École de gestion TELFER School of Management

LIÉE au LINKED with LEADERSHIP

ADM 2304 – Final Examination

APPLIED STATISTICAL METHODS IN BUSINESS

20 April 2012

PRINT YOUR NAME: ____

STUDENT NUMBER: ___________________________________

SECTION (please circle one): M N P Q R

Time allowed: 3 hours

Question booklet: 10 pages (including front cover)

Appendices: 4 pages of Minitab output and 5 pages of statistical tables

Instructions:

Complete all tests with hypotheses, test statistic, critical value, decision, and conclusion. Use the .05 level of significance unless otherwise indicated. Explain your answers where asked.

Calculators and one sheet of notes (8.5 x 14 in.) are allowed.

You must hand back all exam materials (exam question booklet, appendices and tables), but please keep your personal sheet of notes for future use.

Question	Value	Mark
1	8	
2	11	
3	13	
4	24	
5	14	
Total	**70**	

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ____

Question 1. [8 marks]

Happy Puppies Golden Retriever Breeding Kennels Ltd. wants to assess the quality of two types of puppy food: A, which has been used for years with good results, and B, a new-age organic product. To do this assessment, Happy Puppies compares 9 pairs of Golden Retriever puppies; each pair consists of two littermates (as close to identical twins as dogs can be) and each pair is from a different mother. At random, Food A is assigned to one puppy within each pair and Food B to the other. The weight gains of the puppies, in pounds, after five months are given in Appendix A, along with some summary statistics. State any reasonable assumptions needed to support your calculations.

(a) Use a hypothesis test, with a significance level of 0.05, to assess whether Food B results in <u>greater</u> weight gain than Food A.

[3]

(b) Dandy Dalmatians Breeders Ltd. is also interested in this new-age organic puppy food but approaches its assessment of the two foods differently. It takes 18 randomly selected puppies, none of whom are littermates, and randomly assigns Food A to nine of them and Food B to the other nine. Assume the weight gain data from the Dalmatian puppies are the same as that of the Golden Retrievers in part (a). Use a hypothesis test, again with a significance level of 0.05, to assess whether Food B results in <u>greater</u> weight gain than Food A.

[4]

(c) Compare the results of your two tests in parts a) and b) (not the numbers, just the conclusion "accept/reject"). Why are the results different? (No marks awarded for citing “different breeds” as a reason.)

[1]

Question 2. [11 marks]

Random samples of consumers of new television sets were obtained in four provinces. Consumer preferences for different types of new television sets are given for the four provinces below in the table. See Appendix B for some partial Minitab analysis.

TV Type	Alberta	Ontario	Quebec	Newfoundland	Total:
HD LCD	23	43	30	12	108
HD LCD/LED	64	83	53	14	214
HD 3D	22	25	38	11	96
Plasma/Other	16	24	24	13	77
Total:	125	175	145	50	495

a. State the hypotheses appropriate for the tabulated data above, and calculate the appropriate test statistic.

[3]

b. Complete the test and reach an appropriate conclusion by using the critical value approach. Use the .05 level of significance. Now calculate the approximate p-value.

[3]

p-value = ______________

c. What can you learn about marketing strategy from the conclusion above? Explain briefly.

[1]

Question 2. (cont'd)

d. If p1, p2, p3, and p4 represent the proportions of the four types of TVs sold, test the hypothesis that p1 = p3 = p4 = 0.5 * p2. Use the .05 level of significance and the critical value approach.

[4]

Question 3. [13 marks]

Measuring the amount of "leptin" hormone in circulation in the body and then measuring the percentage of fat in the human body is considered to be the "gold standard" test for determining the presence of obesity in human beings. However, these tests are quite expensive and performed infrequently. Generally, "Body Mass Index (BMI)" and "Waist to Hip Ratio (WHR)" are the two measures which are used to determine the presence of obesity. Recently after careful measurements, it has been suggested that BMI test incorrectly categorises people as obese 48% of the times. Two independent samples were taken of people categorized as obese by using BMI and WHR respectively and the results are given below.

