 (b) From $K = \frac{1}{2}mv^2$, we get $v = (2K/m)^{1/2}$. Thus,
 person: $v_p = [2(39 \text{ J})/80 \text{ kg}]^{1/2} = \boxed{1.0 \text{ m/s}}$.
 bullet: $v_b = [2(39 \text{ J})/(10 \times 10^{-3} \text{ kg})]^{1/2} = \boxed{89 \text{ m/s}}$.
 (c) $K_2 = \frac{1}{2}mv_2^2 = 2K = 2(\frac{1}{2}mv^2)$;
 $v_2 = v\sqrt{2} = (0.28 \text{ m/s})\sqrt{2} = \boxed{0.40 \text{ m/s}}$.
2. (a) Because there is no change in kinetic energy, $W_{\text{net}} = \Delta K = \boxed{0}$.
 (b) Because there is no acceleration, the contact force must have the same magnitude as the gravity force. Thus,
 $W_N = F_N \Delta x = (mg) \Delta x = (85 \text{ kg})(9.8 \text{ m/s}^2)(42 \text{ m}) = \boxed{3.5 \times 10^4 \text{ J}}$.
 (c) $W_{\text{grav}} = -(mg) \Delta x = -(85 \text{ kg})(9.8 \text{ m/s}^2)(42 \text{ m}) = \boxed{-3.5 \times 10^4 \text{ J}}$.
3. (a) There is the force of gravity (down) $= (10 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{98 \text{ N}}$ and the upward pull $= \boxed{98 \text{ N}}$; $F_{\text{net}} = \boxed{0}$.
 (b) $W_{\text{net}} = \Delta K = \boxed{0}$.
 (c) $W_F = F \Delta x = mg \Delta x = (10 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m}) = \boxed{98 \text{ J}}$.
4. (a) The men must provide a force with the same magnitude as the gravity force.
 $W = F \Delta x = mg \Delta x = (38 \text{ kg})(9.8 \text{ m/s}^2)(130 \text{ m}) = \boxed{4.8 \times 10^4 \text{ J}}$.
 (b) The normal force must also have the same magnitude as the gravity force, so the same work must be done:
 $W = \boxed{4.8 \times 10^4 \text{ J}}$.
5. The work done on the crate increases its kinetic energy:
 $W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(66 \text{ kg})[(63 \text{ km/h})(10^3 \text{ m/km})/(3600 \text{ s/h})]^2 - 0 = \boxed{1.0 \times 10^4 \text{ J}}$.
6. (a) Because pushing the piano out the window does not involve a distance, $W = \boxed{0}$.
 (b) The tension has the same magnitude as the gravity force:
 $W_T = -T \Delta x = -(mg) \Delta x = -(180 \text{ kg})(9.8 \text{ m/s}^2)(25 \text{ m}) = \boxed{-4.4 \times 10^5 \text{ J}}$.
 (c) $W_g = (mg) \Delta x = \boxed{4.4 \times 10^4 \text{ J}}$.
7. Because there is no acceleration, we have $F_N = mg$ and $F = f_k = \mu_k mg$.
 (a) The work done by the man is
 $W_p = F \Delta x = 0.4(40 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = \boxed{2.4 \times 10^2 \text{ J}}$.
 (b) The friction force also does work (negative).
 (c) $W_{\text{net}} = W_p + W_f = \Delta K = \boxed{0}$.
8. (a) $W_g = -mg \Delta y = -(37 \text{ kg})(9.8 \text{ m/s}^2)(7.5 \text{ m}) = \boxed{-2.7 \times 10^3 \text{ J}}$.
 (b) The tension has the same magnitude as the gravity force:
 $W_T = (37 \text{ kg})(9.8 \text{ m/s}^2)(7.5 \text{ m}) = \boxed{+2.7 \times 10^3 \text{ J}}$. (Net work is zero.)
 (c) The person does no work directly on the load.
 The work is done on the rope as it passes through the hands.



9. The work done by the woman is

$$W_T = T \Delta y = (365 \text{ N})(2 \text{ m}) = \boxed{730 \text{ J}}.$$

We want to find the distance the lineman is raised. If the woman pulls a length L , the four segments supporting the two moveable pulleys must shorten by L . Thus each segment shortens by $\frac{1}{4}L$, which is the distance the lineman is raised:

$$W_g = mg \Delta y = (149 \text{ kg})(9.8 \text{ m/s}^2)\frac{1}{4}(2 \text{ m}) = \boxed{-730 \text{ J}}.$$

10. (a) There are two tensions. The tension in the rope being pulled is $T_1 = F$. Because all objects are considered to be in equilibrium, from the middle pulley we have

$$T_2 = 2T_1 = 2F.$$

From the right pulley we have

$$Mg = 2T_2 = 4T_1 = 4F;$$

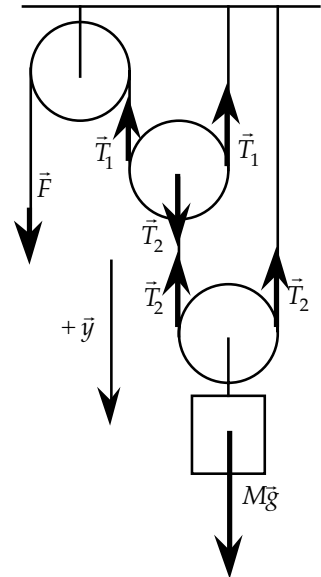
$$(40 \text{ kg})(9.8 \text{ m/s}^2) = 4F, \text{ which gives } F = \boxed{98 \text{ N}}.$$

- (b) We want to find the distance the rope is pulled. If the mass is raised a height h , the right side of the rope supporting the right pulley shortens by h , and the middle pulley must rise a height $2h$. If both sides of the rope supporting the middle pulley shorten by $2h$, the rope must be pulled a distance of $4h$. The work done by the force is

$$W_F = F 4h = (98 \text{ N})4(4 \text{ m}) = \boxed{1.6 \times 10^3 \text{ J}}.$$

- (c) The work done by gravity is

$$W_g = mgh = (40 \text{ kg})(9.8 \text{ m/s}^2)(-4 \text{ m}) = \boxed{-1.6 \times 10^3 \text{ J}}.$$



11. We convert the speed units:

$$(95.3 \text{ mi/h})(1.61 \times 10^3 \text{ m/mi})/(3600 \text{ s/h}) = 42.6 \text{ m/s}; \quad 96.6 \text{ mi/h} = 43.2 \text{ m/s}.$$

The work done by friction is the net work:

$$W_f = W_{\text{net}} = \Delta K = \frac{1}{2}m(v^2 - v_0^2) = \frac{1}{2}(0.145 \text{ kg})[(42.6 \text{ m/s})^2 - (43.2 \text{ m/s})^2] = \boxed{-3.73 \text{ J}}.$$

12. (a) $W_g = mg \Delta y = (0.24 \text{ kg})(9.8 \text{ m/s}^2)(2 \text{ m}) = \boxed{4.7 \text{ J}}.$

- (b) $W_g = -mg \Delta y = -(0.24 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = \boxed{-3.5 \text{ J}}.$

13. To find the work done we need to find the distance the load moves, which we can find by analyzing the forces on the worker and on the load, $\Sigma F_y = ma_y$:

$$\text{worker: } m_w g - T = (75 \text{ kg})(9.8 \text{ m/s}^2) - T = (75 \text{ kg})a;$$

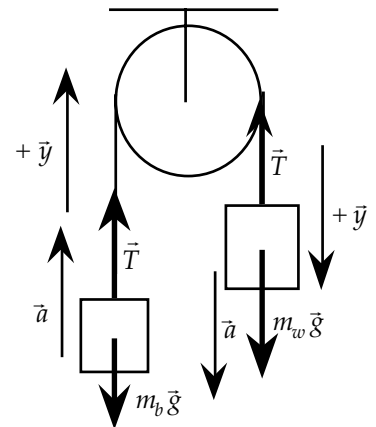
$$\text{load: } T - m_b g = T - (42 \text{ kg})(9.8 \text{ m/s}^2) = (42 \text{ kg})a.$$

By eliminating T , we find $a = 2.8 \text{ m/s}^2$.

Then the distance traveled is $y = \frac{1}{2}at^2 = \frac{1}{2}(2.8 \text{ m/s}^2)(2.0 \text{ s})^2 = 5.6 \text{ m}$.

The total work done by gravity is

$$W_g = m_w g \Delta y - m_b g \Delta y = (75 \text{ kg} - 42 \text{ kg})(9.8 \text{ m/s}^2)(5.6 \text{ m}) = \boxed{1.8 \times 10^3 \text{ J}}.$$



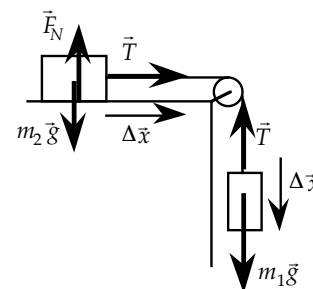
14. (a) When we take the two blocks as the system, the tension becomes an internal force. The only force that does work is the work done by gravity on block m_1 . For the work-energy theorem, we have

$$\begin{aligned} W_{\text{net}} &= \Delta K \\ &= \frac{1}{2}m_1 v^2 - \frac{1}{2}m_1 v_0^2 + \frac{1}{2}m_2 v^2 - \frac{1}{2}m_2 v_0^2 \\ &= \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 - 0; \\ m_1 g \Delta x &= \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 - 0, \text{ which gives} \\ v &= \boxed{[2m_1 g \Delta x / (m_1 + m_2)]^{1/2}}. \end{aligned}$$

- (b) Because all of the forces are constant, the acceleration will be constant.

