Econ 401/501 Midterm Examination

15 October 2014

Instructions: *Please answer all questions. The exam lasts 120 minutes. The total number of points is 80. Show the work with which you obtain all your answers. If you cannot completely solve a problem, you can still obtain points by providing graphs or equations that explain how you would approach the problem. Begin answers to each question on a new page. Indicate before each answer which question you are answering.*

1. (25 points) Consider the utility function $u(x_1, x_2) = \ln x_1 + x_2$, where x_1 is food and x_2 is clothing.

   (a) (10 points) Find the demand functions of the consumer, $x_1(p, m)$ and $x_2(p, m)$. Derive the indirect utility function $v(p, m)$. You may assume interior solutions throughout.

   (b) (15 points) Suppose the government provides the consumer a subsidy, σ, for each unit of food he buys. Therefore, instead of paying p_1 for a unit of food, now the consumer only has to pay $p_1' = p_1 - \sigma$. Decompose the change in the consumer's demand for food into income and substitution effects, using the the Hicksian method.

2. (20 points) Consider a two-commodity model. Suppose that the price of the two commodities are p_1 and p_2. Find the expenditure function in each of the following cases:

   (a) $u(x_1, x_2) = (x_1 - k_1)^{\alpha}(x_2 - k_2)^{1-\alpha}$, where $k_1 > 0$ and $k_2 > 0$.

   (b) $u(x_1, x_2) = \max\{x_1, x_2\}$

3. (21 points) TRUE or FALSE. Determine if each of the following statements is true or false. Explain your reasoning. Make additional assumptions if necessary.

   (a) Garfield is strictly happier when he eats more lasagna and has more sleep. Based on this information, Garfield's preferences over lasagna and sleep are complete.

   (b) Ms. Undecided is a US voter. She prefers Obama to McCain, McCain to Hillary, and Hillary to Obama. Her preferences over presidential candidates can be represented by a utility function.

(c) If Ms. A's and Mr. B's von Neumann-Morgenstern utility functions v^A and v^B satisfy $v^B = \alpha v^A + \beta$, where $\alpha > 0$, then when asked to choose between two prospects, P and Q, they must make the same choice.

4. (14 points) Ms A and Mr L have vNM utility functions $v(y) = \sqrt{y}$ and $v(y) = y^{3/2}$, respectively. Each of them has an income of $\bar{y} = 1000$ initially.

   (a) Each of them is now offered a fair bet $(\pi; z) = (1/7; 600, -100)$, i.e., with probability $\pi = 1/7$ the bettor wins and gets \$600, and with probability $1-\pi = 6/7$ the bettor loses and has to pay \$100. Represent this bet as a prospect. Who will take the bet and who will not?

   (b) For the person who declines to take the bet, how much is he or she willing to pay to avoid the bet? How much must the other person be paid to give up the bet?

Mid-Term Answer Key:-

1 (a) $u(x_1, x_2) = \ln x_1 + x_2$ [quasi-linear preferences]

The Lagrangian is

$$L(x_1, x_2, \lambda) = u(x_1, x_2) + \lambda[m - p_1 x_1 - p_2 x_2]$$

We have been asked to assume interior solutions

Therefore the F.O.C are.

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow \frac{1}{x_1} - p_1 = 0. \quad -(1)$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 1 - p_2 = 0. \quad -(2)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow m = p_1 x_1 + p_2 x_2. \quad -(3)$$

From (2) you get $p_2 = 1$ [the idea of good 2 being the numeraire]

$$\therefore x_1 = \frac{1}{p_1}. \quad -(A)$$

$$\therefore x_2 = \frac{m-1}{p_2} \quad -(B)$$

Plugging A and B in the utility function we get the expression for $v(p,m)$:

$$v(p,m) = \ln\left(\frac{1}{p_1}\right) + \frac{m-1}{p_2}.$$

B (b) Govt imposes a subsidy of σ on commodity 1. Now, $u(x_1,x_2) = \ln x_1 + x_2$ is quasi-linear preference. This implies that the income effect will be ZERO. So the entire change is the substitution effect.