Category Obese:	Using BMI	Using WHR
Correctly Categorized:	145	222
Incorrectly Categorized:	155	178
Total:	300	400

a. Test the hypothesis that people categorized as obese using the BMI are incorrectly categorized more than 48% of the time. Use the .05 level of significance.

[3]

b. Calculate the appropriate 95% one-sided/asymmetrical confidence interval. Is this interval consistent with your conclusion in part 'a' above? Explain briefly.

[2]

c. What should be the sample size for finding a 95% two-sided/symmetrical confidence interval for the proportion of incorrectly categorized people using BMI, if the margin of error were ± 1.5%?

[2]

d. Test the hypothesis that the proportion of incorrectly categorized people by using BMI and WHR are different. Use a level of significance of 5%.

[4]

e. Calculate the appropriate 95% confidence interval for the test in part 'd' above. Is it consistent with the conclusion reached? Explain briefly.

[2]

Question 4. [24 marks]

You recently used multiple regression analysis to explain the variability in the salaries of U of O administrative staff using four independent variables: $X_{1,}$ hours worked; $X_{2,}$ hours worked by the worker's spouse; $X_{3,}$ age; and $X_{4,}$ years of education. You were discussing the implications with several utterly fascinated friends at the local pub when one of them spilled beer on your printed output. Appendix C is what was left of your results after the ink, the beer and your tears stopped running.

a) Write the regression equation that estimates the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon$.
[2]

b) Replace the lost ANOVA table by filling in the seven boxes below. Round off your final answers to, at most, three decimal places but ***retain all precision throughout your calculations***. (Hint: Start by calculating the MSE.)
[3]

Source of Variation	D.F.	S.S.	M.S.	F
Regression Model				27.547
Error				
Total		171.168		

S = 1.105

c) Test whether the years of education variable X_4 is helpful in predicting wage rates. Use $\alpha = 0.05$.
[3]

d) Explain the meaning of the estimated coefficient of the years of education variable X_4.
[2]

e) What is the R-square of the model?
[1]

f) Carry out a hypothesis test to determine whether this model is useful for explaining the variability in wage rate or in predicting the wage rate. Use the 5% significance level. Make sure to state the hypotheses and a final conclusion!
[3]

g) The data were then used to estimate a simpler model, using only $X_{4,}$ Years of Education, as a predictor variable. Here are some results:
[3]

Estimated slope	1.607	Estimated intercept	–1.478
Model Sum of Squares	94.065	Error Sum of Squares	77.103
Standard Error	1.529		

Predict the hourly wage for an employee with 12 years of education and calculate an <u>approximate</u> 95% prediction interval.

h) What evidence is there that a model containing only X_4 is not the best model?
[2]

i) Explain how you would detect a problem of multicollinearity with the years of education variable X_4, if your software program were not able to calculate measures of multicollinearity (this means that the years of education variable is close to being a linear combination of the other variables).
[2]

j) What subset model might be better? Based on the partial analyses given, explain briefly how you would select a better model.
[2]

k) Describe a general method of selecting the best subset model.
[1]

Question 5. [14 marks]

To test the effects of the type of Display (regular, Special) and the Price (low, medium, high) on the total sales of a new product, a product manager tested all the combinations for a month at a time in various stores. Appendix D displays, summarizes and analyzes the data.

(a) Discuss whether the basic assumptions of the analysis of variance are warranted.
[2]

(b) The tabulated statistics give the cell means, the sample standard deviations in each cell, and the number of observations in each cell. Write down a formula (using numbers) that shows how you could use these standard deviations to calculate the MSE.
[2]

(c) Explain what the interactions plot suggests about the possibility of interaction between the two factors or the possibility of main effects.
[2]

(d) Test at the .05 level of significance whether the effect of Price on Sales depends on the Type of Display.

[3]

(e) Does part (d) support your observations in part(c). Explain why or why not.

[1]

(f) Using the Bonferroni method and a family error rate of 5%, calculate confidence intervals to compare the pairwise differences between the price levels. What do you conclude?

[4]

Appendix A.

