Urban Economics (ECO333Y1Y) Midterm Exam

June 25th, 2013 – 1:00-4:00pm

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(Examination aids: non-programmable calculators, drawing instruments.)

Questions 1 and 2 are MANDATORY questions, and there is a CHOICE of EITHER Question 3 or Question 4. Only three questions should be answered in total. (Answers combining part of Question 3 and part of Question 4 are not acceptable.)

5 MARKS of the 100-mark total for this exam will be allocated to exam answers complying with the instructions outlined in this paragraph. You must use separate books for each question: for example one book for Question 1, a second book for Question 2, and a third book for whichever optional question you choose. You may use additional books if needed, but no book may contain answers to more than one question. In addition to your name and student number, the cover page of each book must identify the question number answered in that book. The "Book No." line on the cover page can be used for this purpose, but you must write "Q1", "Q2", "Q3" or "Q4" on that line – not just a number without the "Q". Alternatively you can write "Q1", "Q2", "Q3" or "Q4" elsewhere on the cover page. Either way, the question number answered in a book must be immediately identifiable by looking at the book's cover page. One last instruction already appears on the cover page (right under the "Name" line): Your last name must be printed legibly on the left side of the line (where you see "surname" under the line). Your first name must be printed legibly on the right side (where you see "given names" under the line), and must be the same first name shown on the registration list next to your surname.

All answers should be briefly and clearly explained, using diagrams where appropriate. Correct numerical answers will not earn full marks without an appropriate explanation. If you are not able to answer a numerical question, provide a brief description of the relevant theory to obtain partial credit. Questions 1 and 2 are each worth 33 marks; questions 3 and 4 (answer either one, but not both) are each worth 29 marks. As was noted in the preceding paragraph, the remaining 5 marks are allocated to exam answers complying with instructions outlined in that paragraph.

Question 1 (33 marks)

Consider a rectangular city model. Identical businesses manufacture bicycles that are exported out of the City via a port located along the City's western edge; this will be the location where land rents paid by businesses are highest. Businesses face constant marginal freight costs per mile as they transport their product from their location to the city port. The land occupied by manufacturing firms is called the central business district (CBD).

Residents are employed by manufacturing businesses at a common wage, face constant commuting costs per mile, and rent housing from housing firms at the market housing price. The output of housing firms is residential floor area in square feet. Commuting is costless for workers once inside the CBD.

The city has four boundaries. The western boundary is where land ends and water begins (the port). The northern and southern boundaries are fixed by zoning, and the eastern edge can move inward or outward depending on conditions in the land market. The wage rate is endogenous in the model. There is no factor substitution and no consumer substitution in the entire question. Throughout the question, let x be the distance in miles to the city port. Consider the following data, where all numbers with a time dimension are on a monthly basis:

<u>Business Sector</u>

The following data apply to each manufacturing firm; a firm:

- Produces a fixed output of q = 200 units
- Faces an exogenous output price of p = $200/unit
- Incurs constant freight costs of T = $10/unit of output/mile
- Hires E = 10 employees at market wage $w/employee
- Occupies L_b = 0.25 hectares of land for production
- Has a bid rent function $$R_b(x,w)$/hectare
- Incurs capital cost of C = $20,000 (including any costs not already listed)

<u>Residential Sector</u>

The following data apply to each housing firm; a firm:

- Produces a fixed output of Q = 40,000 sq.ft. of floor area
- Faces a market housing price function of $P(x,w)/sq.ft.
- Builds housing on L_r = 4 hectares of land
- Has a bid rent function for land $$R_r(x,w)$/hectare
- Incurs capital cost of K = $52,000 (including any costs not already listed)

The following data apply to each resident; a resident:

- Earns the market wage $w via employment at a manufacturing firm
- Faces a market housing price function of $P(x,w)/sq.ft.
- Incurs a commuting cost of t = $40/mile
- Consumes housing h = 1,000 sq.ft. and non-housing goods worth G = $100
- Spends all wage income $w on housing, non-housing goods and commuting

