CHAPTER 5 PROPERTIES OF FUNCTIONS

Section 5.1 Shifting Graphs

Example
consider the simple demand and supply functions

$$Q_d = 100 - P$$

$$Q_s = 10 + 2P$$

$$Q_d = Q_s$$

The equilibrium Price and quantity are:

$$100 - P = 10 + 2P$$

$$3P = 90$$

$$\bar{P} = \frac{90}{3} = 30$$

$$Q_d = 100 - P$$

$$= 100 - 30$$

$$\bar{Q} = 70$$

Consider a shift to the right in the supply function

The new supply at price p is

$$Q_s = 16 + 2P$$

The new equilibrium price is determined using the original demand function and the new supply equation.

$Q_d = 100 - P$ original demand function

$100 - P = 16 + 2P$

$84 = 3P$

$\bar{P} = \frac{84}{3} = 28$

The new equilibrium quantity

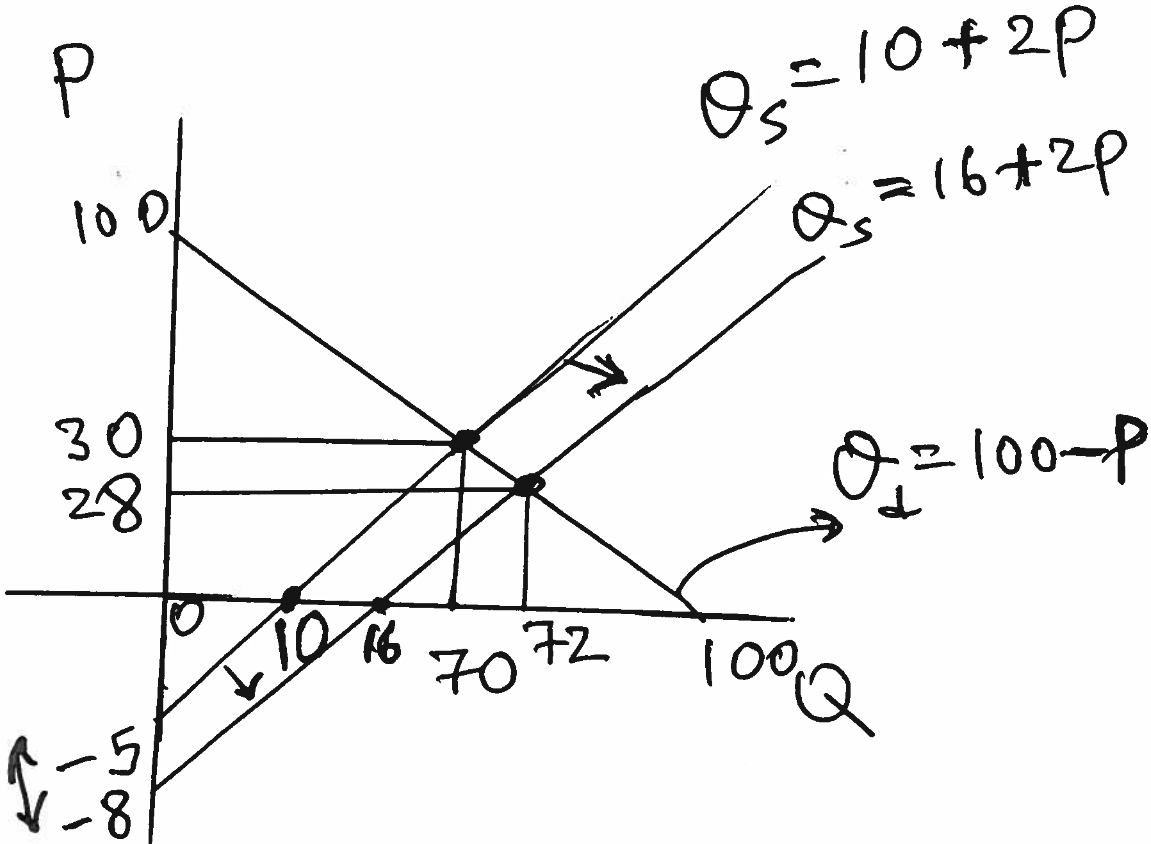
$Q_d = 100 - P$

$= 100 - 28$

$\bar{Q} = 72$

The new equilibrium price is lower than the old one, while the quantity is higher.

