# UNIVERSITY OF TORONTO
# DEPARTMENT OF ECONOMICS

# ECO358: FINANCIAL ECONOMICS

## Sample Test-1(Solutions)

Instructions: This is a closed book examination. A fomual sheet is attached. Show all your work otherwise **you will not get full credit**.

**Make sure you allocate time appropriately**.

**You have 90 minutes.**

**Good Luck!**



NAME: _____

ID#: _____

| | | |
|---|---|---|
| **Problem-1** | _____ | **(15 points)** |
| **Problem-2** | _____ | **(20 points)** |
| **Problem-3** | _____ | **(15 points)** |
| **Problem-4** | _____ | **(15 points)** |
| **Total** | _____ | **(65 points)** |

**Problem-1 [15 Points]: Answer the following short questions.**

a) **[5 Points]** Microbust is trading at NASDOCK for 50 with a price-earning ratio (e.g. $P_t/E_{t+1}$) of 10. Assume you believe in the discounted dividend mode (DDM). If the ROE on this stock is 12% and if its discount rate is 10%, what would be the retention ratio for this stock.

**Solution:**

$P_t/E_{t+1}$= 10 => P=10E => E = 5

**P = Div/(r-g)= E*(1-RR) /(0.10-ROE*RR)**
**50 = 5*(1-RR)/ (.10– .12*RR)**

**=> RR = 0**



b) **[5 Points]** If your wealth doubles and you keep your portion of wealth invested in risky assets constant, then you are characterized by Constant Absolute Risk Aversion.

**False, this individual exhibits decreasing ARA. An individual with constant ARA will keep his *absolute investment amount* in the risky asset constant following an increase in the wealth.**

d) **[5 Points]** For a rising yield curve, the liquidity premium theory of term structure implies that the short rate is expected to rise next period. True/False explain your answer.

**The liquidity Premium of theory implies that the forward rate is equal to the expected rate plus the liquidity premium: $f_{ij} = E(r_{ij}) + \Pi_{ij}$. A rising yield implies that the forward rate is higher than the spot short rate but that may or may not imply that the expected rate is higher than the current spot rate because the forward rate carries a "non-negative" liquidity premium.**

**Problem-2: [20 points]** You are given the following information about bonds A, B, and C:

| *Bond* | *Type* | *Face Value* | *Maturity* | *Price* |
|---|---|---|---|---|
| A | Zero Coupon | $1000 | 1 years | $970.874 |
| B | 6% Coupon | $1000 | 2 years | $1038.282 |
| C | 8% Coupon | $1000 | 3 years | $1084.579 |

a) **[5 Points]** Find the one -year, two-year, and the three-year spot rates.

**Solution:**

$$(1+r_1)=\frac{1000}{970.874} \Rightarrow r_1=0.03$$

$$\frac{60}{(1+0.03)}+\frac{1060}{(1+r_2)^2}=1038.282 \Rightarrow r_2=0.04$$

$$\frac{80}{(1+0.03)}+\frac{80}{(1+0.04)^2}+\frac{1080}{(1+r_3)^3}=1084.579 \Rightarrow r_3=0.05$$



b) **[5 Points]** Calculate all the forward rates embedded in these prices? In a few sentences explain what forward rates mean.

**Solution:**

$$f_{j,k}=\left(\frac{(1+r_k^{spot})^k}{(1+r_j^{spot})^j}\right)^{\frac{1}{k-j}}-1$$

$$f_{1,2}=\frac{1.04^2}{1.03}-1\cong 5\%$$

$$f_{2,3}=\frac{1.05^3}{1.04^2}-1\cong 7.02\%$$



$$f_{1,3}=\left[\frac{1.05^3}{1.02}\right]^{(1/2)}-1=60.01\%$$

**Note: 1 point deduction is f1,3 not found:**

Explanation **[2 Points]:** Forward rate **is the implied rate from the spot rates**. If you believe in the expectation theory it means the expected future spot rate (f=E(r)) otherwise it means the expected spot rate plus a premium. Unless one can quantify the premium, it would be hard to analyze the forward rate. [1 point deduction if one only says it is the expected rate]

c) **[5 Points]** Assume that the current rate in the market for a one-year loan, **two-years** from today is 6% per annum. Assuming you can borrow and lend at the spot rates: Is there an arbitrage opportunity? If so, use a table and show how you would take advantage of such an opportunity.

**Solution:** Since the forward rate is 7.02%, there is an arbitrage opportunity. Borrow short and lend long to replicate the forward rate at 7.02% and gain the difference (7%-6%)

Fill in the table and explain.

| action | Cash flow$_0$ | Cash flow$_2$ | Cash flow$_3$ |
|---|---|---|---|
| Borrow two-year | $(1000/1.04^2)$ | -1000 | |
| Lend for 3 period | $-(1000/1.04^2)$ | 0 | $(1000/1.04^2)*(1.05)^3=$ $1000*(1.0702)$ |
| total | 0 | -1000 | 1070 |

Which means you can generate 7.02% for your year-2 investment, if you can borrow at the same time at 6%, 1.02% will be your arbitrage.

d) **[5 Points] Ignore part (C),** A 3-year zero coupon bond D with a face value of $1,000 is trading at $1000. Show that this bond is mis-priced and then demonstrate how you could take advantage the arbitrage opportunity by buying and selling bonds A, B, C. In doing so, make sure your arbitrage profits are realized today.

**Solution:**

d) The price of this bond should be given by:

$$P = \frac{1000}{(1+0.05)^3} = 863.84$$

Since this is smaller than 1000, we could sell bond D for $1000, and buy a portfolio of bonds A, B, and C which would replicate bond D.