Other Variables

- Agricultural bid rent per hectare is constant at R_a = $1,600/hectare
- The CBD/residential boundary (also known as the zero-commute location) is $x_1(w)$
- The residential/agricultural boundary (also known as the city limit) is $x_2(w)$

<u>Note:</u> Parts (a) and (b) give you the answers and ask you to show they are correct. If you can't show they are correct, you can still use the answers given for Part (c). All of the equilibrium values you are required to derive in this question are either whole numbers or terminal decimals. You will <u>NOT</u> need the conversion 1 mile^2 = 260 hectares.

a) Show that business bid rent as a function of x and w is:

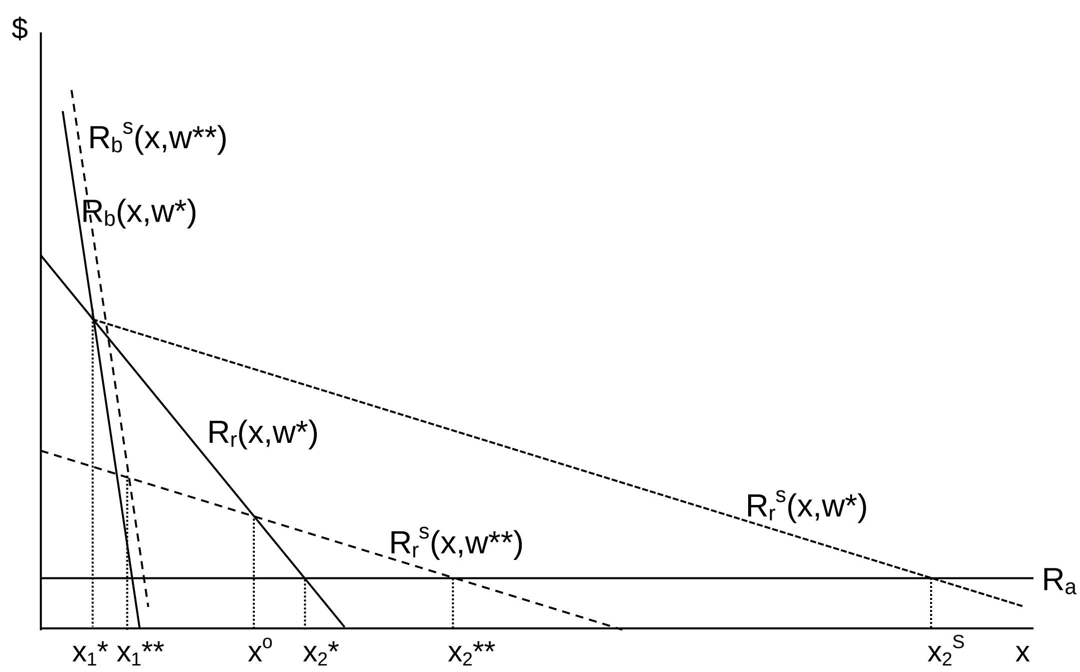
$$R_b(x,w) = 80{,}000 - 8{,}000x - 40w$$

b) Show that residential bid rent as a function of x and w is:

$$R_r(x,w) = -\,9{,}300 - 400x + 7.5w$$

c) Find the equilibrium wage w*, CBD/residential boundary x_1^*, and residential/agricultural boundary x_2^*.

d) Suppose a streetcar is introduced which cuts the exogenous per-mile commuting cost t down to t^s. Consider the land market diagram appearing below, which illustrates adjustment to the streetcar.



Not all areas of the diagram are necessarily drawn to scale; the diagram is representative only. Functional labels refer to the nearest function appearing to the left of the labels. The superscript "s" indicates a post-streetcar function or exogenous value, and the superscript double asterisk (**) indicates a post-streetcar endogenous value. As shown in the diagram, the adjustment has taken place in two stages: Stage 1 assumed the wage remained unchanged while Stage 2 allowed the wage to change.

The values of x_1*, x_2*, and w* are those you were asked to find in Part (c). You are told that $P(x_1^{**},w^{**})$ = $1.524. Clearly showing your work, find the following values: x_1^{**}, x_2^{**}, w**, x^o, x_2^S, and t^S.