We find it from

$$\begin{aligned} v^2 &= v_0^2 + 2a \Delta x; \\ 2m_1 g \Delta x / (m_1 + m_2) &= 0 + 2a \Delta x, \text{ which gives} \\ a &= \boxed{m_1 g / (m_1 + m_2)}. \end{aligned}$$



15. $W = mg \Delta y = (200 \times 10^3 \text{ kg/s})(1 \text{ h})(3600 \text{ s/h})(9.8 \text{ m/s}^2)(40 \text{ m}) = \boxed{2.8 \times 10^{11} \text{ J}}.$

16. The work done by the force of gravity on the ball as it falls through a distance Δy_1 ($=3.00 \text{ m}$) is given by

$$W_g = mg \Delta y_1 = (0.085 \text{ kg})(9.8 \text{ m/s}^2)(3.00 \text{ m}) = 2.499 \text{ J},$$

which according to the work-energy theorem is equal to K_1 , the kinetic energy of the ball just before it makes contact with the ground. After the contact the ball rebounds upwards by Δy_2 ($=2.75 \text{ m}$) before losing all of its kinetic energy, during which the work done by gravity on the ball is

$$W_g = -mg \Delta y_2 = (0.085 \text{ kg})(9.8 \text{ m/s}^2)(2.75 \text{ m}) = -2.291 \text{ J},$$

and so the kinetic energy of the ball just after the rebound must be

$$K_2 = -W_g = 2.291 \text{ J}. \text{ The change in kinetic energy in question is then}$$

$$\Delta K = K_2 - K_1 = 2.291 \text{ J} - 2.499 \text{ J} = \boxed{-0.21 \text{ J}}.$$

17. Let the kinetic energy of the ball before and after the n -th bounce be K_n and K_{n+1} , respectively. Then the loss in kinetic energy as a result of the n -th bounce is

$$\Delta K_n = K_{n+1} - K_n = c K_n, \text{ where } c \text{ is determined from the information of the first bounce } (n = 1)$$

studied in the previous problem:

$$\Delta K_1 = -0.21 \text{ J} = c K_1 = c (2.499 \text{ J}), \text{ which gives } c = -0.084.$$

The kinetic energy of the ball after the second bounce is then

$$K_3 = K_2 + \Delta K_2 = K_2 + c K_2 = (1 + c) K_2 = (1 - 0.084) (2.291 \text{ J}) = 2.01 \text{ J}.$$

Set $W_g = -mg \Delta y_3 = -K_3$ to obtain the height to which the ball will reach after the second bounce:

$$\Delta y_3 = K_3 / mg = 2.01 \text{ J} / [(0.084 \text{ kg})(9.8 \text{ m/s}^2)] = \boxed{2.5 \text{ m}}.$$

In general, just after the n -th bounce the kinetic energy of the ball is

$$\begin{aligned} K_{n+1} &= K_n + \Delta K_n = K_n + c K_n = (1 + c) K_n = (1 + c)^2 K_{n-1} = \dots \\ &= (1 + c)^n K_1 = (1 - 0.084)^n K_1, \end{aligned}$$

so the ball will reach a height of

$$\begin{aligned} \Delta y_{n+1} &= K_{n+1} / mg = (1 - 0.084)^n K_1 / mg \\ &= (1 - 0.084)^n (2.499 \text{ J}) / [(0.084 \text{ kg})(9.8 \text{ m/s}^2)] \\ &= \boxed{(0.916)^n (3.00 \text{ m})} \text{ after the } n\text{-th bounce.} \end{aligned}$$

18. (a) The required constant acceleration can be found from

$$v^2 = v_0^2 + 2a \Delta y; (2.0 \text{ m/s})^2 = 0 + 2a(4 \text{ m}), \text{ which gives } a = 0.5 \text{ m/s}^2.$$

We find the required tension from $\Sigma F_y = ma_y$:

$$T - mg = ma; T = (106 \text{ kg})(9.8 \text{ m/s}^2) + (106 \text{ kg})(0.5 \text{ m/s}^2) = \boxed{1.1 \times 10^3 \text{ N}}.$$

The work done is $W_T = T \Delta y = (1.1 \times 10^3 \text{ N})(4 \text{ m}) = \boxed{4.4 \times 10^3 \text{ J}}$.

- (b) During the accelerated motion, we have

$$v^2 = v_0^2 + 2a \Delta y; (2.0 \text{ m/s})^2 = 0 + 2a(1 \text{ m}), \text{ which gives}$$

$$a = 2.0 \text{ m/s}^2.$$

We find the required tension from $\Sigma F_y = ma_y$:

$$T_1 - mg = ma; T_1 = (106 \text{ kg})(9.8 \text{ m/s}^2) + (106 \text{ kg})(2 \text{ m/s}^2) = \boxed{1.3 \times 10^3 \text{ N}}.$$

The work done during the first 5 m is

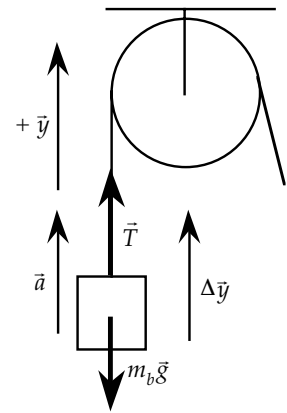
$$W_1 = T_1 \Delta y = (1.3 \times 10^3 \text{ N})(1 \text{ m}) = \boxed{1.3 \times 10^3 \text{ J}}.$$

During the uniform speed motion, the tension must have the same magnitude as the gravity force:

$$T_2 = mg = (106 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{1.0 \times 10^3 \text{ N}}.$$

The work done during this motion is $W_2 = T_2 \Delta y = (1.0 \times 10^3 \text{ N})(3 \text{ m}) = \boxed{3.0 \times 10^3 \text{ J}}$.

The total work done is $W_1 + W_2 = \boxed{4.3 \times 10^3 \text{ J}}$, the same as part (a) (to one significant figure).



19. Because the net work must be zero, the work that the child must do will have the same magnitude as the work done by gravity. For each block this work is mg times the distance the center is raised (zero for the first block, one block-height for the second block, etc.).

$$W_1 = (0.036 \text{ kg})(9.8 \text{ m/s}^2)(0 + 1 + 2)(0.12 \text{ m}) = \boxed{0.127 \text{ J}}.$$

$$W_2 = (0.018 \text{ kg})(9.8 \text{ m/s}^2)(0 + 1 + 2 + 3 + 4 + 5)(0.06 \text{ m}) = \boxed{0.159 \text{ J}}.$$

$$W_3 = (0.009 \text{ kg})(9.8 \text{ m/s}^2)(0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11)(0.03 \text{ m}) = \boxed{0.175 \text{ J}}.$$

20. The work done by the force of gravity on m_1 as it is lowered through a distance $\Delta y_1 (= l_1 \theta)$ is given by

$W_{g1} = m_1 g \Delta y_1 = m_1 g l_1 \theta$, while that done on m_2 as it is lifted through a distance $\Delta y_2 (= l_2 \theta)$ is

$W_{g2} = -m_2 g \Delta y_2 = -m_2 g l_2 \theta$. The kinetic energy of the rod must then change by

$$\Delta K = W_{g1} + W_{g2} = m_1 g l_1 \theta - m_2 g l_2 \theta = (m_1 l_1 - m_2 l_2) g \theta.$$

But since $m_1 l_1 = m_2 l_2$ we have $\Delta K = 0$, so $K_f = K_i + \Delta K = 0$, meaning that the rod is stationary after the slight rotation.

21. As the mass swings upward it slows down, meaning that it is losing kinetic energy. From the work-energy theorem we know that negative work must be done on it. In fact, two forces are exerted on the mass: the tension in the string and the weight of the mass. Since the tension always points along the string while the mass moves perpendicularly to the string the tension does not do any work on the mass. The (negative) work done by the weight of the mass is

$$W_g = -m g \Delta y = \boxed{-mgR(1 - \cos \theta)}.$$

22. We think of the rope as having n segments of length $\Delta y = L/n$. Using the analysis from Problem 17, we have

$$\begin{aligned} W &= (Mg/n)[0 + 1 + 2 + 3 + \dots + (n-1)] \Delta y \\ &= (Mg/n)[1 + 2 + 3 + \dots + (n-1)](L/n) \\ &= (Mg/n)\left[\frac{1}{2}(n)(n-1)\right](L/n) = \frac{1}{2}MgL[(n-1)/n]. \end{aligned}$$

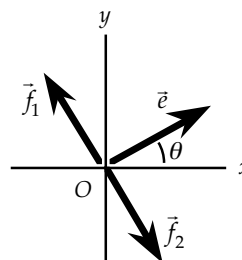
$$\text{As } n \rightarrow \infty, W \rightarrow \boxed{\frac{1}{2}MgL}.$$

23. $\vec{A} = -2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{B} = 5\hat{i} + \hat{j} - 2\hat{k}$. Then

$$\vec{A} \cdot \vec{B} = (-2)(5) + (3)(1) + (-5)(-2) = \boxed{+3}.$$

24. For a constant force the work is

$$\begin{aligned}
 W_F &= \vec{F} \cdot \vec{r} \\
 &= [(-3.1 \text{ N})\hat{i} + (2.7 \text{ N})\hat{j}] \cdot [(0.50 \text{ m})\hat{i} - (0.75 \text{ m})\hat{j}] \\
 &= (-3.1)(0.5) + (2.7)(-0.75) \\
 &= \boxed{-3.6 \text{ J}}.
 \end{aligned}$$



25. We find the final speed from the work-energy theorem:

$$\begin{aligned}
 W_{\text{net}} &= \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2; \\
 1.0 \text{ J} &= \frac{1}{2}(2.6 \text{ kg})v_f^2 - 0, \text{ which gives } v_f = \boxed{0.89 \text{ m/s}}.
 \end{aligned}$$

26. Because the scalar product will be zero for two vectors that are perpendicular, we evaluate

$$\vec{v} \cdot \vec{u} = (-y\hat{i} + x\hat{j}) \cdot (x\hat{i} + y\hat{j}) = -yx + xy = 0.$$

27. Because the scalar product will be zero for two orthogonal vectors, we evaluate

$$\vec{u} \cdot \vec{v} = (3\hat{i} - 4\hat{j} + 7\hat{k}) \cdot (-2\hat{i} + 3\hat{j} + z\hat{k}) = -6 - 12 + 7z, \text{ which gives } z = \boxed{2.57}.$$

28. Because the speeds at beginning and end are zero, there is no change in kinetic energy. Work will be done by the person and gravity. There will be work by gravity only for the vertical lift, so we have

