Comp 232 Assignment # 2 Feb 15^{th} 2015

1) a) $P(x)$: x is a cheater
$Q(x)$: x sits in the back row

1) $\forall x \, (P(x) \rightarrow Q(x))$ [Premise]
2) $P(a) \rightarrow Q(a)$ [Universal Instantiation] 1
3) $Q(\text{George})$ [Premise]
4) cannot conclude

↳ invald : converse error

b) $P(x)$: x studies discrete math
$Q(x)$: x is good at logic

1) $\forall x \, (P(x) \rightarrow Q(x))$ [Premise]
2) $P(a) \rightarrow Q(a)$ [Universal Instantiation] 1
3) $P(\text{dawn})$ [Premise]
4) $Q(\text{dawn}$ [Modes Ponens] 2 3

↳ Valid

c) $P(x)$: program produces errors
$Q(x)$: program is correct
$R(x)$: program is faulty

$$\begin{array}{l} P(x) \rightarrow (\neg Q(x) \vee R(x)) \\ \neg P(x) \\ \hline \therefore \; Q(x) \wedge \neg R(x) \end{array}$$

↳ Invalid : Inverse error

d.) $P(x)$: x does their homework

$Q(x)$: x studies course material

$R(x)$: x gets good grades

1) $\forall x\ ((\neg P(x) \land \neg Q(x)) \rightarrow \neg R(x))$ [Promise]

2) $\neg P(a) \land \neg Q(a) \rightarrow \neg R(a)$ [Universal Instantiation]

3) $R(\text{John})$ [Promise]

4) $\neg(\neg P(\text{John}) \land \neg Q(\text{John}))$ [Modus Tollens] 2

5) $P(\text{John}) \lor Q(\text{John})$ [De morgen]

↳ Valid

2) a) [$\neg p \vee q \to r$, $s \vee \neg q$, $\neg t$, $p \to t$, $\neg p \wedge r \to \neg s$] Conclusion : $\neg q$

1) $\neg t$	[Premise]	
2) $p \to t$	[Premise]	
3) $\neg p$	[Modus Tollens]	1 2
4) $\neg p \vee q \to r$	[Premise]	
5) $\neg p \vee q$	[addition]	3
6) r	[Modus Ponens]	4 5
7) $\neg p \wedge r$	[Conjunction]	3 6
8) $\neg p \wedge r \to \neg s$	[Premise]	
9) $\neg s$	[Modus Ponens]	7 8
10) $s \vee \neg q$	[Premise]	
11) $\neg q$	[Disjunctive syllogism]	9 10

b. [$\neg p \to r \wedge \neg s$, $t \to s$, $u \to \neg p$, $\neg w$, $u \vee w$] Conclusion: $\neg t \vee w$

1) $\neg w$ [Premise]
2) $u \vee w$ [Premise]
3) u [Disjunctive Syllogism] 1 2
4) $u \to \neg p$ [Premise]
5) $\neg p$ [Modus Ponens] 3 4
6) $\neg p \to (r \wedge \neg s)$ [Premise]
7) $r \wedge \neg s$ [Modus Ponens] 5 6
8) $\neg s$ [Simplification] 7
9) $t \to s$ [Premise]
10) $\neg t$ [Modus Tollens] 8 9
11) $\neg t \vee w$ [addition] 10

c.) [$p \vee q$, $q \to r$, $p \wedge s \to t$, $\neg r$, $\neg q \to u \wedge s$) Conclusion: t

1) $\neg r$ [Premise]
2) $q \to r$ [Premise]
3) $\neg q$ [Modus Tollens] 1 2
4) $\neg q \to u \wedge s$ [Premise]
5) $u \wedge s$ [Modus Ponens] 3 4
6) s [Simplification] 5
7) $p \vee q$ [Premise]
8) p [Disjunctive Syllogism] 3 7
9) $p \wedge s$ [conjunction] 6 8
10) $p \wedge s \to t$ [Premise]
11) t [Modus Ponens] 9 10

4) Prove that the following statements are equivilant.

a) n^2 is odd b) $1-n$ is even c) n^3 is odd d) n^2+1 is even

- $a \Rightarrow b$: n^2 is odd then $1-n$ is even
  proof by contrapositive : $1-n$ is odd $\rightarrow$ n^2 is even

↳ $1-n = 2k+1$ $\therefore n = -2k$ $\therefore n^2 = 4k^2 = 2(2k^2)$
= even by definition