Question 2 (33 marks)

a) The following information is given about firms in a bicycle manufacturing model (the usual assumptions apply):

- Output of a firm per day (b) = 10 bicycles.
- Land input of a firm (L) = 2.5 hectares
- Freight cost per km per bicycle (t) = $2.5
- North-south dimension of the rectangular city (y) = 0.5 km.
- Demand curve equation is P = 120 – 0.05 B.
- Bid rent equation is R (x) = 80 – 10x.
- The zoning law does not allow agriculture to compete for land with manufacturing.

i. Calculate equilibrium price, total output, and non-land production cost of a firm (NLPC), explaining your answers briefly.

ii. Use a diagram to show total cost of non-land production inputs, total freight cost and total land cost (you should calculate dollar amounts for these variables).

iii. In a map diagram show the total land area in manufacturing use.

b) In this part of the question there is no change to the exogenous variables in Part (a) except the city reduces y to 0.25 km and the demand curve equation in Part (a) no longer applies. You are told that with y = 0.25 km, the bid rent function is R (x) = 160 – 10x. Calculate equilibrium price and total output. In a diagram show supply curves for both Part (a) and for Part (b) with their equations. Your diagram should also show demand curves for both Part (a) and Part (b). Diagrams like those requested in Part (a) (ii) and (iii) are not required in Part (b).

Reminder: Answer Question 3 or Question 4 (not both)

Question 3 (29 marks)

This question is based on the general equilibrium rectangular city model, and should be answered in 1.5 pages or less single-spaced (3 pages if double-spaced). Using the letter notation (but not the numbers) of Question 1, answer the following questions:

a) Show that for any resident that does not move during adjustment to the streetcar the change in housing costs as a function of x is given by:

$$[P(x,w^*) – P(x,w^{**})]h = (w^* – w^{**}) – [(x_1^{**} – x_1^*)t + (x – x_1^{**})t – (x – x_1^{**})t^S]$$

Note that the equation compares pre-streetcar equilibrium with Stage 2 equilibrium. Stage 1 equilibrium is not relevant to the equation. In explaining your answer, you should refer to each of the four terms with round brackets on the right side of the equation. Also, you should refer both to values of x where the streetcar causes housing prices to increase and to values of x where the streetcar causes housing prices to decrease.

b) Using only the equation in Part (a), derive the condition under which $x^o = x_1^{**}$, where x^o is the location where residential bid rent is unaffected by the streetcar. This condition should be expressed as an equation. You should provide a brief explanation of your equation.

Question 4 (29 marks)

Consider a rectangular land market model with manufacturing and agricultural industries. Manufacturing firms produce bicycles and ship them along local roads to a highway running north-south. A zoning law restricts manufacturing to a land area east of the highway between two parallel east-west lines. The output price for bicycles is endogenous and there is no factor substitution. Bid rent for the agricultural industry R_a does not vary with distance from the highway.

a) Derive equations for the manufacturing industry's supply and bid rent curves. You should clearly define all variables you use, and briefly explain your derivation.

b) Consider the following two zoning policies:

Zoning Policy A: Agriculture is permitted on all land in the model.

Zoning Policy B: Agriculture is permitted only on land outside of the area where manufacturing is permitted.

The manufacturing industry receives a number of complaints from bicycle consumers about the high price of bicycles. The response from the manufacturing industry is that the price is high because manufacturers have to pay high land rent. Briefly evaluate the accuracy of this response separately for the two zoning policies. One half page single spaced, or one page double spaced will be sufficient to answer.

June 2013 Midterm Answers

Question 1:

a) **(2 marks)** $R_b(x,w)$ is derived as follows:

$(200)(200) = 20{,}000 + (10)(200)x + 10w + 0.25R_b(x,w)$
$R_b(x,w) = 80{,}000 – 8{,}000x – 40w$

b) **(5 marks – 3 marks for deriving $x_1(w)$ and 2 marks for deriving $R_r(x,w)$)**

First set $P(x,w) = (w – 100 – 40(x – x_1(w)))/1000$ and $(40{,}000)P(x,w) = 52{,}000 + 4R_r(x,w)$. These are the budget constraint for residents and ZEP condition for housing firms, respectively.