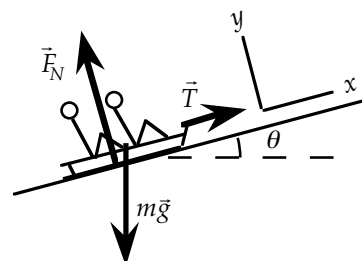
$$\begin{aligned}
 W_{\text{net}} &= W_{\text{person}} + W_g = \Delta K = 0, \text{ or} \\
 W_{\text{person}} &= -W_g = -(-mg)h = (11.5 \text{ kg})(9.8 \text{ m/s}^2)(0.6 \text{ m}) = \boxed{68 \text{ J}}.
 \end{aligned}$$

29. We can find the force
- T
- exerted by the man from
- $\Sigma \vec{F} = m\vec{a}$
- , using the force diagram for the sled:

$$\begin{aligned}
 y\text{-component: } F_N &= mg \cos \theta; \\
 x\text{-component: } T &= mg \sin \theta \\
 &= (43 \text{ kg})(9.8 \text{ m/s}^2) \sin 15^\circ = 1.09 \times 10^2 \text{ N}.
 \end{aligned}$$

Because the displacement is parallel to the force, the work done is

$$W_T = T \Delta x = (1.09 \times 10^2 \text{ N})(36 \text{ m}) = \boxed{3.9 \times 10^3 \text{ J}}.$$



30. We assume there is no change in elevation. We find the net work from the change in kinetic energy:

$$\begin{aligned}
 W_{\text{net}} &= \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) \\
 &= \frac{1}{2}(1300 \text{ kg})[(0.21 \text{ m/s})^2 - (0.15 \text{ m/s})^2] \\
 &= \boxed{14 \text{ J}}.
 \end{aligned}$$

31. We can find the acceleration from
- $\Sigma \vec{F} = m\vec{a}$
- , using the force diagram for the skier:

$$\begin{aligned}
 y\text{-component: } F_N &= mg \cos \theta; \\
 x\text{-component: } mg \sin \theta - \mu_k mg \cos \theta &= ma.
 \end{aligned}$$

From this we get

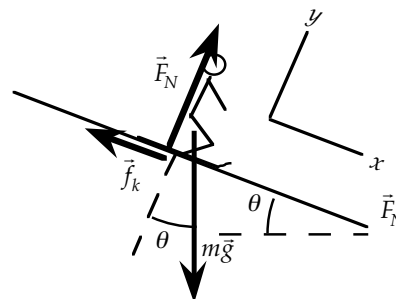
$$\begin{aligned}
 a &= g(\sin \theta - \mu_k \cos \theta) \\
 &= (9.8 \text{ m/s}^2)[\sin 18^\circ - (0.12) \cos 18^\circ] = 1.9 \text{ m/s}^2.
 \end{aligned}$$

The final speed is $v = v_0 + at = 0 + (1.9 \text{ m/s}^2)(7.0 \text{ s}) = 13 \text{ m/s}$.

We find the net work from the change in kinetic energy:

$$W_{\text{net}} = \Delta K = \frac{1}{2}m(v^2 - v_0^2) = \frac{1}{2}(72 \text{ kg})[(13 \text{ m/s})^2 - 0] = \boxed{6.1 \times 10^3 \text{ J}}.$$

This net work is from the positive work done by gravity and the negative work done by friction.



32. We write the vectors as

$$\vec{A} = A_1 \hat{i} + A_2 \hat{j}; \vec{B} = B_1 \hat{i} + B_2 \hat{j} \text{ or}$$

$$\vec{A} = A \cos \theta \hat{i} + A \sin \theta \hat{j}; \vec{B} = B \cos \phi \hat{i} + B \sin \phi \hat{j}.$$

For the vectors to be perpendicular, we have

$$\vec{A} \cdot \vec{B} = A_1 B_1 + A_2 B_2 = 0 \quad \text{or} \quad \phi = \frac{1}{2}\pi + \theta, \text{ which means}$$

$$\cos \phi = -\sin \theta \text{ and } \sin \phi = \cos \theta.$$

From the first we get $B_2 = -(A_1/A_2)B_1 = -(B_1/A_2)A_1 = -[(B \cos \phi)/(A \sin \theta)]A_1$.

Using the trigonometric relations, this becomes $B_2 = +(B/A)A_1 = cA_1$.

Similarly we can write

$$B_1 = -(A_2/A_1)B_2 = -(B_2/A_1)A_2 = -[(B \sin \phi)/(A \cos \theta)]A_2.$$

Using the trigonometric relations, this becomes $B_1 = -(B/A)A_2 = -cA_2$, $|c| = B/A$.

33. The magnitude of
- $\vec{e} = \cos \theta \hat{i} + \sin \theta \hat{j}$
- is found from

$$e^2 = \vec{e} \cdot \vec{e} = \cos^2 \theta + \sin^2 \theta = 1.$$

There are two vectors perpendicular to \vec{e} :

$$\vec{f}_1 = [(-\sin \theta)\hat{i} + (\cos \theta)\hat{j}] \text{ and}$$

$$\vec{f}_2 = [(\sin \theta)\hat{i} - (\cos \theta)\hat{j}].$$

34. We write the vector that is in the
- yz
- plane perpendicular to
- $\vec{A} = 7\hat{i} + 3\hat{j} - 6\hat{k}$
- as
- $\vec{B} = B_y \hat{j} + B_z \hat{k}$
- .

Then $\vec{A} \cdot \vec{B} = 0 + (3)B_y + (-6)B_z = 0$, which gives $B_z = \frac{1}{2}B_y = c$, where c is an arbitrary constant. So the most general vector is $2c\hat{j} + c\hat{k}$.

35. We find the magnitude of the projection of
- $\vec{A} = 3\hat{i} - 2\hat{j}$
- onto the unit vector
- $\vec{e} = -0.6\hat{i} + 0.8\hat{j}$
- from

$$|\vec{A} \cdot \vec{e}| = |(3)(-0.6) + (-2)(0.8)| = \boxed{3.4}.$$

36. (a) Because the force of gravity is vertical, we use the vertical displacement to find its work.

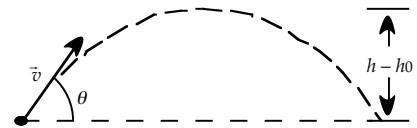
On the way up to the maximum height h , the work is negative, and on the way down, the work is positive:

$$W_g = -mg(h - h_0) + mg(h - h_0) = \boxed{0}.$$

- (b) Because the force of gravity is the only force acting on the stone,

$$W_{\text{net}} = W_g = 0 = \Delta K;$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv^2 = 0, \quad \text{or} \quad v_2 = v.$$



37. The work is
- $W_F = \vec{F} \cdot \Delta \vec{r} = (2\hat{i} - 5\hat{j}) \text{ N} \cdot [(5\hat{i} - 4\hat{j} + 5\hat{k}) - (7\hat{i} - 8\hat{j} + 2\hat{k})] \text{ m}$
- , which gives

$$W_F = [(2)(5 - 7) + (-5)(-4 + 8) + (0)(5 - 2)] \text{ J} = \boxed{-24 \text{ J}}.$$

38. The normal force does no work, so the work-energy theorem gives

$$W_{\text{net}} = \Delta K, \text{ or } W_g + W_f = \frac{1}{2}m(v^2 - v_0^2), \text{ which becomes}$$

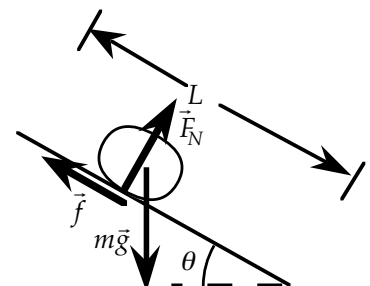
$$(32 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m}) \sin 17^\circ + W_f = \frac{1}{2}(32 \text{ kg})[(2.5 \text{ m/s})^2 - 0],$$

from which we get $W_f = \boxed{-8.2 \times 10^2 \text{ J}}$.

Because $W_f = -m_k(mg \cos \theta)L$, we have

$$-8.2 \times 10^2 \text{ J} = -m_k(32 \text{ kg})(9.8 \text{ m/s}^2)(\cos 17^\circ)(10 \text{ m}), \text{ which gives}$$

$$\mu_k = \boxed{0.27}.$$



39. The work done on the object as it undergoes an infinitesimal displacement $d\vec{r} = (dx)\hat{i}$ along the x axis is

$$dW_F = \vec{F} \cdot d\vec{r} = (F_x \hat{i} + F_y \hat{j}) \cdot (dx)\hat{i} = (F_x dx)\hat{i} \cdot \hat{i} + (F_y dx)\hat{i} \cdot \hat{j} = F_x dx.$$

Integrate over a range of x of length L , from an arbitrary starting point x_i to $x_f = x_i + L$, to obtain

$$W_F = \int_{x_i}^{x_i+L} F_x dx = F_x(x_i + L - x_i) = F_x L.$$

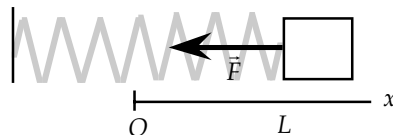
The final kinetic energy of the object is then found from the work-energy theorem to be

$$K_f = K_i + \Delta K = 0 + W_F = \boxed{F_x L}.$$

40. The force of the spring is $F = -kx$, so the work done is

$$W_{sp} = \int_L^0 (-kx) dx = -\frac{1}{2}k(0)^2 + \frac{1}{2}k(L)^2.$$

Thus $W_{sp} = \frac{1}{2}(12 \text{ N/m})(0.50 \text{ m})^2 = \boxed{1.5 \text{ J}}.$



41. We can find the total work by adding the works from each segment:

$$W = W_1 + W_2 = F_1 \Delta x_1 + F_2 \Delta x_2 = (0.3 \text{ N})[0 - (-0.06 \text{ m})] + (0.7 \text{ N})[(0.07 \text{ m}) - 0] = \boxed{0.067 \text{ J}}.$$