Puppy Pair	1	2	3	4	5	6	7	8	9
Food A	53	39	29	41	47	50	56	47	52
Food B	60	38	39	49	49	62	53	42	58

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
A	9	0	46.00	2.80	8.41	29.00	40.00	47.00	52.50	56.00
B	9	0	50.00	3.00	9.00	38.00	40.50	49.00	59.00	62.00
diff	9	0	-4.00	2.00	6.00	-12.00	-9.00	-6.00	2.00	5.00

Appendix B.

Data Display

Row	TV Type	Alberta	Ontario	Quebec	Newfoundland
1	HD LCD	23	43	30	12
2	HD LCD/LED	64	83	53	14
3	HD 3D	22	25	38	11
4	Plasma/Other	16	24	24	13

Chi-Square Test: Alberta, Ontario, Quebec, Newfoundland

Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

	Alberta	Ontario	Quebec	Newfoundland	Total
1	23	43	30	12	108
	-----	38.18	31.64	10.91	
	-----	0.608	0.085	0.109	
2	64	83	53	14	214
	54.04	-----	62.69	21.62	
	1.836	-----	1.497	2.683	
3	22	25	38	11	96
	24.24	33.94	-----	9.70	
	0.207	2.355	-----	0.175	
4	16	24	24	13	77
	19.44	27.22	22.56	7.78	
	0.610	0.381	0.093	3.506	
Total	125	---	145	50	495

Chi-Sq = -----, DF = --, P-Value = ----

Appendix C.

REGRESSION MODEL STATISTICS

R Square	______
Standard Error	1.105
Observations	35

ANOVA	**df**	**SS**	**MS**	**F**	**P-value**
Model/Regression					
Error/Residual					
Total					

REGRESSION MODEL ESTIMATES

	Coefficients	**Std Error**	**t Stat**	**P-value**
Intercept	–10.500	6.525	-1.61	0.118
X1 (Hours worked – worker)	–0.002	0.008	-0.25	0.804
X2 (Hours worked – spouse)	0.001	0.001	1.04	0.307
X3 (Age)	0.198	0.043	4.60	0.000
X4 (Years of education)	1.917	0.225		

Appendix D.

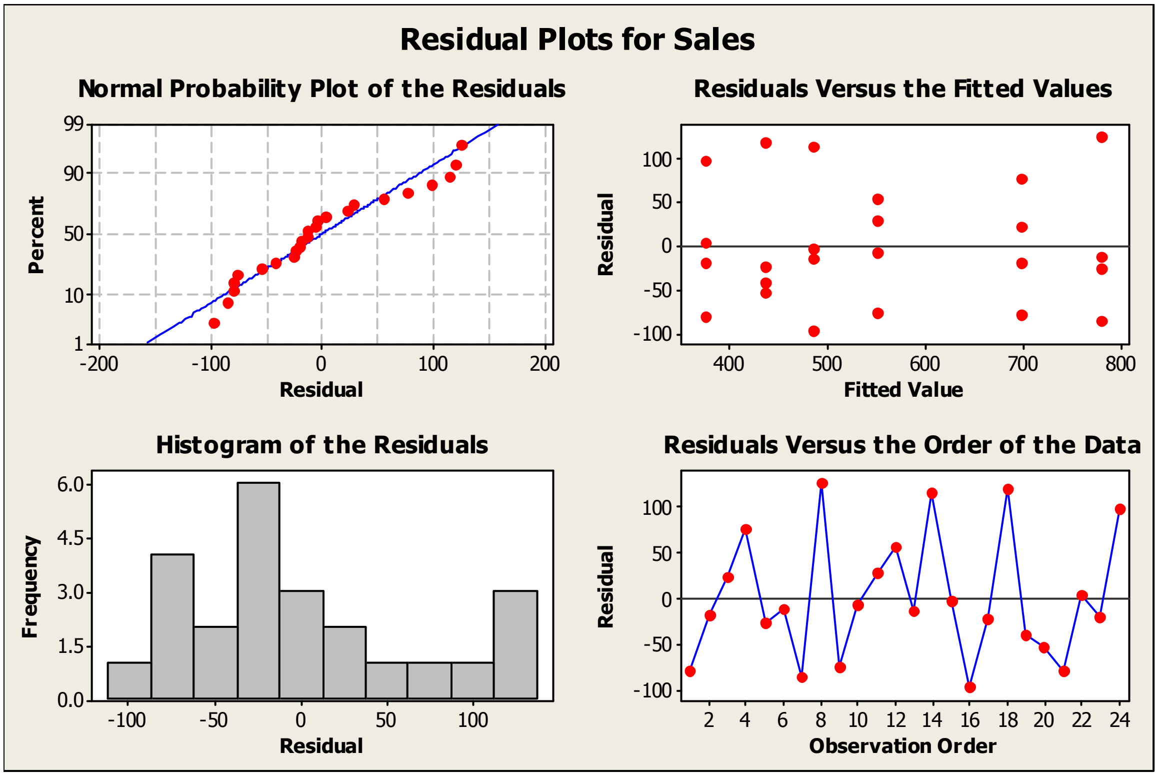
Display	Price		
	High	Low	Medium
Regular	413	620	475
Regular	556	680	544
Regular	395	723	579
Regular	382	776	606
Special	294	755	472
Special	378	769	601
Special	355	696	482
Special	473	908	388