$1{,}000 \cdot n_A + 60\, n_B + 80 \cdot n_C = 0$
$1{,}060 \cdot n_B + 80\, n_C = 0$ and
$1{,}080 \cdot n_C = 1000$

Solving for the portfolio weights results in: $n_A = -0.069881$, $n_B = -0.069881$, and $n_C = 0.9259$.

Then consider the following strategy:

| Strategy | $C_0$ | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|---|
| **Sell Bond D** | **1000.0** | **0** | **0** | **-1000** |
| Sell 0.069881 A-bond | 67.85 | -69.88 | 0 | 0 |
| Sell 0.069881 B-bond | 72.56 | -4.19 | -74.07 | 0 |
| Buy 0.9259 C-bond | -1004.24 | 74.07 | 74.07 | 1,000 |
| **Strategy cash flow** | -863.84 | 0 | 0 | 1,000 |
| Total | 136.16 | 0 | 0 | 0 |

**Problem-3 [15 Points]:** The expected next-period earnings of XYZ Co. are $8mil. The company is currently retaining 60% of the earnings and is expected to decrease its retention ratio to 20% in 3 years. XYZ Co. has 1 mil. shares outstanding and its historical ROE is 12%. The appropriate discount rate for the company's cash flows is r = 11%.

a) **[8 Points]** What is the price of the common stock of the company today?

**Solution:**

**$Div_1$ = (1-0.6) * (8 million) / 1 million = 3.2**

**$g_1$ = 0.6 (0.12) = 0.072**
**$g_2$ = 0.20 (0.12) = 0.024**

**First three years:**

$$\frac{3.2}{0.11-0.072}\left(1-\left(\frac{1.072}{1.11}\right)^3\right)=\$8.3559$$

Perpetuity: $$\left(\frac{0.8\times 8\times(1.072)^2(1.024)}{0.11-0.024}\right).\frac{1}{1.11^3}=\$64.03$$

ABC = $8.356+$64.03a=$72.39

b) **[7 Points]** Assume you bought the stock today and held it for 3 full years before selling it. Find the capital gain and dividend yield of your investment.

**Solution:**

The future value of the investment is:

$$FV = 1.11^3 \times 72.39 = \$99$$

This is the total gain. But the sock will be sold for:

$$P_3 = 87.57$$

Therefore:

$$Total = \frac{99 - 72.39}{72.39} = \frac{87.57 - 72.39}{72.39} + DividendYierld = 36.76\%$$

$$CaitalGain = 20.98\%$$

$$DYield = 15.79\%$$

**Question-4 [15 Points]:** An individual is characterized by the following utility function: U(W) = 5W-0.002W$^2$, where W represents the wealth. Suppose the initial wealth is \$1,000. Furthermore, suppose the individual is faced with a gamble that has 50% chance of increasing the wealth by \$100 and 50% chance of decreasing the wealth by \$100.

a) **[5 Points]** Use Markowitz approach to show how much the individual will pay to avoid the gamble.

**Expected utility without the gamble is:**

**E(U) = 3,000**

**With the gamble:**

$$E(U) = 5E(W) - 0.002E(W^2)$$
$$E(U) = 5000 - 0.002\left[0.5\times(1000+100)^2 + 0.5(1000-100)^2\right]$$
$$E(U) = 5000 - 2020 = 2980$$

**CE;**

**2980= 5CE-0.002CE$^2$**

**CE ~980.74 => RP = 1000-980.74 = 19.26**

b) **[5 Points]** Use Arrow-Pratt approach to show how much the individual will pay to avoid the gamble.

$$\pi = \frac{1}{2}\sigma^2\left[-\frac{U''}{U'}\right]$$
$$\sigma^2 = 0.5(1100-1000)^2 + 0.5(900-1000)^2 = 100^2$$
$$\pi = \frac{1}{2}100^2\left[-\frac{-0.004}{5-0.004\times1000}\right] = \frac{1}{2}100^2\times\frac{0.004}{1} = 20$$

c) **[5 Points]** Suppose the individual initially invested $100 in risky assets. If the individual inherits $200, based on your calculation of ARA & RRA can you speculate what will happen to his investment?

**Solution:**

$$ARA_{before} = \left[\frac{0.004}{5 - 0.004W}\right] = 0.004(W = 1000)$$

$$ARA_{After} = \left[\frac{0.004}{5 - 0.004W}\right] = 0.02(W = 1200)$$

$$RRA_{Before} = ARA \times W = 0.004 \times 1000 = 4$$

$$RRA_{After} = 0.02 \times 1200 = 24$$

Both ARA & RRA are rising.

(i) A rising ARA implies that the individual becomes more risk averse in absolute term => he will decrease his $ investment.

**(ii)** A rising RRA implies that the investor is relatively more risk averse => the share of risky investment will decline

# Formula sheet



## 1. Basics of Valuation

$$NPV = -C_0 + \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^N} = -C_0 + \sum_{t=1}^{N} \frac{C}{(1+r)^t}$$

$$\text{Perpetuity}: PV = \frac{C}{r}$$

$$\text{Annuity}: PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$



$$\text{Growing Annuity}: PV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{(1+r)}\right)^T\right]$$

g = (Retention Ratio) (ROE)

## 2. Bond//Term Structure of Interest Rate

Forward Rate:

$$f_{i,j} = \left(\frac{(1+r_j^{spot})^j}{(1+r_i^{spot})^i}\right)^{\frac{1}{j-i}} - 1$$



## 3. Expected Utility Basics of Portfolio

Risk Premium:

Markowitz Risk Premium: RP = E(W) – CE
CE (Certainty Equivalent): U(CE) = E(U(W))

Arrow Pratt Risk:

Absolute Risk Aversion: ARA = $-\frac{U''(W)}{U'(W)}$

Relative Risk Aversion: RRA $= W \times ARA = W \times -\frac{U''(W)}{U'(W)}$