Graph



The outward shift in the supply curve implies that the equilibrium point moves down to the right along the unchanged demand curve.

A shift to the right in the supply curve can result from decreased taxation or decreased cost.

Shifts in the demand curve can be analyzed in the same way.

A tax on the producer

consider the demand and supply curves

$p = 65 - \frac{7}{100} q$ demand equation

$p = 50 + \frac{8}{100} q$ supply equation

the equilibrium price $\bar{p} = 58$

the equilibrium quantity $\bar{q} = 100$

(a). If a tax of \$1.50 per unit is to be imposed on the manufacturer, find the new supely equation

before the tax the manufacturer supplies q units at a price of $p = 50 + \frac{8}{100}q$ per unit.

After the tax the manufacturer will sell the same q units for an additional \$1.50 per unit. The price per unit will then be

$$\left(\frac{8}{100}q + 50\right) + 1.50.$$

The new supply equation will be

$$p = 51.50 + \frac{8}{100}q$$

find the equilibrium price and the equilibrium quantity after the imposition of the tax on the manufacturer

Now solve the system

$p = 65 - \frac{7}{100} q$ original Demand equation

$p = 51.50 + \frac{8}{100} q$ new supply equation with the tax set them equal

$$65 - \frac{7}{100}q = 51.50 + \frac{8}{100}q$$

$$\frac{15}{100}q = 13.50$$

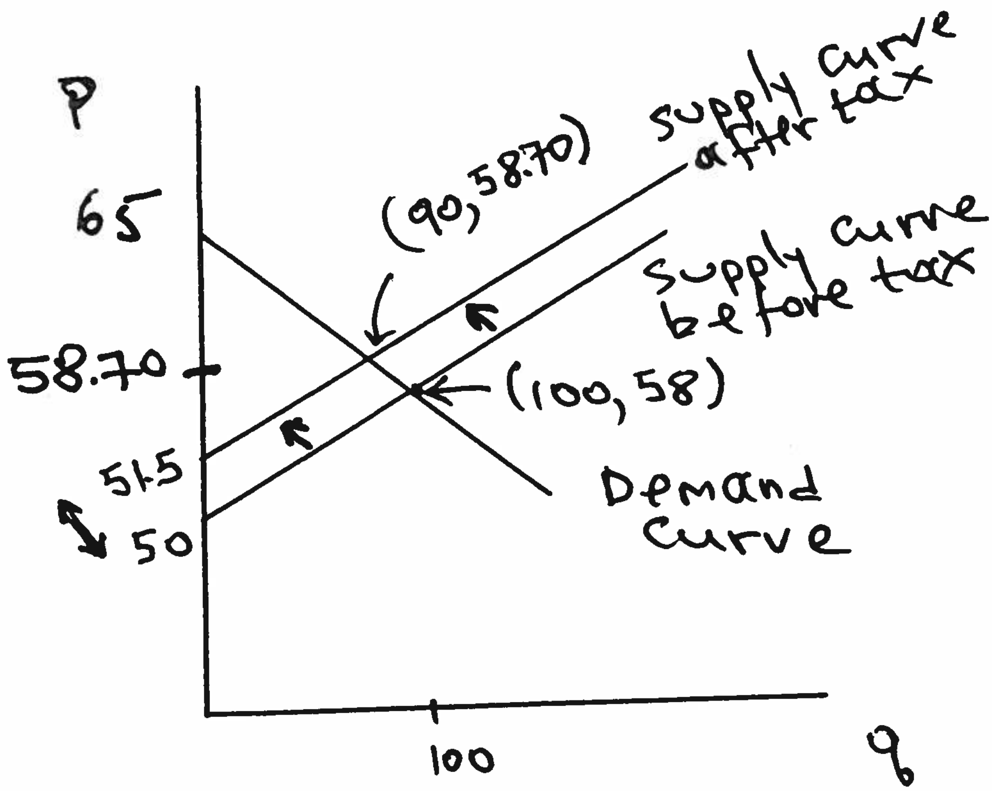
$$15q = 1350$$

$$\bar{q} = \frac{1350}{15} = 90$$

$$\bar{P} = 51.50 + \frac{8}{100}(90)$$

$$= 58.70$$

Graph



The tax of \$1.50 per unit increases the equilibrium price by \$0.70. There is also a decrease in the equilibrium quantity from q =100 to q= 90 due to the change in the equilibrium price.

A subsidy to the producer

(b) Determine how the original equilibrium price will be affected if the Company is given a government subsidy of \$1.50 per unit.

A Subsidy will reduce the price of the product. Thus,

$$P = 50 + \frac{8}{100}q - 1.50$$

$$P = 48.5 + \frac{8}{100}q$$

The new supply equation (with the subsidy).

solving the system

$P = 65 - \frac{7}{100}q \rightarrow$ original demand equation

$P = 48.5 + \frac{8}{100}q \rightarrow$ The new supply equation with the subsidy

set them equal

$$48.5 + \frac{8}{100}q = 65 - \frac{7}{100}q$$

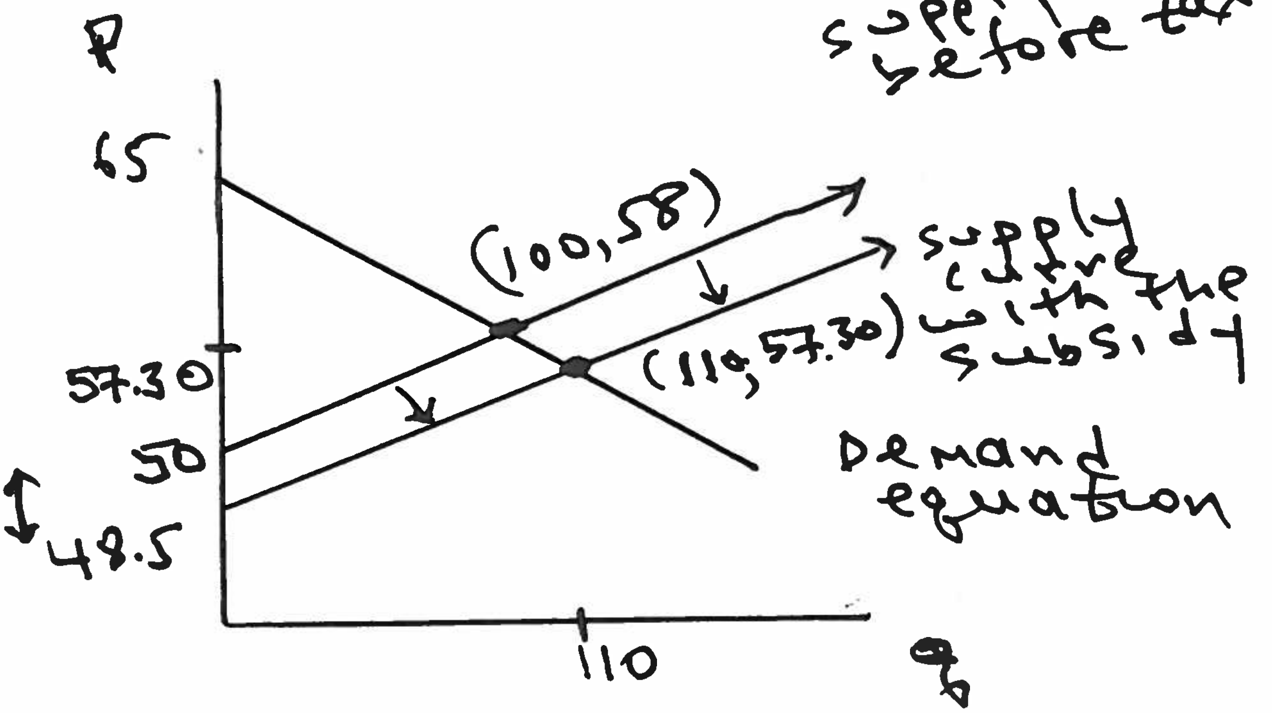
$$\frac{15}{100}q = 16.5$$

$$\bar{q} = \frac{1650}{15} = 110$$

$$\bar{P} = 48.5 + \frac{8}{100}(110)$$

$$= 57.30$$

Graph



The equilibrium price will decrease by $ 0.70 (= 58 − 57.30)