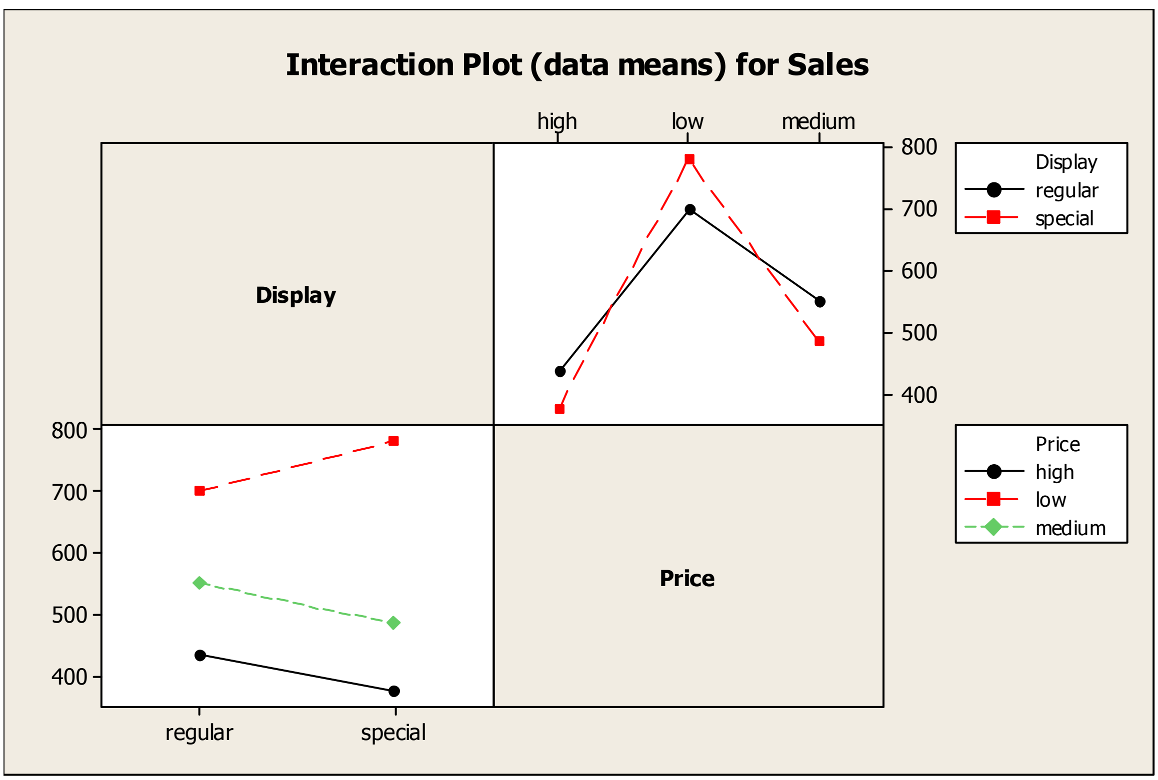
Tabulated statistics: Display, Price

```
Rows: Display   Columns: Price

            high     low  medium      All

regular    436.5   699.8   551.0    562.4
           80.67   66.09   56.67   128.50
               4       4       4       12

special    375.0   782.0   485.8    547.6
           74.33   89.76   87.64   194.94
               4       4       4       12

All        405.8   740.9   518.4    555.0
           78.98   85.19   76.71   161.65
               8       8       8       24

Cell Contents:  Sales  :  Mean
                Sales  :  Standard deviation
                          Count
```