Now the new demanded amount for commodity 1 is

$$x_1' = \frac{1}{p_1'} = \frac{1}{p_1 - \sigma}.$$

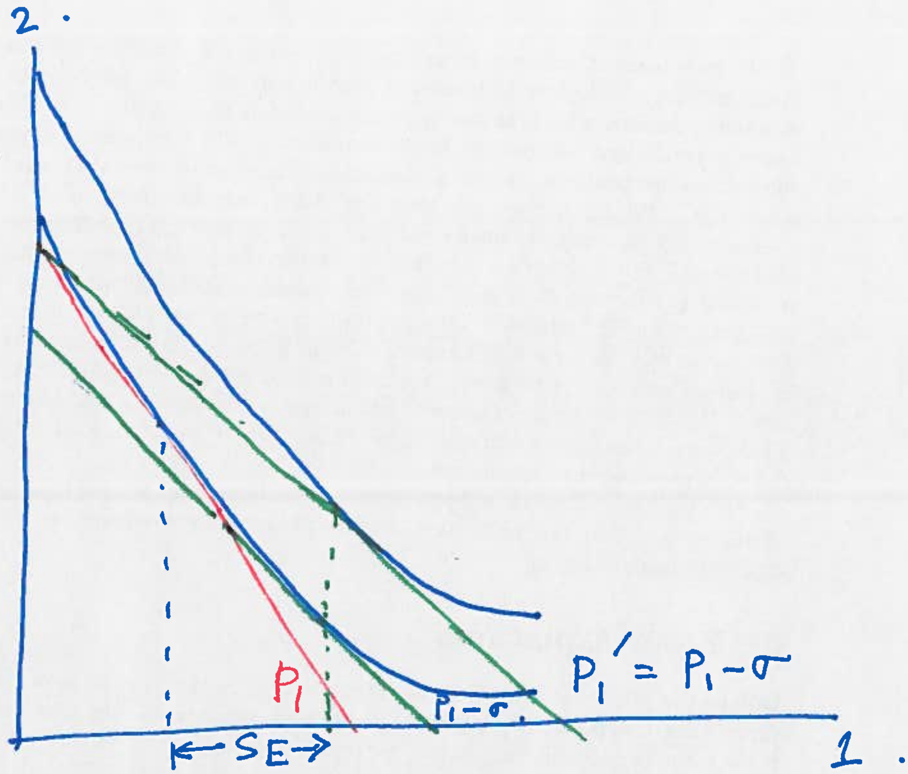
$$\therefore \quad S.E = \left(\frac{1}{p_1} - \frac{1}{p_1-\sigma}\right).$$

$$= \frac{-\sigma}{p_1(p_1-\sigma)}.$$

and

$$I.E = 0.$$

DIAGRAM:-



$$IE = 0.$$

2(a) $u(x_1, x_2) = (x_1 - k_1)^{\alpha}(x_2 - k_2)^{1-\alpha}$

Expenditure minimization problem.

$$\underset{x_1, x_2}{\text{minimize}} \quad p_1 x_1 + p_2 x_2 \quad \text{s.t.}$$

$$u(x_1, x_2) = u^0.$$

<u>The Lagrangian:</u>

$$L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 + \lambda\left[u^0 - (x_1 - k_1)^{\alpha}(x_2 - k_2)^{1-\alpha}\right]$$

let $z_1 = (x_1 - k_1)$.

$z_2 = (x_2 - k_2)$.

From the first order conditions we get.

$$\frac{p_1}{p_2} = \frac{\alpha}{(1-\alpha)} \frac{(x_1 - k_1)^{\alpha - 1}(x_2 - k_2)^{1-\alpha}}{(x_1 - k_1)^{\alpha}(x_2 - k_2)^{-\alpha}}$$

$$= \left(\frac{\alpha}{1-\alpha}\right) \cdot \left(\frac{x_2 - k_2}{x_1 - k_1}\right)$$

$$= \left(\frac{\alpha}{1-\alpha}\right) \cdot \frac{z_2}{z_1}$$

$$\Rightarrow \left(\frac{1-\alpha}{\alpha}\right)\frac{p_1 z_1}{p_2} = z_2.$$

$$\therefore \quad u^0 = z_1^{\alpha} z_2^{1-\alpha}.$$

$$= \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \left(\frac{p_1}{p_2}\right)^{1-\alpha} \cdot z_1$$