The equilibrium quantity will increase by 10 units to 110 units

Section 5.2 New Functions from old

Example 1

The cost of producing Q units of a commodity is $C(Q)$. The cost per unit of output, $A(Q) = C(Q)/Q$, is called the average cost.

$$A(Q) = C(Q)/Q \qquad \textbf{(average cost)}$$

If, in particular, $C(Q) = aQ^3 + bQ^2 + cQ + d$ is a cubic cost function of the type shown in Fig. 4.7.2, the average cost is

$$A(Q) = aQ^2 + bQ + c + d/Q, \qquad Q > 0$$

Thus $A(Q)$ is a sum of a quadratic function $y = aQ^2 + bQ + c$ and the hyperbola $y = d/Q$. Figure 2 shows how the graph of the average cost function $A(Q)$ is obtained by piling the graph of the hyperbola $y = d/Q$ onto the graph of the parabola $y = aQ^2 + bQ + c$.

Note that for small values of Q the graph of $A(Q)$ is close to the graph of $y = d/Q$, while for large values of Q, the graph is close to the parabola (since d/Q is small when Q is large).

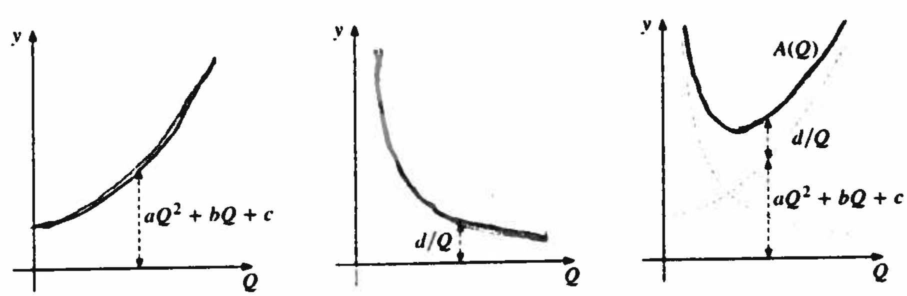


Figure 2 $A(Q) = (aQ^2 + bQ + c) + d/Q = C(Q)/Q$

Example 2

Let $R(Q)$ denote the **revenues** obtained by producing (and selling) Q units. Then the **profit** $\pi(Q)$ is given by

$$\pi(Q) = R(Q) - C(Q)$$

An example showing how to construct the graph of the profit function $\pi(Q)$ is given in Fig. 3. In this case the firm gets a fixed price p per unit, so that the graph of $R(Q)$ is a straight line through the origin. The graph of $-C(Q)$ must be added to that of $R(Q)$. The production level which maximizes profit is Q^*.

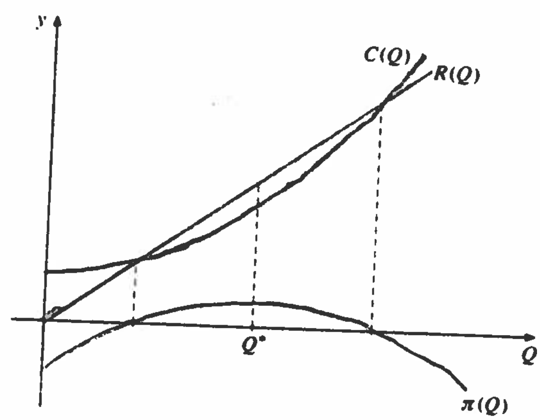


Figure 3 $\pi(Q) = R(Q) - C(Q)$

$R(Q) = pQ$