Two-way ANOVA: Sales versus Display, Price

```
Source       DF      SS      MS      F      P
Display       1    1320    1320   0.22  _____
Price         2  465332  232666  39.49  _____
Interaction   2   28290   14145   2.40  _____
Error        18  106049    5892
Total        23  600990

S = 76.76   R-Sq = 82.35%   R-Sq(adj) = 77.45%
```


Residual Plots for Sales
Normal Probability Plot of the Residuals
Percent
Residual
Residuals Versus the Fitted Values
Residual
Fitted Value
Histogram of the Residuals
Frequency
Residual
Residuals Versus the Order of the Data
Residual
Observation Order


Interaction Plot (data means) for Sales
high
low
medium
Display
Price
regular
special
Display
regular
special
Price
high
low
medium

Standard Normal Distribution

P(Z < z)　(z negative)

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) (z positive)

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

t_α

$\alpha = P(t > t_\alpha)$ = one-tail probability

df	0.100	0.050	0.025	0.010	0.009	0.008	0.005	0.001	0.0005	0.0004	0.0001
1	3.08	6.31	12.71	31.82	35.36	39.78	63.66	318.31	636.62	837.66	3183.10
2	1.89	2.92	4.30	6.96	7.35	7.81	9.92	22.33	31.60	36.25	70.70
3	1.64	2.35	3.18	4.54	4.72	4.93	5.84	10.21	12.92	14.18	22.20
4	1.53	2.13	2.78	3.75	3.87	4.01	4.60	7.17	8.61	9.25	13.03
5	1.48	2.02	2.57	3.36	3.46	3.57	4.03	5.89	6.87	7.29	9.68
6	1.44	1.94	2.45	3.14	3.23	3.32	3.71	5.21	5.96	6.28	8.02
7	1.41	1.89	2.36	3.00	3.07	3.16	3.50	4.79	5.41	5.67	7.06
8	1.40	1.86	2.31	2.90	2.97	3.04	3.36	4.50	5.04	5.26	6.44
9	1.38	1.83	2.26	2.82	2.89	2.96	3.25	4.30	4.78	4.98	6.01
10	1.37	1.81	2.23	2.76	2.83	2.89	3.17	4.14	4.59	4.77	5.69
11	1.36	1.80	2.20	2.72	2.78	2.84	3.11	4.02	4.44	4.60	5.45
12	1.36	1.78	2.18	2.68	2.74	2.80	3.05	3.93	4.32	4.47	5.26
13	1.35	1.77	2.16	2.65	2.71	2.77	3.01	3.85	4.22	4.37	5.11