42. The work of a variable force is found by integrating. Because the force changes at $x = 0$, the work will be the sum of two integrals:

$$\begin{aligned} W &= \int_{-1.50\text{m}}^0 (-3.00 \text{ N/m})x dx + \int_0^{+1.50\text{m}} (+7.00 \text{ N/m})x dx \\ &= (-3.00 \text{ N/m})\frac{1}{2}\left[x^2\right]_{-1.50\text{m}}^0 + (+7.00 \text{ N/m})\frac{1}{2}\left[x^2\right]_0^{+1.50\text{m}} \\ &= 11.3 \text{ J}. \end{aligned}$$

43. Because the force is variable, we must integrate:

$$W = \int_0^{2.0\text{m}} (g_1 x - g_2 x^3) dx = g_1 \frac{1}{2} x^2 \Big|_0^{2.0\text{m}} - g_2 \frac{1}{4} x^4 \Big|_0^{2.0\text{m}} = 2g_1 - 4g_2.$$

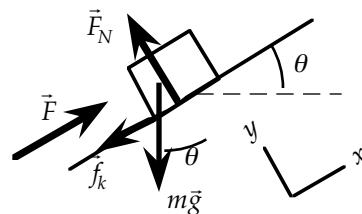
44. Because the velocity is uniform, for $\Sigma \vec{F} = m\vec{a}$ for the crate, we can write:

$$x\text{-component: } F - mg \sin \theta - \mu_k F_N = 0;$$

$$y\text{-component: } -mg \cos \theta - F_N = 0.$$

From these we get $F_N = mg \cos \theta$ and $F = mg \sin \theta + \mu_k mg \cos \theta$.

Because μ_k is a function of position, $\mu_k = \mu_1 + (\mu_2 - \mu_1)s/L$, F is a variable force. We must find the work by integrating:



$$\begin{aligned} W &= \int_0^s \left\{ mg \sin \theta + \left[\mu_1 + (\mu_2 - \mu_1) \frac{s'}{L} \right] mg \cos \theta \right\} ds' \\ &= \left[mg (\sin \theta + \mu_1 \cos \theta) s' \right]_0^s + \left[mg (\mu_2 - \mu_1) \cos \theta \right] \frac{s'^2}{2L} \Big|_0^s = \left[mg (\sin \theta + \mu_1 \cos \theta) \right] s + \left[mg (\mu_2 - \mu_1) \cos \theta \right] \frac{s^2}{2L} \\ &= \left[(50 \text{ kg})(9.8 \text{ m/s}^2) (\sin 30^\circ + (0.2) \cos 30^\circ) \right] s + \left[(50 \text{ kg})(9.8 \text{ m/s}^2) (0.3 - 0.2) \cos 30^\circ \right] \frac{s^2}{2(10 \text{ m})} \\ &= 330s + 2.12s^2 \text{ J, with } s \text{ in meters.} \end{aligned}$$

45. The work of the spring changes the kinetic energy:

$$W_{sp} = \int_L^0 (-kx) dx = -\frac{1}{2} k(0)^2 + \frac{1}{2} k(L)^2 = \frac{1}{2} mv^2 - 0; \text{ or } \frac{1}{2} kL^2 = \frac{1}{2} mv^2; .$$

$$(60 \text{ N/m})(0.07 \text{ m})^2 = (4 \times 10^{-3} \text{ kg})v^2, \text{ which gives } v = \boxed{8.6 \text{ m/s}}.$$

46. Consider the work done by the Sun on the asteroid as it moves towards the Sun over an infinitesimal displacement $d\vec{r}$:

$$dW = \vec{F} \cdot d\vec{r} = F(-\vec{r}/r) \cdot d\vec{r} (\vec{r}/r) = -F dr = -(\text{Constant}/r) dr.$$

Note that \vec{r}/r is the unit vector in the radial direction. Also, dr is negative and dW is positive. The total work done by the Sun as it pulls the asteroid from infinity to its surface is then

$$W = -\int_{\infty}^{R_{sun}} F dr = -\int_{\infty}^{R_{sun}} \frac{\text{Constant}}{r^2} dr = -\text{Constant} \left[-\frac{1}{r} \right]_{\infty}^{R_{sun}} = \frac{\text{Constant}}{R_{sun}},$$

which equals the change in kinetic energy of the asteroid. Since its initial kinetic energy is zero when the asteroid is at infinity, its final kinetic energy upon reaching the surface of the Sun is

$$K_f = \Delta K = W = \boxed{\text{Constant}/R_{sun}}.$$

47. The force required to stretch the spring must be opposite to the spring force and thus is

$$F = +k_1 x + k_2 x^3.$$

The work done by this force is

$$W = \int_{0.10 \text{ m}}^{0.20 \text{ m}} F dx = \int_{0.10 \text{ m}}^{0.20 \text{ m}} (k_1 x + k_2 x^3) dx = k_1 \frac{1}{2} x^2 \bigg|_{0.10 \text{ m}}^{0.20 \text{ m}} + k_2 \frac{1}{4} x^4 \bigg|_{0.10 \text{ m}}^{0.20 \text{ m}}$$

$$= (5.0 \text{ N/m}) \frac{1}{2} \left[(0.20 \text{ m})^2 - (0.10 \text{ m})^2 \right] + (15 \text{ N/m}^3) \frac{1}{4} \left[(0.20 \text{ m})^4 - (0.10 \text{ m})^4 \right] = 8.1 \times 10^{-2} \text{ J}.$$

48. We assume the kinetic energy is zero at launch and essentially zero when the rocket (of the same mass) leaves the gravitational force of the earth. The rocket starts at the surface of the earth, $r = R$. The net work is then zero, so the work done by the rocket engine must be the negative of the work done by the attractive (negative) gravitational force:

$$W_{\text{engine}} = \int_R^{\infty} F_{\text{engine}} dr = \int_R^{\infty} -F_{\text{gravitation}} dr = \int_R^{\infty} \left(\frac{K}{r^2} \right) dr = -\frac{K}{r} \bigg|_R^{\infty} = +\frac{K}{R}.$$

49. There is no change in elevation. Even though the force variation and path may be complicated, we find the net work from the change in kinetic energy:

$$W_{\text{net}} = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2) = \frac{1}{2} m(r_f \omega_f)^2 - 0$$

$$= \frac{1}{2} (0.085 \text{ kg})[(1.45 \text{ m})(2 \text{ rev/s})(2\pi \text{ rad/rev})]^2 - (0.01 \text{ m/s})^2 = \boxed{14 \text{ J}}.$$

50. Because the work done by gravity is independent of the path, it will be the same for both balls:

$$W_g = -mg(h - h_0) = -(0.074 \text{ kg})(9.8 \text{ m/s}^2)(-0.60 \text{ m}) = \boxed{0.44 \text{ J}}.$$

51. Work is done by two forces, the tension provided by the movers and gravity. Because gravity is a conservative force, its work is independent of the path and thus the same for the two paths. The net work is zero, and thus the work done by the tension will also be the same for each path:

$$W_{\text{net}} = 0 = W_T + W_g;$$

$$W_T = -W_g = -mg(-H) = (54 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) = \boxed{2.1 \times 10^3 \text{ J}}.$$

52. $W_g = \int m\vec{g} \cdot d\vec{r} = (-mg)(-\Delta y) = [-(0.600 \text{ kg})(9.8 \text{ m/s}^2)](-0.40 \text{ m}) = \boxed{2.4 \text{ J}}.$

53. $W_{0 \rightarrow 1} = \int F dx = F \Delta x$
 $= (10 \text{ N})\{[11 \text{ m} - (2 \text{ m/s})(1 \text{ s}) + (0.5 \text{ m/s}^2)(1 \text{ s})^2] - [11 \text{ m} - (2 \text{ m/s})(0 \text{ s}) + (0.5 \text{ m/s}^2)(0 \text{ s})^2]\} = \boxed{-15 \text{ J}}.$
 $W_{1 \rightarrow 2} = (10 \text{ N})\{[11 \text{ m} - (2 \text{ m/s})(2 \text{ s}) + (0.5 \text{ m/s}^2)(2 \text{ s})^2] - [11 \text{ m} - (2 \text{ m/s})(1 \text{ s}) + (0.5 \text{ m/s}^2)(1 \text{ s})^2]\} = \boxed{-5 \text{ J}}.$

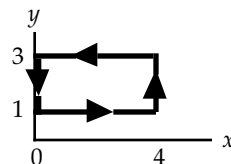
The work done by a constant force is independent of the path and thus conservative.

54. In general the work of a force is $W_F = \int \vec{F} \cdot d\vec{r}$. If we write $d\vec{r} = dx \hat{i} + dy \hat{j}$, this becomes

$\int (F_x dx + F_y dy)$. For path 1, $dy = 0$,

$$W_1 = \int_{(0,1)}^{(4,1)} (axy - by^2) dx = \left(\frac{1}{2} ax^2 y - by^2 x \right) \Big|_{(0,1)}^{(4,1)}$$

$$= \frac{1}{2} (2 \text{ N/m}^2) \left[(4 \text{ m})^2 (1 \text{ m}) - (0 \text{ m})^2 (1 \text{ m}) \right] - (2 \text{ N/m}^2) \left[(1 \text{ m})^2 (4 \text{ m}) - (1 \text{ m})^2 (0 \text{ m}) \right] = 8 \text{ J}.$$



For path 2, $dx = 0$, so

$$W_2 = \int_{(4,1)}^{(4,3)} (-axy + bx^2) dy = \left(-\frac{1}{2} axy^2 + bx^2 y \right) \Big|_{(4,1)}^{(4,3)}$$

$$= -\frac{1}{2} (2 \text{ N/m}^2) \left[(4 \text{ m})(3 \text{ m})^2 - (4 \text{ m})(1 \text{ m})^2 \right] + (2 \text{ N/m}^2) \left[(4 \text{ m})^2 (3 \text{ m}) - (4 \text{ m})^2 (1 \text{ m}) \right] = 32 \text{ J}.$$

