(A)

MAT 1320 A   Fall 2010   October 6th, 8:30   Prof. Desjardins

TEST #1

Max = 20

Solutions

Student Number: ______________________________

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite. Graphs and sketches may be drawn in pencil.
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

Ⓐ

1. [2 points] Solve for x: $e^{2x+5} = 3$.

if $e^{2x+5} = 3$

then $\ln(e^{2x+5}) = \ln 3$

so $2x+5 = \ln 3$

thus $2x = \ln 3 - 5$

or $\boxed{x = \frac{1}{2}(\ln 3 - 5)} \simeq \boxed{-1.9507}$

2. [2 points] Find a formula for the inverse of $f(x) = \ln(x+6)$.

if $y = \ln(x+6)$

$e^y = e^{\ln(x+6)} = x+6$

so $x = e^y - 6$

$\therefore$ $\boxed{f^{-1}(x) = e^x - 6}$

3. [2 points] Using the known derivatives of $\sin x$ and $\cos x$, show that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$.

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{(0)(\cos x) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)$$

$$= \sec x \tan x$$

A

4. [4 points] If the position of a moving object is given by $s(t) = 5t^2 + t$,

(a) find the average velocity on

(i) $[1, 1.1]$

$$\frac{s(1.1) - s(1)}{1.1 - 1} = \frac{7.15 - 6}{0.1} = \boxed{11.5}$$

(ii) $[1, 1.01]$

$$\frac{s(1.01) - s(1)}{1.01 - 1} = \frac{6.1105 - 6}{0.01} = \boxed{11.05}$$

(iii) $[1, 1.001]$

$$\frac{s(1.001) - s(1)}{1.001 - 1} = \frac{6.011005 - 6}{0.001} = \boxed{11.005}$$

(b) estimate the instantaneous velocity at $t = 1$

$$\boxed{11}$$

A

5. [5 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{3x}{x-1}$. And then use the Quotient Rule to verify your answer.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{3(x+h)}{x+h-1} - \dfrac{3x}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{3(x+h)(x-1) - 3x(x+h-1)}{(x+h-1)(x-1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{3(x^2 - x + xh - h) - 3x^2 - 3xh + 3x}{(x+h-1)(x-1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{-3h}{(x+h-1)(x-1)}\right)$$

$$= \lim_{h\to 0} \frac{-3}{(x+h-1)(x-1)}$$

$$= \boxed{\frac{-3}{(x-1)^2}}$$

Check: $\dfrac{d}{dx}\left(\dfrac{3x}{x-1}\right) = \dfrac{3(x-1) - 3x(1)}{(x-1)^2} = \dfrac{-3}{(x-1)^2}$

Ⓐ

6. [5 points] Differentiate the following functions:

(a) $f(x) = 3e^x + x^2 - 7$

(b) $g(t) = 5\cos t + 3t^3$

(c) $y = 2x^{5/2} - x^{1/3}$

(d) $f(x) = 2x^2 e^x \sin x$

a) $\boxed{f'(x) = 3e^x + 2x}$

b) $\boxed{g'(t) = -5\sin t + 9t^2}$

c) $\boxed{\frac{dy}{dx} = 5x^{3/2} - \frac{1}{3}x^{-2/3}}$

d) $\boxed{f'(x) = 4xe^x \sin x + 2x^2 e^x \sin x + 2x^2 e^x \cos x}$

(B)

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- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

(B)

1. [2 points] Solve for x: $e^{3x+2} = 4$.

if $e^{3x+2} = 4$

so $3x+2 = \ln 4$

$\boxed{x = \frac{1}{3}(\ln 4 - 2)} \simeq \boxed{-0.2046}$

2. [2 points] Find a formula for the inverse of $f(x) = \ln(x+4)$.

$y = \ln(x+4)$

$e^y = x+4$

$x = e^y - 4$

$\boxed{f^{-1}(x) = e^x - 4}$

3. [2 points] Using the known derivatives of $\sin x$ and $\cos x$, show that $\frac{d}{dx}(\csc x) = -\csc x \cot x$.

$\frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{(0)(\sin x) - (1)(\cos x)}{\sin^2 x}$

$= -\frac{\cos x}{\sin^2 x}$

$= \left(-\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)$

$= -\csc x \cot x$

(B)

4. [4 points] If the position of a moving object is given by $s(t) = 3t^2 + 2t$,

(a) find the average velocity on

(i) $[1, 1.1]$

$$\frac{s(1.1) - s(1)}{1.1 - 1} = \frac{5.83 - 5}{0.1} = \boxed{8.3}$$

(ii) $[1, 1.01]$

$$\frac{s(1.01) - s(1)}{1.01 - 1} = \frac{5.0803 - 5}{0.01} = \boxed{8.03}$$

(iii) $[1, 1.001]$

$$\frac{s(1.001) - s(1)}{1.001 - 1} = \frac{5.008003 - 5}{0.001} = \boxed{8.003}$$

(b) estimate the instantaneous velocity at $t = 1$ $\boxed{8}$

(B)

5. [5 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{2x}{x-2}$. And then use the Quotient Rule to verify your answer.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{2(x+h)}{x+h-2} - \frac{2x}{x-2} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{2(x+h)(x-2) - 2x(x+h-2)}{(x+h-2)(x-2)} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{2(x^2 + xh - 2x - 2h) - 2x^2 - 2xh + 4x}{(x+h-2)(x-2)} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{-4h}{(x+h-2)(x-2)} \right)$$

$$= \lim_{h \to 0} \frac{-4}{(x+h-2)(x-2)}$$

$$= \boxed{\frac{-4}{(x-2)^2}}$$

check: $\dfrac{d}{dx}\left(\dfrac{2x}{x-2} \right) = \dfrac{(2)(x-2) - 2x(1)}{(x-2)^2} = \dfrac{-4}{(x-2)^2}$

(B)

6. [5 points] Differentiate the following functions:

(a) $f(x) = 2e^x - x^3 + 5$

(b) $g(t) = 7\sin t + 4t^2$

(c) $y = 3x^{4/3} - x^{1/2}$

(d) $f(x) = 2x^3 e^x \cos x$

a) $\boxed{f'(x) = 2e^x - 3x^2}$

b, $\boxed{g'(t) = 7\cos t + 8t}$

c, $\boxed{\frac{dy}{dx} = 4x^{1/3} - \frac{1}{2}x^{-1/2}}$

d, $\boxed{f'(x) = 6x^2e^x\cos x + 2x^3e^x\cos x - 2x^3e^x\sin x}$