↳ we have shown that $a \Rightarrow b$

- $b \Rightarrow c$: $1-n$ is even then n^3 is odd
  direct proof :

↳ $1-n = 2k$ $\therefore n = -2k$ $\therefore n^3 = (-2k)^3$ ↵
$n^3 = 2(-4k^3 + 6k^2 - 3k) + 1$ which is odd by definition

↳ we have shown that $b \Rightarrow c$

- $c \Rightarrow d$: n^3 is odd then n^2+1 is even
  proof by cases :

If n is even, n^3 is even : $n = 2k$ $n^3 = 2(4k^3) =$ even
If n is even, n^2+1 is even : $n = 2k$ $n^2 = 2(2k^2)$ $n^2+1 =$ ↵
$n^2+1 = 2(2k^2)+1$ = odd by definition

If n is odd, n^3 is odd : $n = 2k+1$ $n^3 = (2k+1)^2$
$n^3 = 2(4k^3 + 6k^2 + 3k) + 1$
$n^3 =$ odd by definition
If n is odd, n^2+1 is even : $n = 2k+1$ $n^2 = (2k+1)^2$
$n^2 = 2(2k^2+1)$
$n^2+1 = 2(2k^2+1)$
$n^2+1 =$ even by definition

- $d \Rightarrow a$: n^2+1 even then n^2 odd

  Direct proof :

↳ $n^2+1 = 2k$ = even by definition

$n^2 = 2k-1$ = odd by definition

$\therefore$ All statements are equivilant because

$$\begin{tikzcd} a \arrow[r] & b \arrow[d] \\ d \arrow[u] & c \arrow[l] \end{tikzcd}$$

5) a. If x is an odd integer and y is an even integer, then $x+y$ is odd.

↳ Direct proof:

$x = 2K + 1$ = odd by definition
$y = 2K$ = even by definition

$x+y = 2K+1 + 2K = 4K+1 = 2(2K)+1 = \text{odd}$

∴ the sum of two integers, odd and even, gives an odd integer.

b. If n is an odd integer, then n^2 is odd.

↳ proof by contradiction $p \to \neg q$

Assume n^2 is even and n is odd

$n = 2K+1$ = odd by definition
$n^2 = (2K+1)^2$
$n^2 = 4K^2 + 4K + 1 = 2(2K^2+2K)+1$
which is odd by definition
therefore contradicting our assumption

c. If n is an odd integer, then $n+2$ is odd
$(n: \text{odd}) \rightarrow (n+2: \text{odd})$

↳ Indirect proof : contraposition

If $n+2$ is even then n is even

↳ $n+2 = 2k$ = even by definition

$n = 2k - 2$

$n = 2(k-1)$ = even by definition

* where $(k-1)$ is an integer

6) for all positive $x, y \in \mathbb{R}$, if x is irrational and y is irrational, then $x+y$ is irrational

↳ False : counterexample

$x = \frac{10}{2} + \sqrt{2}$ $\qquad y = \frac{10}{2} - \sqrt{2}$

$x + y = \frac{10}{2} + \sqrt{2} + \frac{10}{2} - \sqrt{2}$

$x + y = \frac{20}{2} = 10$ = rational number

∴ adding two irrational numbers can produce a rational number contradicting our initial statement.

7) If $m+n$ is even, then $m-n$ is even

a.) direct Proof : $((m+n : \text{even}) \rightarrow (m-n : \text{even}))$

- $m+n = 2K$ by definition of even
- $m = 2K - n$
- $(2K-n)-n = m-n$ substitute for m
- $2K - 2n = 2(K-n) = \text{even}$

* where $(K-n)$ is an integer

b) Indirect Proof : contrapositive $((m-n : \text{odd}) \rightarrow (m+n : \text{odd}))$

- $m-n = 2K+1$ by definition of odd
- $m = 2K+1+n$
- $(2K+1+n)+n = m+n$ Substitute for m
- $2(K+n)+1 = \text{odd}$

$\neg(m-n) \rightarrow \neg(m+n)$

* where $(K+n)$ is an integer

c) Proof by Contradiction: $((m+n : \text{even}) \rightarrow (m-n : \text{odd}))$

- $m-n = 2K+1$ by definition of odd
- $m = 2K+1+n$
- $m+n = (2K+n+1)+n$
- $m+n = 2(K+n)+1$

* where $K+n$ is an integer

$\therefore$ $m+n = 2(K+n)+1$ which is odd which contradicts our assumption that $m+n$ is even.