Along with $R_b(x,w)$, these equations imply that $x_1(w) = 11.75 – 0.00625w$, and substituting this back into the housing firm's ZEP equation yields $R_r(x,w) = – 9{,}300 – 400x + 7.5w$.

c) **(8 marks – 2 marks for deriving $x_2(w)$ and 2 marks for each value found)** Set $R_r(x,w) = 1{,}600$ to get $x_2(w) = – 27.25 + 0.01875w$. Then use densities to show that $x_2(w) = 5x_1(w)$. Combine these two equations with that found for $x_1(w)$ previously to get $w^* = \$1{,}720$, $x_1^* = 1$, and $x_2^* = 5$.

d) **(18 marks – 1 mark for each equation set up and 2 marks for each value found)** There are six unknowns and there are six equations to set up as follows:

i. $x_2^{**} = 5x_1^{**}$ (constant densities)

ii. $x_1^{**} = 11.75 – 0.00625w^{**}$ ($x_1(w)$ equation still holds post-streetcar)

iii. $1{,}524 = w^{**} – 100$ (it is given in the question that $P(x_1^{**},w^{**}) = \$1.524$)

iv. $160 = t^s(x_2^S – 1)$ (Boundary Betty maintains her total commuting costs during Stage 1 of adjustment)

v. $1{,}720 – w^{**} = t^s(x_2^S – 1) – t^s(x_2^{**} – x_1^{**})$ (Boundary Betty's commuting costs fall by the same amount as the wage during Stage 2 of adjustment)

vi. $1{,}720 – w^{**} = 40(x^o – 1) – t^s(x^o – x_1^{**})$ (bid rent and therefore house price does not change at $x = x^o$, implying the fall in the wage equals the fall in commuting costs at this location)

Solving these equations yields:

$x_1^{**} = 1.6$, $x_2^{**} = 8$, $w^{**} = \$1{,}624$, $x^o = 4$, $x_2^S = 17$ and $t^s = 10$.

See the July 2012, December 2012 and December 2011 midterm answers for the general substitution method used to find the solution to such a set of equations.

Question 2

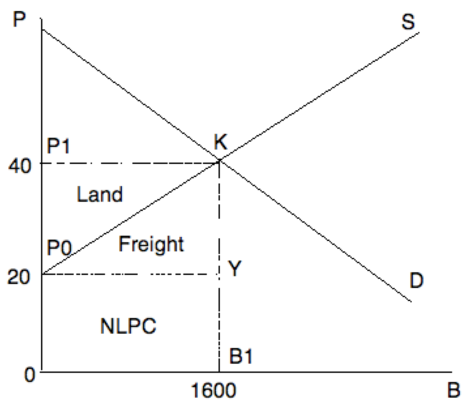
a) 18 marks

i. (6 marks)

From the bid rent equation, manufacturing land extends from x=0 to x=8km (where bid rent=0). The north-south dimension of the rectangular land market is 0.5km. Therefore, land area for manufacturing use is 4 sq. km, or 400 hectares (4 = 8 x 0.5).

To find total output:
Output/land ratio = b/L = 10/2.5 = 4 bicycles per hectare
→ Total Output = (output per hectare)x(total hectares)
→ B=4 x 400 = 1600 bicycles per day
To find price:
From the demand curve equation: P=120 – 0.05B
→ P=120-0.05(1600) = $40
To find NLPC:
From the bid rent equation: R(x) = (Pb – NLPC)/L
→ at x=0, R(x) = 80 – 10(0) = 0
→ 80 = (40(10) – NLPC)/2.5
→ NLPC = 200

ii. (6 marks)