14	1.35	1.76	2.14	2.62	2.68	2.74	2.98	3.79	4.14	4.28	4.99
15	1.34	1.75	2.13	2.60	2.66	2.71	2.95	3.73	4.07	4.21	4.88
16	1.34	1.75	2.12	2.58	2.64	2.69	2.92	3.69	4.01	4.15	4.79
17	1.33	1.74	2.11	2.57	2.62	2.67	2.90	3.65	3.97	4.09	4.71
18	1.33	1.73	2.10	2.55	2.60	2.66	2.88	3.61	3.92	4.04	4.65
19	1.33	1.73	2.09	2.54	2.59	2.64	2.86	3.58	3.88	4.00	4.59
20	1.33	1.72	2.09	2.53	2.58	2.63	2.85	3.55	3.85	3.97	4.54
21	1.32	1.72	2.08	2.52	2.57	2.62	2.83	3.53	3.82	3.93	4.49
22	1.32	1.72	2.07	2.51	2.56	2.61	2.82	3.50	3.79	3.91	4.45
23	1.32	1.71	2.07	2.50	2.55	2.60	2.81	3.48	3.77	3.88	4.42
24	1.32	1.71	2.06	2.49	2.54	2.59	2.80	3.47	3.75	3.85	4.38
25	1.32	1.71	2.06	2.49	2.53	2.58	2.79	3.45	3.73	3.83	4.35
26	1.31	1.71	2.06	2.48	2.53	2.58	2.78	3.43	3.71	3.81	4.32
27	1.31	1.70	2.05	2.47	2.52	2.57	2.77	3.42	3.69	3.79	4.30
28	1.31	1.70	2.05	2.47	2.51	2.56	2.76	3.41	3.67	3.78	4.28
29	1.31	1.70	2.05	2.46	2.51	2.56	2.76	3.40	3.66	3.76	4.25
30	1.31	1.70	2.04	2.46	2.50	2.55	2.75	3.39	3.65	3.75	4.23
31	1.31	1.70	2.04	2.45	2.50	2.55	2.74	3.37	3.63	3.73	4.22
32	1.31	1.69	2.04	2.45	2.49	2.54	2.74	3.37	3.62	3.72	4.20
33	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.36	3.61	3.71	4.18
34	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.35	3.60	3.70	4.17
35	1.31	1.69	2.03	2.44	2.48	2.53	2.72	3.34	3.59	3.69	4.15
36	1.31	1.69	2.03	2.43	2.48	2.53	2.72	3.33	3.58	3.68	4.14
37	1.30	1.69	2.03	2.43	2.48	2.52	2.72	3.33	3.57	3.67	4.13
38	1.30	1.69	2.02	2.43	2.47	2.52	2.71	3.32	3.57	3.66	4.12
39	1.30	1.68	2.02	2.43	2.47	2.52	2.71	3.31	3.56	3.65	4.10
40	1.30	1.68	2.02	2.42	2.47	2.52	2.70	3.31	3.55	3.65	4.09
50	1.30	1.68	2.01	2.40	2.45	2.49	2.68	3.26	3.50	3.59	4.01
60	1.30	1.67	2.00	2.39	2.43	2.48	2.66	3.23	3.46	3.55	3.96
100	1.29	1.66	1.98	2.36	2.41	2.45	2.63	3.17	3.39	3.47	3.86
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.58	3.10	3.30	3.38	3.73