For path 3, $dy = 0$, so

$$W_3 = \int_{(4,3)}^{(0,3)} (axy - by^2) dx = \left(\frac{1}{2} ax^2 y - by^2 x \right) \Big|_{(4,3)}^{(0,3)}$$

$$= \frac{1}{2} (2 \text{ N/m}^2) \left[(0 \text{ m})^2 (3 \text{ m}) - (4 \text{ m})^2 (3 \text{ m}) \right] - (2 \text{ N/m}^2) \left[(3 \text{ m})^2 (0 \text{ m}) - (3 \text{ m})^2 (4 \text{ m}) \right] = 24 \text{ J}.$$

For path 4, $dx = 0$, so

$$W_4 = \int_{(0,3)}^{(0,1)} (-axy + bx^2) dy = \left(-\frac{1}{2} axy^2 + bx^2 y \right) \Big|_{(0,3)}^{(0,1)}$$

$$= -\frac{1}{2} (2 \text{ N/m}^2) \left[(4 \text{ m})(3 \text{ m})^2 - (4 \text{ m})(1 \text{ m})^2 \right] + (2 \text{ N/m}^2) \left[(0 \text{ m})^2 (1 \text{ m}) - (0 \text{ m})^2 (3 \text{ m}) \right] = 0 \text{ J}.$$

Thus, $W_{\text{total}} = W_1 + W_2 + W_3 + W_4 = \boxed{+64 \text{ J}}.$

55. We are given $|F(x)| = Ax^2$ with the force always toward the origin.

(a) For this part the force is toward positive x , so the work is

$$W_a = \int_{-5.0 \text{ cm}}^0 (+Ax^2) dx = +\frac{1}{3} Ax^3 \Big|_{-5.0 \text{ cm}}^0 = +\frac{1}{3} (1500 \text{ N/m}^2) \left[(0)^3 - (-0.05 \text{ m})^3 \right] = 6.3 \times 10^{-2} \text{ J}.$$

(b) In addition to the work from part (a), there is additional work done while the force is toward negative x , which is

$$W_{b2} = \int_0^{5.0 \text{ cm}} (-Ax^2) dx = -\frac{1}{3} Ax^3 \Big|_0^{5.0 \text{ cm}} = -\frac{1}{3} (1500 \text{ N/m}^2) \left[(0.05 \text{ m})^3 - (0)^3 \right] = -6.3 \times 10^{-2} \text{ J}.$$

Thus the total work now is $6.3 \times 10^{-2} \text{ J} - 6.3 \times 10^{-2} \text{ J} = \boxed{0}.$

(c) Now the force is toward negative x , so the work is

$$W_c = \int_{5.0 \text{ cm}}^{20 \text{ cm}} (-Ax^2) dx = -\frac{1}{3} Ax^3 \Big|_{5.0 \text{ cm}}^{20 \text{ cm}} = -\frac{1}{3} (1500 \text{ N/m}^2) \left[(0.02 \text{ m})^3 - (0.05 \text{ m})^3 \right] = 5.9 \times 10^{-2} \text{ J}.$$

(d) Now the force is toward positive x , so the work is

$$W_d = \int_{-2.0 \text{ cm}}^{-5.0 \text{ cm}} (+Ax^2) dx = +\frac{1}{3} Ax^3 \Big|_{-2.0 \text{ cm}}^{-5.0 \text{ cm}} = +\frac{1}{3} (1500 \text{ N/m}^2) \left[(-0.05 \text{ m})^3 - (-0.02 \text{ m})^3 \right] = -5.9 \times 10^{-2} \text{ J}.$$

56. From the work-energy theorem, the required work must equal the increase in the kinetic energy. The four seats and the child start from rest and reach a speed of $v = R\omega$. Thus

$$\begin{aligned} W &= \Delta K = \frac{1}{2}(m_c + 4m_s)[(R\omega)^2 - 0^2] \\ &= \frac{1}{2}[21 \text{ kg} + 4(12 \text{ kg})][(1.8 \text{ m})(0.6 \text{ rev/s})(2\pi \text{ rad/rev})]^2 \\ &= \boxed{1.6 \text{ kJ}}. \end{aligned}$$

57. For a variable force, the work is found by integrating:

$$W_x = \int_0^x (F_0 + Cx') dx' = \left(F_0 x' + \frac{1}{2} C x'^2 \right) \bigg|_0^x = F_0 x + \frac{1}{2} C x^2.$$

(a) $W_1 = F_0 x + \frac{1}{2} C x^2 = (5 \text{ N})(1 \text{ m}) + \frac{1}{2}(-2 \text{ N/m})(1 \text{ m})^2 = \boxed{4 \text{ J}}.$

$$W_2 = (5 \text{ N})(2 \text{ m}) + \frac{1}{2}(-2 \text{ N/m})(2 \text{ m})^2 = \boxed{6 \text{ J}}.$$

$$W_3 = (5 \text{ N})(3 \text{ m}) + \frac{1}{2}(-2 \text{ N/m})(3 \text{ m})^2 = \boxed{6 \text{ J}}.$$

$$W_4 = (5 \text{ N})(4 \text{ m}) + \frac{1}{2}(-2 \text{ N/m})(4 \text{ m})^2 = \boxed{4 \text{ J}}.$$

(b) $W = 0 = F_0 x + \frac{1}{2} C x^2 = x(F_0 + \frac{1}{2} C x)$, which gives

$$x = 0 \quad \text{and} \quad x = -2F_0/C = -2(5 \text{ N})/(-2 \text{ N/m}) = \boxed{+5 \text{ m}}.$$

- (c) In one dimension, a force that is a function of position only is **conservative**.

58. (a) The work done by gravity is the sum of the work done for each leg:

$$W_g = W_{g1} + W_{g2}.$$

During leg 1 the force and displacement are parallel:

$$W_{g1} = (-mg)(-h) = mgh;$$

during leg 2 the force and displacement are perpendicular:

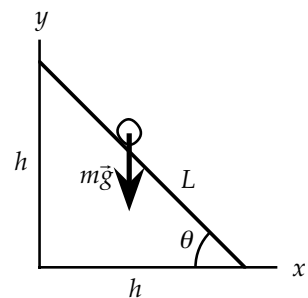
$$W_{g2} = mg(0) = 0. \text{ Thus}$$

$$W_g = mgh.$$

- (b) For the motion along the incline, the work done by gravity is the weight component along the incline times the displacement:

$$W_g = (mg \sin \theta) L = mg(L \sin \theta) = mgh,$$

which is the same as in part (a).



59. In one dimension, the work done by a force is $W = \int F dx$. If the force is a function of position only, $F(x)$, this becomes $\int F(x) dx$. Because there is only one variable, $F(x)$ is the derivative of some other function of x , which we call $G(x)$; i. e.,

$$F(x) = dG(x)/dx, \text{ or } F(x) dx = dG(x).$$

Let us determine the work done while the body moves from position $x = A$ to position $x = B$, by some path. Then the work is

$$W = \int_A^B F(x) dx = \int_A^B dG(x) = G(B) - G(A).$$

We see that this result does not depend on the path taken but only on the initial and final points. If we select a different path, the work is still $G(B) - G(A)$. If the path is closed, so that $A = B$, the work done is zero. These are different ways of stating the requirement for the force to be conservative.

Constant forces are included, since this is a special type of position-dependent force.

A friction force, even though of constant magnitude, is nonconservative, because its direction depends on the direction of the velocity, thus it is not a function of position only.

60. (a) Because the tangential speed is constant, we have

$$\Sigma F_t = 0 = F - mg \sin \theta, \text{ or } F = mg \sin \theta.$$

Because the force is variable, we find the work by integrating over the path, with a differential tangential displacement of $ds = R d\theta$:

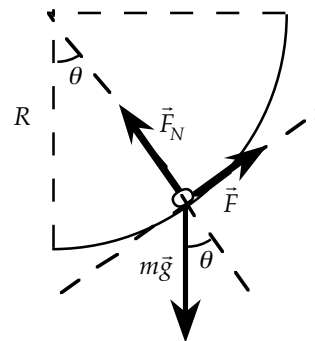
$$\begin{aligned} W_F &= \int -\vec{F} \cdot d\vec{r} = \int F ds = \int_0^{\pi/2} (mg \sin \theta) R d\theta \\ &= mgR \int_0^{\pi/2} \sin \theta d\theta = mgR (-\cos \theta) \Big|_0^{\pi/2} \\ &= mgR[0 - (-1)] = mgR. \end{aligned}$$

- (b) Because the speed does not change, from the work-energy theorem we have
- $W_{\text{net}} = \Delta K = 0$
- .

The normal force is always perpendicular to the path and thus does no work, so we have

$$W_{\text{net}} = W_N + W_F + W_g = 0 + W_F + W_g = 0, \text{ or}$$

$$W_F = -W_g = -(-mg) \Delta h = \boxed{mgR}.$$



61. We are given
- $F(x) = C|x|$
- , which can be expressed as

$$F(x) = C(-x) \text{ for negative } x\text{-values and}$$

$$F(x) = C(+x) \text{ for positive } x\text{-values.}$$

For a variable force we integrate to find the work.