$$\Rightarrow z_1(p_1, p_2, u^0) = u^0 \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \cdot \left(\frac{p_2}{p_1}\right)^{1-\alpha}.$$

$$\& \quad z_2(p_1, p_2, u^0) = u^0 \cdot \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \cdot \left(\frac{p_1}{p_2}\right)^{\alpha}.$$

$$\therefore \cancel{e(p_1, p_2, u^0) =}$$

$$\therefore x_1(p_1, p_2, u^0) = u^0 \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{p_2}{p_1}\right)^{1-\alpha} + k_1$$

$$\& \quad x_2(p_1, p_2, u^0) = u^0 \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(\frac{p_1}{p_2}\right)^{\alpha} + k_2.$$

So

$$e(p_1, p_2, u^0) = p_1 x_1(p_1, p_2, u^0) + p_2 x_2(p_1, p_2, u^0)$$

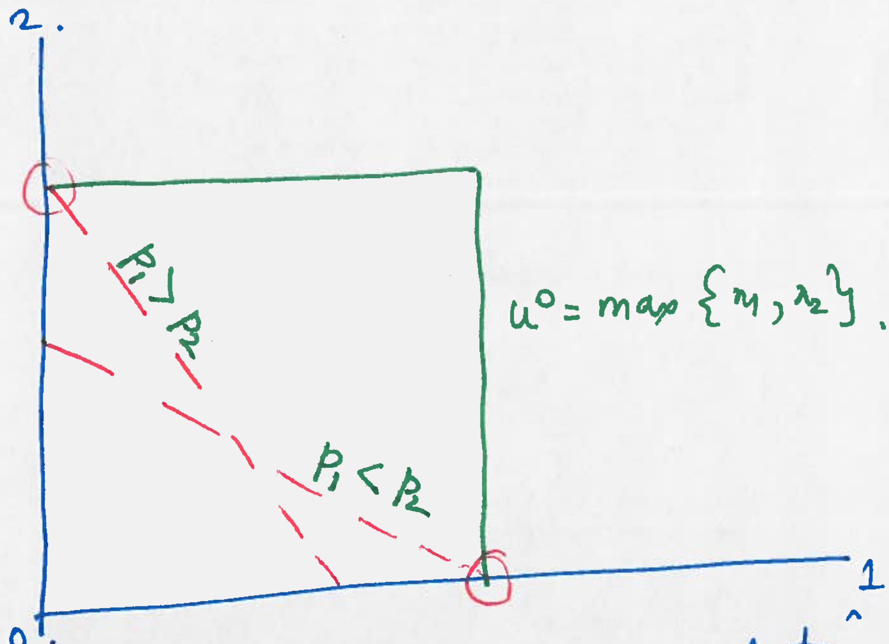
I leave it to you to simplify the expression.

2 b $\quad u(x_1, x_2) = \max\{x_1, x_2\}$

Let us fix a level of utility u^0.

$$\therefore \quad u^0 \equiv \max\{x_1, x_2\}.$$

Now let us draw the indifference curves.



The diagram makes it clear the solution will be on one of the corners.

If $p_1 < p_2$ then $x_1(p_1, p_2, u^0) = u^0$.
$x_2(p_1, p_2, u^0) = 0$.

If $p_1 > p_2$ then $x_1(p_1, p_2, u^0) = 0$.
$x_2(p_1, p_2, u^0) = u^0$.

$$\therefore\ e(p_1, p_2, u^0) = \min\{p_1, p_2\} \cdot u^0.$$

—x—

3. True or False.

(a) FALSE. Let us denote the quantities of Lasagna and sleep by l & s respectively. Consider now bundles (l_1, s_1) & (l_2, s_2) where $l_1 > l_2$ & $s_1 < s_2$.

These two bundles are not comparable according to the information given. Hence FALSE.

(b) FALSE:- The preference violates transitivity.

(c) TRUE. The utility functions are affine transformations of one another. Hence they represent the same preferences. Hence

⑧

they would make the same choice over prospects P & Q.

④: The solution to ④ is given in the answer key to assignment ②.