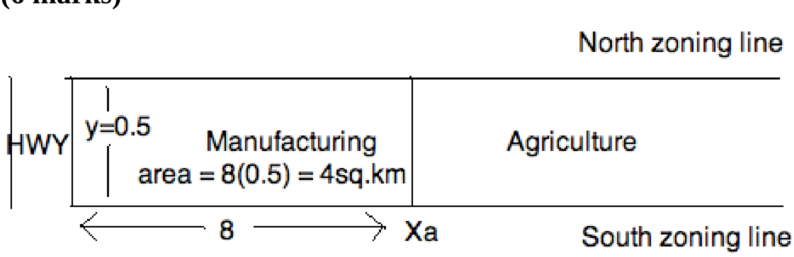
Supply Equation:

$$P = \frac{NLPC =}{b} + \left[\frac{tL}{100by}\right]B = \frac{200}{10} + \left[\frac{2.5(2.5)}{100(10)(0.5)}\right]B = 20 + 0.0125B$$

Total NLPC = Area($0P_0YB_1$) = 20 x 1600 = 32,000
Total Freight Cost = Area(P_0KY) = 0.5(40-20)1600 = 16,000
Total Land Cost = Area(P_0P_1 K) = 0.5(40-20)1600 = 16,000

(Note: max 3 marks if only have correct diagram, including intercepts and zero opportunity cost (Ra=0))

iii. (6 marks)



(Note: 6 marks for correct diagram with y=0.5, x=8, and TA=4sq.km

b) 15 marks (6 marks for equilibrium and supply equation, 9 marks for diagram)

From the bid rent equation, manufacturing land area now extends from x=0 to x=16km (where bid rent=0). The north-south dimension of the rectangular land market is 0.25km. Therefore, land area for manufacturing use is unchanged at 4 sq. km, or 400 hectares (4 = 16 x 0.25). Therefore, total output (B) is unchanged (B=1600). A less elastic supply curve and an unchanged output, implies a perfectly inelastic demand curve.

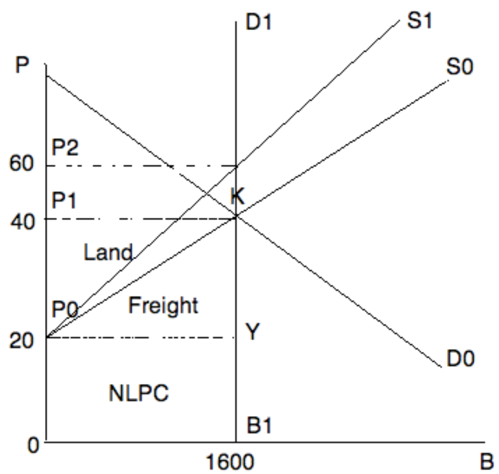
Supply curve equations:

Part (a):

$$P = \frac{NLPC =}{b} + \left|\frac{tL}{100by}\right| B = \frac{200}{10} + \left[\frac{2.5(2.5)}{100(10)(0.5)}\right] B = 20 + 0.0125B$$

Part (b):

$$P = \frac{NLPC =}{b} + \left|\frac{tL}{100by}\right| B = \frac{200}{10} + \left[\frac{2.5(2.5)}{100(10)(0.25)}\right] B = 20 + 0.025B$$



New equilibrium:
→ at B=1600, P=60

(Note: on graph, 4 marks for supply equations, 2 marks for demand from part (a), 3 marks for demand from part (b))

<u>Question 3:</u>

a) **(14 marks – 6 marks for derivation and 2 marks for each term of explanation)** Begin with differencing the pre-streetcar and post-streetcar house price functions:

$P(x,w^*) – P(x,w^{**}) = [w^* – G – [(x – x_1^*)t]/h – [w^{**} – G – [(x – x_1^{**})t^s]]/h$

Multiply both sides by h and rearrange to get:

$[P(x,w^*) - P(x,w^{**})]h = (w^* - w^{**}) - (x - x_1^*)t + (x - x_1^{**})t^s$

Within the brackets of the second term on the right hand side of the equation, add and subtract x_1^{**} to get:

$[P(x,w^*) - P(x,w^{**})]h = (w^* - w^{**}) - (x + x_1^{**} - x_1^{**} - x_1^*)t + (x - x_1^{**})t^s$

$[P(x,w^*) - P(x,w^{**})]h = (w^* - w^{**}) - (x_1^{**} - x_1^*)t - (x - x_1^{**})t + (x - x_1^{**})t^s$

Dividing (i.e. factoring) -1 out of the second, third and fourth right hand terms yields the desired result:

$[P(x,w^*) - P(x,w^{**})]h = (w^* - w^{**}) - [(x_1^{**} - x_1^*)t + (x - x_1^{**})t - (x - x_1^{**})t^s]$

Note: There are other equally-valid methods for obtaining this result. These methods are given full credit where due.