- (a) This motion has positive and negative values for
- x
- , so we use two integrals:

$$\begin{aligned} W_a &= \int_{-4.0 \text{ cm}}^0 C(-x) dx + \int_0^{4.0 \text{ cm}} Cx dx = -\frac{1}{2} Cx^2 \Big|_{-4.0 \text{ cm}}^0 + \frac{1}{2} Cx^2 \Big|_0^{4.0 \text{ cm}} \\ &= -\frac{1}{2} C (\text{N/cm}) (100 \text{ cm/m}) \left\{ \left[(0)^2 - (-0.04 \text{ m})^2 \right] - \left[(0.04 \text{ m})^2 - (0)^2 \right] \right\} = 0.16C \text{ J.} \end{aligned}$$

If $F = Cx$, the direction of the force is given by the sign of x , so we get

$$\begin{aligned} W &= \int_{-4.0 \text{ cm}}^{4.0 \text{ cm}} Cx dx = +\frac{1}{2} Cx^2 \Big|_{-4.0 \text{ cm}}^{4.0 \text{ cm}} \\ &= +\frac{1}{2} C (\text{N/cm}) (100 \text{ cm/m}) \left\{ \left[(0.04 \text{ m})^2 - (-0.04 \text{ m})^2 \right] \right\} = 0. \end{aligned}$$

- (b) Because the motion has only positive values of
- x
- , both force laws give the same result:

$$\begin{aligned} W_b &= \int_0^{8.0 \text{ cm}} Cx dx = +\frac{1}{2} Cx^2 \Big|_0^{8.0 \text{ cm}} \\ &= +\frac{1}{2} C (\text{N/cm}) (100 \text{ cm/m}) \left\{ \left[(0.08 \text{ m})^2 - (0)^2 \right] \right\} = 0.32C \text{ J.} \end{aligned}$$

62. From the diagram we see that
- $x = r \cos \theta$
- and
- $y = r \sin \theta$
- . By differentiating, we obtain, with
- $r = 1$
- :

$$dx = -\sin \theta d\theta \text{ and } dy = \cos \theta d\theta.$$

This allows us to write the force as

$$F(x, y) = k_1 x \hat{i} + k_2 y \hat{j} = k_1 \cos \theta \hat{i} + k_2 \sin \theta \hat{j}$$

and the work after moving through an angle θ as

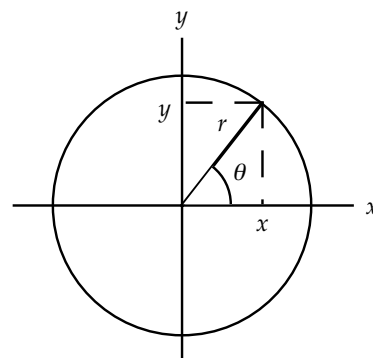
$$\begin{aligned} W &= \int F_x dx + \int F_y dy = \int k_1 (-\cos \theta \sin \theta d\theta) + \int k_2 \sin \theta \cos \theta d\theta \\ &= (k_2 - k_1) \int \sin \theta \cos \theta d\theta = \frac{1}{2} (k_2 - k_1) \sin^2 \theta + C. \end{aligned}$$

The initial position corresponds to $\theta = 270^\circ$, so $C = -\frac{1}{2}(k_2 - k_1)$.

$$(a) \quad W = \frac{1}{2}(k_2 - k_1) \sin^2 0^\circ - \frac{1}{2}(k_2 - k_1) = \boxed{\frac{1}{2}(k_1 - k_2)}.$$

$$(b) \quad W = \frac{1}{2}(k_2 - k_1) \sin^2 90^\circ - \frac{1}{2}(k_2 - k_1) = \boxed{0}.$$

$$(c) \quad W = \frac{1}{2}(k_2 - k_1) \sin^2 270^\circ - \frac{1}{2}(k_2 - k_1) = \boxed{0}.$$



63. We can let the units lead us through the solution:

$$[(\$26.00/\text{month})/(\$0.08/\text{kWh})](10^3 \text{ W/kW})(1 \text{ month}/30 \text{ d})(1 \text{ d}/5 \text{ h})(1 \text{ bulb}/100 \text{ W}) = \boxed{22 \text{ bulbs}}.$$

64. The energy is $Pt = [(100 \text{ cars})(80 \text{ hp}/\text{car})(746 \text{ W}/\text{hp})][(30 \text{ d})(24 \text{ h}/\text{d})(3600 \text{ s}/\text{h})] = \boxed{1.5 \times 10^{13} \text{ J}}.$

65. The amount of work that is done is the negative of the work by gravity: $W = mg \Delta y$.

If P represents your total power output and you climb the four flights in one minute,

$$0.20P = W/t = (70 \text{ kg})(9.8 \text{ m/s}^2)(4)(3 \text{ m})/(60 \text{ s}), \text{ which gives}$$

$$P \approx 0.7 \times 10^3 \text{ W} = \boxed{0.7 \text{ kW}}.$$

66. (a) $W_1 = F_1 x = (0.05 \text{ N})(2.5 \text{ m}) = \boxed{0.13 \text{ J}}.$

$$W_2 = F_2 x = (0.75 \text{ N})(2.5 \text{ m}) = \boxed{1.9 \text{ J}}.$$

(b) We could find either the time of the motion or the average speed. From the work-energy theorem,

$$W_{\text{net}} = \Delta K;$$

$$0.13 \text{ J} + 1.9 \text{ J} = \frac{1}{2}(80 \text{ kg})(v_f^2 - 0), \text{ which gives } v_f = 0.224 \text{ m/s}.$$

Because the forces are constant, the acceleration is also constant and thus

$$v_{\text{av}} = \frac{1}{2}(v_i + v_f) = \frac{1}{2}(0.224 \text{ m/s} + 0) = 0.112 \text{ m/s}; \text{ so}$$

$$P_1 = F_1 v_{\text{av}} = (0.05 \text{ N})(0.112 \text{ m/s}) = \boxed{6 \times 10^{-3} \text{ W}} \text{ and}$$

$$P_2 = F_2 v_{\text{av}} = (0.75 \text{ N})(0.112 \text{ m/s}) = \boxed{8.4 \times 10^{-2} \text{ W}}.$$

67. We convert the speed units: $(15 \text{ mi/h})(1.6 \text{ km/mi})/(3.6 \text{ ks/h}) = 6.7 \text{ m/s}.$

$$\text{The net work done is } \Delta W = \Delta K = 0 - \frac{1}{2}(700 \text{ kg})(6.7 \text{ m/s})^2 = -1.6 \times 10^4 \text{ J}.$$

$$\text{The average power is then } P_{\text{av}} = |\Delta W|/\Delta t = (1.6 \times 10^4 \text{ J})/0.3 \text{ s} = +5.2 \times 10^4 \text{ W} = \boxed{52 \text{ kW}}.$$

68. We convert the speed units: $(100 \text{ km/h})/(3600 \text{ s/h}) = 27.8 \text{ m/s}.$

$$\text{The required work is } \Delta W = \Delta K = \frac{1}{2}m(v^2 - 0);$$

From $P = \Delta W/\Delta t$ we find

$$\Delta t = \Delta W/P = \frac{1}{2}(1200 \text{ kg})(27.8 \text{ m/s})^2 / [(80 \text{ hp})(746 \text{ W}/\text{hp})] = \boxed{7.8 \text{ s}}.$$

69. Because the velocity is uniform, for $\Sigma \vec{F} = m\vec{a}$ for the sled, we can write

$$x\text{-component: } F - \mu_k F_N = 0;$$

$$y\text{-component: } F_N - mg = 0, \text{ which gives}$$

$$F = \mu_k mg = (0.03)(5000 \text{ N}) = 150 \text{ N}.$$

The maximum power will produce the maximum speed:

$$P_{\text{max}} = Fv_{\text{max}};$$

$$(1 \text{ hp})(746 \text{ W}/\text{hp}) = (150 \text{ N})v_{\text{max}}, \text{ which gives } v_{\text{max}} = \boxed{5.0 \text{ m/s}}.$$

On an incline, we have:

$$x\text{-component: } F - mg \sin \theta - \mu_k F_N = 0;$$

$$y\text{-component: } F_N - mg \cos \theta = 0, \text{ which gives}$$

$$F = mg (\sin \theta + \mu_k \cos \theta) = (5000 \text{ N})(\sin 5^\circ + 0.03 \cos 5^\circ) = 585 \text{ N}.$$

The maximum power will produce the maximum speed on the incline:

$$P_{\text{max}} = Fv_{\text{max}};$$

$$(1 \text{ hp})(746 \text{ W}/\text{hp}) = (585 \text{ N})v_{\text{max}}, \text{ which gives } v_{\text{max}} = \boxed{1.3 \text{ m/s}}.$$

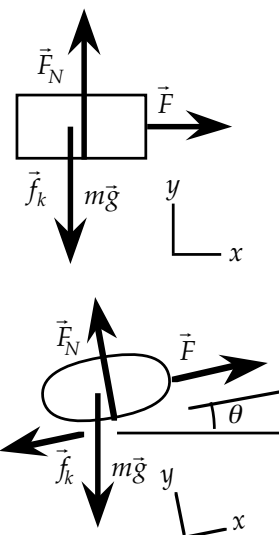
70. For each proton, with $v \approx c$, we have

$$W = \Delta K = mc^2 [1/(1 - v^2/c^2)^{1/2} - 1] - 0 = mc^2 [1/(1 - v^2/c^2)^{1/2} - 1]$$

$$= (1.67 \times 10^{-27} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 [1/(1 - 0.999^2)^{1/2} - 1] = 3.2 \times 10^{-9} \text{ J}.$$

For an efficiency of 5.00% we have

$$0.0500P = (3.2 \times 10^{-9} \text{ J})(6.5 \times 10^{10} / \text{min}) / (60 \text{ s}/\text{min}), \text{ which gives } P = \boxed{69 \text{ W}}.$$



71. The input power from the falling water is $P_{\text{in}} = (dm/dt)g \Delta y$. The output power is

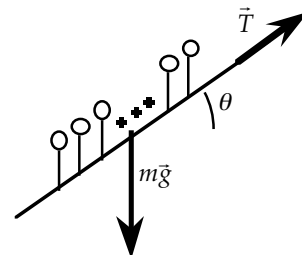
$$P_{\text{out}} = 0.60P_{\text{in}} = 0.60(200 \text{ m}^3)(10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(40 \text{ m}) = \boxed{4.7 \times 10^7 \text{ W}} = 47 \text{ MW}.$$

72. Because the speed is constant, the force provided by the motor corresponds to a tension and must equal the magnitude of the component of the force of gravity along the escalator:

$$T = mg \sin \theta.$$

The power generated by the motor is

$$\begin{aligned} P &= Tv = (mg \sin \theta)v \\ &= (75 \text{ passengers})(75 \text{ kg/passenger})(9.8 \text{ m/s}^2)(\sin 20^\circ)(1.2 \text{ m/s}) \\ &= 2.3 \times 10^4 \text{ W} = 23 \text{ kW} = \boxed{30 \text{ hp}}. \end{aligned}$$



73. (a) At constant speed the force provided by the bicyclist is $F = Av^2$. The power is

$$P = Fv; (0.4 \text{ hp})(746 \text{ W/hp}) = (0.08 \text{ kg/m})v^3, \text{ which gives } v = \boxed{16 \text{ m/s}}.$$

$$(b) \quad t = W/P = mg \Delta y / P = (100 \text{ kg})(9.8 \text{ m/s}^2)(2 \text{ m}) / (5 \text{ hp})(746 \text{ W/hp}) = \boxed{0.5 \text{ s}}.$$

- (c) We assume a power of 1 hp:

$$t = W/P = mg \Delta y / P = (75 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m}) / (1 \text{ hp})(746 \text{ W/hp}) = \boxed{12 \text{ s}}.$$

74. $W = \Delta K = mc^2[1/(1 - v^2/c^2)^{1/2} - 1] - 0$

$$= (1.7 \times 10^{-27} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2[1/(1 - 0.85^2)^{1/2} - 1] = \boxed{1.4 \times 10^{-10} \text{ J}}.$$

75. The value of mc^2 is $(9.1 \times 10^{-31} \text{ kg})(3 \times 10^8 \text{ m/s})^2 = 8.2 \times 10^{-14} \text{ J}$.