Chi-square distribution

$\chi^2_{\alpha;\nu}$

$\alpha = P(\chi^2 \geq \chi^2_{\alpha;\nu})$

ν	0.995	0.100	0.050	0.025	0.010	0.005	0.004	0.003	0.002	0.001	0.0005	0.0001
1	0.00	2.71	3.84	5.02	6.63	7.88	8.28	8.81	9.55	10.83	12.12	15.14
2	0.01	4.61	5.99	7.38	9.21	10.60	11.04	11.62	12.43	13.82	15.20	18.42
3	0.07	6.25	7.81	9.35	11.34	12.84	13.32	13.93	14.80	16.27	17.73	21.11
4	0.21	7.78	9.49	11.14	13.28	14.86	15.37	16.01	16.92	18.47	20.00	23.51
5	0.41	9.24	11.07	12.83	15.09	16.75	17.28	17.96	18.91	20.52	22.11	25.74
6	0.68	10.64	12.59	14.45	16.81	18.55	19.10	19.80	20.79	22.46	24.10	27.86
7	0.99	12.02	14.07	16.01	18.48	20.28	20.85	21.58	22.60	24.32	26.02	29.88
8	1.34	13.36	15.51	17.53	20.09	21.95	22.55	23.30	24.35	26.12	27.87	31.83
9	1.73	14.68	16.92	19.02	21.67	23.59	24.20	24.97	26.06	27.88	29.67	33.72
10	2.16	15.99	18.31	20.48	23.21	25.19	25.81	26.61	27.72	29.59	31.42	35.56
11	2.60	17.28	19.68	21.92	24.72	26.76	27.40	28.22	29.35	31.26	33.14	37.37
12	3.07	18.55	21.03	23.34	26.22	28.30	28.96	29.79	30.96	32.91	34.82	39.13
13	3.57	19.81	22.36	24.74	27.69	29.82	30.49	31.35	32.54	34.53	36.48	40.87
14	4.07	21.06	23.68	26.12	29.14	31.32	32.00	32.88	34.09	36.12	38.11	42.58
15	4.60	22.31	25.00	27.49	30.58	32.80	33.50	34.39	35.63	37.70	39.72	44.26
16	5.14	23.54	26.30	28.85	32.00	34.27	34.98	35.89	37.15	39.25	41.31	45.92
17	5.70	24.77	27.59	30.19	33.41	35.72	36.44	37.37	38.65	40.79	42.88	47.57
18	6.26	25.99	28.87	31.53	34.81	37.16	37.89	38.83	40.14	42.31	44.43	49.19
19	6.84	27.20	30.14	32.85	36.19	38.58	39.33	40.29	41.61	43.82	45.97	50.80
20	7.43	28.41	31.41	34.17	37.57	40.00	40.76	41.73	43.07	45.31	47.50	52.39
21	8.03	29.62	32.67	35.48	38.93	41.40	42.17	43.16	44.52	46.80	49.01	53.96
22	8.64	30.81	33.92	36.78	40.29	42.80	43.58	44.58	45.96	48.27	50.51	55.52
23	9.26	32.01	35.17	38.08	41.64	44.18	44.98	45.99	47.39	49.73	52.00	57.07
24	9.89	33.20	36.42	39.36	42.98	45.56	46.37	47.39	48.81	51.18	53.48	58.61
25	10.52	34.38	37.65	40.65	44.31	46.93	47.75	48.78	50.22	52.62	54.95	60.14
26	11.16	35.56	38.89	41.92	45.64	48.29	49.12	50.17	51.63	54.05	56.41	61.66
27	11.81	36.74	40.11	43.19	46.96	49.64	50.48	51.55	53.02	55.48	57.86	63.16
28	12.46	37.92	41.34	44.46	48.28	50.99	51.84	52.92	54.41	56.89	59.30	64.66
29	13.12	39.09	42.56	45.72	49.59	52.34	53.19	54.28	55.79	58.30	60.73	66.15
30	13.79	40.26	43.77	46.98	50.89	53.67	54.54	55.64	57.17	59.70	62.16	67.63
35	17.19	46.06	49.80	53.20	57.34	60.27	61.19	62.35	63.95	66.62	69.20	74.93
40	20.71	51.81	55.76	59.34	63.69	66.77	67.72	68.94	70.62	73.40	76.09	82.06
50	27.99	63.17	67.50	71.42	76.15	79.49	80.53	81.84	83.66	86.66	89.56	95.97
100	67.33	118.50	124.34	129.56	135.81	140.17	141.52	143.23	145.58	149.45	153.17	161.32
150	109.14	172.58	179.58	185.80	193.21	198.36	199.95	201.96	204.72	209.26	213.61	223.11
200	152.24	226.02	233.99	241.06	249.45	255.26	257.06	259.33	262.43	267.54	272.42	283.06
224	173.24	251.52	259.91	267.35	276.16	282.27	284.15	286.53	289.79	295.14	300.26	311.39
250	196.16	279.05	287.88	295.69	304.94	311.35	313.32	315.81	319.23	324.83	330.18	341.83
500	422.30	540.93	553.13	563.85	576.49	585.21	587.89	591.26	595.88	603.45	610.65	626.24

Fisher's F distribution

$F_{.05}$

$P(F \geq F_{.05}) = 0.050$

ν_1

ν_2	1	2	3	4	5	6	7	8	9	10	11	12
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.0	243.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	2.11	2.08
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	2.10	2.07
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	2.09	2.06
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	2.08	2.05
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.07	2.04
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80