<u>Explanation</u>: 1st Term: The fall in the wage.

2nd Term: The savings in commuting costs due to the elimination of $x_1^{**} - x_1^*$ from the residential area (i.e. the savings from the commuting destination (x_1) moving closer to home (x) by the distance $x_1^{**} - x_1^*$).

3rd Term: The pre-streetcar commuting cost from home (x) to the new zero-commute location (x_1^{**}).

4th Term: The post-streetcar commuting cost from home (x) to the new zero-commute location (x_1^{**}).

b) **(15 marks – 10 marks for derivation, 5 marks for explanation)** As required, start with the result of Part (a):

$[P(x,w^*) - P(x,w^{**})]h = (w^* - w^{**}) - [(x_1^{**} - x_1^*)t + (x - x_1^{**})t - (x - x_1^{**})t^s]$

At $x = x^o$, by definition (and as given in the question) we know that residential bid rent has been unaffected by the streetcar. Therefore, the house price at this location must also be unaffected by the streetcar, implying pre-streetcar and post-streetcar house prices are the same at $x = x^o$:

$0 = (w^* - w^{**}) - [(x_1^{**} - x_1^*)t + (x^o - x_1^{**})t - (x^o - x_1^{**})t^s]$ **(5 marks)**

The question also states that $x^o = x_1^{**}$, indicating that there is no change in the house price at the new zero-commute location:

$0 = (w^* - w^{**}) - [(x_1^{**} - x_1^*)t + (0)t - (0)t^s]$

$0 = (w^* - w^{**}) - (x_1^{**} - x_1^*)t$

$(x_1^{**} - x_1^*)t = (w^* - w^{**})$ **(5 marks)**

<u>Explanation</u>: This is a special result. When the streetcar reduces the per-mile commuting costs such that $x^o = x_1^{**}$ (i.e. residents at the new zero-commute location pay the a post-streetcar house price equal to the pre-streetcar house price), the fall in the wage for all residents must be equal to the savings in commuting costs due to the commuting destination (x_1) moving closer to home (x) by the distance $x_1^{**} - x_1^*$. **(5 marks)**

Question 4

a) 14 marks

- 6 marks for variable definitions (b, NLPC, L, R(x), t, x)
- 4 marks for supply equation (from the zero economic profit condition):

$$MR = P = \frac{NLPC}{b} + \frac{LR(x)}{b} + tbx$$

$$B = 100yx\left(\frac{b}{L}\right) \rightarrow x = \left[\frac{L}{100by}\right]B$$

$$\rightarrow P = \frac{NLPC}{b} + \frac{LR(x)}{b} + \left[\frac{tL}{100by}\right]B$$

- 4 marks the bid rent curve (condition, TR=TC):

$$TR = TC$$

$$Pb = R(x)L + NLPC + tbx$$

$$\rightarrow R(x) = \frac{Pb - NLPC - tbx}{L}$$

b) 15 marks

In Zoning Policy, A there is an associated opportunity cost to land use. Because land can be used for either manufacturing or agriculture, this opportunity cost is factored into the manufacturing firm's costs of operating (higher land rents). A positive opportunity cost increases the marginal cost of operating, which in turn increases the price (under the zero economic profit condition, price equals marginal cost). Therefore, a high agricultural willingness to pay for land can lead to a high manufacturing output price, as the manufacturing industry claims.

In Zoning Policy B, agricultural land is not permitted on manufacturing land. As such, there is no opportunity cost of land. Therefore, marginal costs for the manufacturing firm are unaffected by the land allocation. Therefore, manufacturing output prices cannot be affected by land costs as claimed by the manufacturing industry.