When $v \approx c$, we write $v/c = 1 - x$ and $v^2/c^2 = 1 - 2x$, where x is small.

Then $(1 - v^2/c^2)^{1/2} \approx [1 - (1 - 2x)]^{1/2} = (2x)^{1/2}$.

For $v/c = 0.9999$, $x = 1 \times 10^{-4}$; $K = mc^2\{1/[1 - (v/c)^2]^{1/2} - 1\}$

$$= (8.2 \times 10^{-14} \text{ J})^2\{1/[2(1 \times 10^{-4})]^{1/2} - 1\} = \boxed{5.8 \times 10^{-12} \text{ J}}.$$

For $v/c = 0.9999999999$, $x = 1 \times 10^{-10}$; $K = (8.2 \times 10^{-14} \text{ J})^2\{1/[2(1 \times 10^{-10})]^{1/2} - 1\} = \boxed{5.8 \times 10^{-9} \text{ J}}.$

76. We use the analysis in the solution to Problem 75. $W = \Delta K$.

$$(a) \quad W = mc^2\{1/[1 - (v/c)^2]^{1/2} - 1/[1 - (v_0/c)^2]^{1/2}\}$$

$$= (8.2 \times 10^{-14} \text{ J})[1/(1 - 0.5^2)^{1/2} - 1/(1 - 0.1^2)^{1/2}] = \boxed{1.1 \times 10^{-14} \text{ J}}.$$

$$(b) \quad W = mc^2\{1/[1 - (v/c)^2]^{1/2} - 1/[1 - (v_0/c)^2]^{1/2}\}$$

$$= (8.2 \times 10^{-14} \text{ J})[1/(1 - 0.99^2)^{1/2} - 1/(1 - 0.5^2)^{1/2}] = \boxed{4.9 \times 10^{-13} \text{ J}}.$$

$$(c) \quad W = mc^2\{1/[1 - (v/c)^2]^{1/2} - 1/[1 - (v_0/c)^2]^{1/2}\}$$

$$= (8.2 \times 10^{-14} \text{ J})[1/(1 - 0.999^2)^{1/2} - 1/(1 - 0.99^2)^{1/2}] = \boxed{1.3 \times 10^{-12} \text{ J}}.$$

77. $1/(1 - x)^{1/2} \approx 1 + \frac{1}{2}x + (3/8)x^2$

For $x = 0.01$, LHS = 1.0050378, RHS = 1.0050375.

For $x = \boxed{0.62}$, the approximation is correct to within 10%.

For $x = \boxed{0.31}$, the approximation is correct to within 1%.

78. Energy = $[\$475/(\$0.09/\text{kWh})](10^3 \text{ W/kW})(3600 \text{ s/h}) = \boxed{1.9 \times 10^{10} \text{ J}}$

79. $W_{\text{net}} = \Delta K = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}(0.1 \text{ kg})(2 \text{ m/s})^2 = \boxed{-0.2 \text{ J}}.$

80. (a) Because $W_{\text{net}} = 0$, $W_d = -W_g = -(-mgh) = (6400 \text{ kg})(9.8 \text{ m/s}^2)(2.5 \text{ m}) = \boxed{1.5 \times 10^5 \text{ J}}.$

$$(b) \quad P = W/t = (1.5 \times 10^5 \text{ J})/(0.5 \text{ s}) = \boxed{3 \times 10^5 \text{ W}} = \boxed{4 \times 10^2 \text{ hp}}.$$

81. Fraction = $P_{747}/P_{\text{ww}} = (0.3 \text{ MW})(10^6 \text{ W/MW})/(6000)(2 \text{ hp})(746 \text{ W/hp}) = \boxed{0.034}.$

82. (a) $W_{F1} = \vec{F}_1 \cdot \Delta \vec{r} = (2\hat{i} + 7\hat{j} \text{ N}) \cdot [(10\hat{i} + 5\hat{j}) \text{ m} - (0\hat{i} + 0\hat{j}) \text{ m}] = \boxed{55 \text{ J}}$.
 (b) $P_1 = \vec{F}_1 \cdot \vec{v}_i = (2\hat{i} + 7\hat{j} \text{ N}) \cdot (2\hat{i} + \hat{j} \text{ m/s}) = \boxed{11 \text{ W}}$.
 (c) $W_{F2} = \vec{F}_2 \cdot \Delta \vec{r} = (2\hat{i} - 5\hat{j} \text{ N}) \cdot [(10\hat{i} + 5\hat{j}) \text{ m} - (0\hat{i} + 0\hat{j}) \text{ m}] = \boxed{-5 \text{ J}}$.

From the work-energy theorem,

$$W_{\text{net}} = K_f - K_i; \quad W_{F1} + W_{F2} = K_f - \frac{1}{2}mv_i^2;$$

$$55 \text{ J} - 5 \text{ J} = K_f - \frac{1}{2}(3 \text{ kg})[(2 \text{ m/s})^2 + (1 \text{ m/s})^2], \text{ which gives } K_f = \boxed{58 \text{ J}}.$$

83. From the uniform acceleration, we find the speed attained by the rocket:

$$v = v_0 + at = 0 + (2.0 \text{ m/s}^2)(33 \text{ s}) = 66 \text{ m/s}.$$

- (a) $W = \Delta K = \frac{1}{2}(1 \times 10^4 \text{ kg})[(66 \text{ m/s})^2 - 0] = \boxed{2.2 \times 10^7 \text{ J}}$.
 (b) Because the kinetic energy change is opposite to that in part (a), the work done will be
 $W = \boxed{-2.2 \times 10^7 \text{ J}}$.

84. (a) The acceleration of the box is uniform, so we can find it from

$$v^2 = v_1^2 + 2a \Delta x; \quad (0.55 \text{ m/s})^2 = 0 + 2a(3.50 \text{ m} - 0),$$

which gives $a = 0.043 \text{ m/s}^2$.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the box:

$$y\text{-component: } F_N - mg - F \sin \theta = 0;$$

$$x\text{-component: } F \cos \theta - m_k F_N = ma.$$

By eliminating F_N we find

$$\begin{aligned} \mu_k &= (ma - F \cos \theta) / (-mg - F \sin \theta) \\ &= [(25 \text{ kg})(0.043 \text{ m/s}^2) - (85 \text{ N}) \cos 10^\circ] / [-(25 \text{ kg})(9.8 \text{ m/s}^2) - (85 \text{ N}) \sin 10^\circ] = \boxed{0.32}. \end{aligned}$$

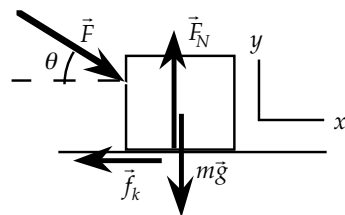
- (b) $W_{\text{net}} = \Delta K = \frac{1}{2}m(v^2 - v_1^2) = \frac{1}{2}(25 \text{ kg})[(0.55 \text{ m/s})^2 - 0] = \boxed{3.8 \text{ J}}$.

- (c) Because the only forces that do work are F and f_k , we have

$$W_f = W_{\text{net}} - W_F = W_{\text{net}} - F \cos \theta \Delta x = 3.8 \text{ J} - (85 \text{ N})(\cos 10^\circ)(3.50 \text{ m}) = \boxed{-2.9 \times 10^2 \text{ J}}.$$

Thus the work done to overcome friction is -0.29 kJ .

- (c) $P_F = \vec{F} \cdot \vec{v} = F \cos \theta v = (85 \text{ N})(\cos 10^\circ)(0.55 \text{ m/s}) = \boxed{46 \text{ W}}$.



85. (a) $mg = (2.6 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{26 \text{ N down}}$.
 $F_N = mg \cos \theta = (26 \text{ N}) \cos 32^\circ = \boxed{22 \text{ N perpendicular to plane (up)}}$.
 $f_k = \mu_k F_N = (0.25)(22 \text{ N}) = \boxed{5.4 \text{ N parallel to plane (down)}}$.

- (b) $W_g = -mg \sin \theta \Delta x = -(26 \text{ N}) \sin 32^\circ (1.3 \text{ m}) = \boxed{-18 \text{ J}}$.

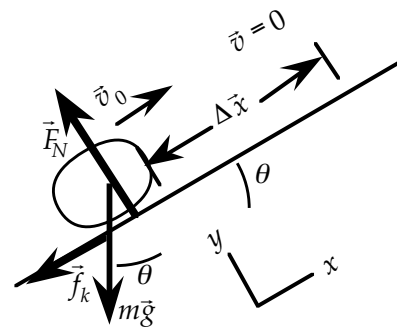
$$W_N = \boxed{0}.$$

$$W_f = -f_k \Delta x = (5.4 \text{ N})(1.3 \text{ m}) = \boxed{-7.0 \text{ J}}.$$

- (c) From the work-energy theorem, $W_{\text{net}} = \Delta K$, we get

$$W_g + W_N + W_f = \Delta K;$$

$$-18 \text{ J} + 0 - 7.0 \text{ J} = 0 - \frac{1}{2}(2.6 \text{ kg})v_0^2, \text{ which gives } v_0 = \boxed{4.4 \text{ m/s}}.$$



86. (a) Because the resistive force must balance the force provided by the engine, we have

$$F = P/v = (45 \text{ hp})(746 \text{ W/hp}) / [(80 \text{ km/h}) / (3.6 \text{ ks/h})] = \boxed{1.2 \times 10^2 \text{ N}}.$$

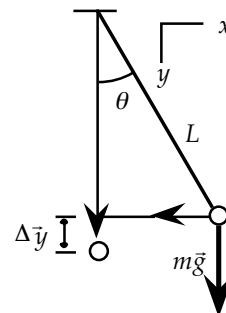
- (b) If the resistive force is proportional to the velocity, the engine force is also proportional to the velocity. Thus $P = Fv = kv^2$, or $P_2 = (v_2/v_1)^2 P_1$.

$$P_2 = [(60 \text{ km/h}) / (80 \text{ km/h})]^2 (45 \text{ hp}) = \boxed{25 \text{ hp}}.$$

- (c) $P_3 = (v_3/v_1)^2 P_1 = [(140 \text{ km/h}) / (80 \text{ km/h})]^2 (45 \text{ hp}) = \boxed{1.4 \times 10^2 \text{ hp}}$.

87. Because the work done is independent of the path, we can go first horizontally to the vertical line and then vertically down. For the horizontal motion, the work done by gravity is zero ($\vec{F}_g \perp \Delta \vec{x}$). For the vertical motion, we have

$$W_g = mg \Delta y = mg(L - L \cos \theta) = mgL(1 - \cos \theta) \\ = (4.0 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m})(1 - \cos 30^\circ) = \boxed{5.3 \text{ J}}.$$



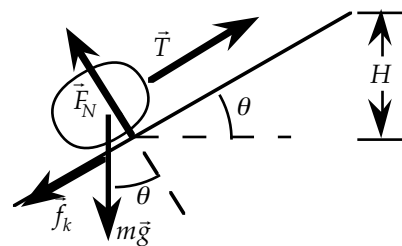
88. (a) $W = \Delta K = \frac{1}{2}m(v^2 - v_0^2) = \frac{1}{2}(5 \text{ kg})[(0.5 \text{ m/s})^2 - 0] = \boxed{0.6 \text{ J}}.$
 (b) $W = \frac{1}{2}(5 \text{ kg})[(10.1 \text{ m/s})^2 - (10 \text{ m/s})^2] = \boxed{5 \text{ J}}.$
 (c) $W = \frac{1}{2}(5 \text{ kg})[(v+1)^2 - v^2] = \boxed{2.5 v^2 + 5v \text{ J}},$ with v in m/s.
 (d) For $v \gg 1 \text{ m/s}$, $(v+1)^2 = v^2 + 2v + 1 \approx v^2 + 2v$. Then $W = \Delta K = \frac{1}{2}m(v^2 + 2v - v^2) = (5 \text{ kg})v = \boxed{5v \text{ J}},$ with v in m/s.

89. (a) Because the tension in the rope is perpendicular to the motion, $W_T = \boxed{0}.$
 (b) With $F_N = mg$, we have $f_k = \mu_k mg$. The friction force is opposite to the velocity and thus tangent to the path. The work done by the friction force is $W_f = -\mu_k mg(2\pi r) = -0.02(0.2 \text{ kg})(9.8 \text{ m/s}^2)(2\pi)(0.8 \text{ m}) = \boxed{-0.20 \text{ J}}.$
 (c) $W_{\text{net}} = \Delta K; -0.20 \text{ J} = K_f - \frac{1}{2}(0.2 \text{ kg})(10 \text{ m/s})^2$, which gives $K_f = \boxed{9.8 \text{ J}}.$

90. (a) With $F_N = mg$, we have $f_k = \mu_k mg = 0.55(1100 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{5.9 \times 10^3 \text{ N}}.$
 (b) Because friction opposes the motion, we have $W_f = -f_k \Delta x = -(5.9 \times 10^3 \text{ N})(48 \text{ m}) = \boxed{-2.8 \times 10^5 \text{ J}}.$
 (c) Friction is the only force that does work, so we have $W_{\text{net}} = W_f = \Delta K; -2.8 \times 10^5 \text{ J} = 0 - \frac{1}{2}(1100 \text{ kg})v_0^2$, which gives $v_0 = \boxed{23 \text{ m/s}} = 51 \text{ mi/h}.$

91. (a) With $f_k = 0$ and no change in the speed (assumed very small) of the mass, we have $W_{\text{net}} = \Delta K = 0;$
 $W_T + W_g = W_T + (-mg)H = 0$, which gives $W_T = mgH$, independent of the angle θ .

- (b) Now we have $W_{\text{net}} = W_T + W_f + W_g = 0;$
 $W_T - \mu_k mg \cos \theta (H/\sin \theta) - mgH = 0$, which gives $W_T = \boxed{mgH(1 + \mu_k \cot \theta)}.$



92. (a) $P = W/\Delta t = (6 \times 10^4 \text{ J})/(0.3 \times 10^{-9} \text{ s}) = 2 \times 10^{14} \text{ W} = \boxed{2 \times 10^{11} \text{ kW}}.$
 (b) $P = W/\Delta t = (6 \times 10^4 \text{ J})/[(20 \text{ min})(60 \text{ s/min})] = \boxed{50 \text{ W}}.$

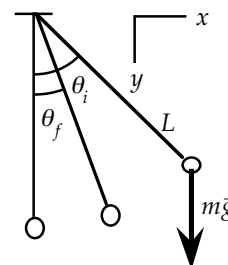
93. From the analysis in the solution to Problem 87:

$$W_g = mg \Delta y = mg(L \cos \theta_f - L \cos \theta_i) = \boxed{mgL(\cos \theta_f - \cos \theta_i)}.$$

At the bottom of the swing, $\theta_f = 0$ and $W = \Delta K$:

$$mgL(1 - \cos \theta_i) = \frac{1}{2}mv^2 - 0, \text{ which gives } v = \boxed{[2gL(1 - \cos \theta_i)]^{1/2}}.$$

The tension can be ignored since $W_T = \boxed{0}$, as the tension in the string is normal to the displacement.



94. From the meaning of efficiency, $P_{\text{elect}} = 0.68 P_{\text{mech}}$;

$$850 \times 10^3 \text{ W} = 0.68 [(\Delta m)gh / \Delta t] = 0.68 (\Delta m / \Delta t)(9.8 \text{ m/s}^2)(18 \text{ m}), \text{ which gives}$$

$$\Delta m / \Delta t = \boxed{7.1 \times 10^3 \text{ kg/s}}.$$

The time to pay for the power plant is $t = \text{cost}/\text{rate}$;

$$t = (\$3.5 \times 10^6) / (\$0.10/\text{kWh})(850 \text{ kW}) = 4.1 \times 10^4 \text{ h} = \boxed{4.7 \text{ y}}.$$

95. If we assume no change in the kinetic energy, we have

$$W_{\text{net}} = \Delta K, \text{ or } W_{\text{body}} + W_{\text{sun}} = 0. \text{ Thus}$$

$$W_{\text{body}} = -W_{\text{sun}}.$$

We take x as positive away from the sun. Because $F(x)$ is toward the sun, we have

$$W_{\text{body}} = -\int F(x) dx = -\int (-mK/x^2) dx = +\int (mK/x^2) dx.$$

For the small variation of $\Delta x = 1\%$ of x , we can take the force to be constant and get

$$W_{\text{body}} \approx -(-mK/x^2) \int dx = + (mK/x^2)(0.01x) = \boxed{0.01mK/x}.$$

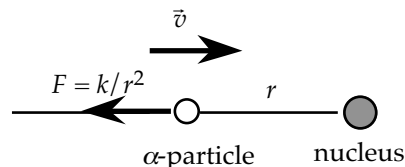
96. With the only force being the repulsive force and a final kinetic energy of zero, we have

$$W_F = \Delta K = 0 - K_i;$$

We take r as positive away from the nucleus, so the repulsive force is positive.

The work done by the force as the α -particle approaches is

$$W_F = \int_{\infty}^R + \left(\frac{k}{r^2} \right) dr = -\frac{k}{r} \Big|_{\infty}^R = -\frac{k}{R}.$$



$$\text{Thus } K_i = -W_F = + (k/R) = (3.65 \times 10^{-26} \text{ N}\cdot\text{m}^2) / (1.00 \times 10^{-14} \text{ m}) = \boxed{3.65 \times 10^{-12} \text{ J}}.$$

97. If F were conservative, then the net work done by F along *any* enclosed path in the xy plane would be zero. This is not true for the force F in this problem, however. Consider, for example, the square path from $(0, 0)$ to $(1, 0)$ to $(1, 1)$ to $(0, 1)$ and back to $(0, 0)$, where all the units are meters. We have

$$dW_F = F_x dx + F_y dy = F_x dx = (3 \text{ N/m})y dx.$$

$$\text{From } (0, 0) \text{ to } (1, 0) \quad y = 0, \text{ so } F_x = 0 \text{ and } F_y = 0 \text{ so } W_F = 0.$$

$$\text{From } (1, 0) \text{ to } (1, 1) \text{ we have } dx = 0, \text{ so } dW_F = (3 \text{ N/m})y dx = 0.$$

$$\text{From } (1, 1) \text{ to } (0, 1) \text{ we have } y = 1 \text{ m, so } dW_F = (3 \text{ N/m})y dx = (3 \text{ N/m})(1 \text{ m}) dx = (3 \text{ N}) dx \text{ and}$$

$$W_F = (3 \text{ N}) \Delta x = (3 \text{ N})(0 \text{ m} - 1 \text{ m}) = -3 \text{ J}.$$

$$\text{Finally, from } (0, 1) \text{ to } (0, 0) \text{ again } dx = 0, \text{ so } dW_F = (3 \text{ N/m})y dx = 0.$$

Summing over the values of W_F over all the four segments of the path, we obtain

$$\Sigma W_F = -3 \text{ J} \neq 0.$$

Thus F is not conservative.