Statistics for Management - ADM2303X

Midterm Exam - Summer 2014

Professor: Davood Astaraky Duration: 2 hours

Last Name: _____ **First Name:** _____

Student #: _____

This exam booklet contains 6 problems. If yours does not, please inform professor now.
Please answer all questions in the exam booklet. Only answers in this exam booklet will be marked. Show all work.

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1. One page (8 ½ by 11 inches) review sheet, both sides, is allowed. Ruler allowed.
2. Calculator permitted for arithmetic use only.
3. NO COMMUNICATION DEVICES (computers, phones, etc.) MAY BE WITHIN SIGHT.

Question	Points	Out of
1		7
2		4
3		6
4		2
5		3
6		3
TOTAL		25

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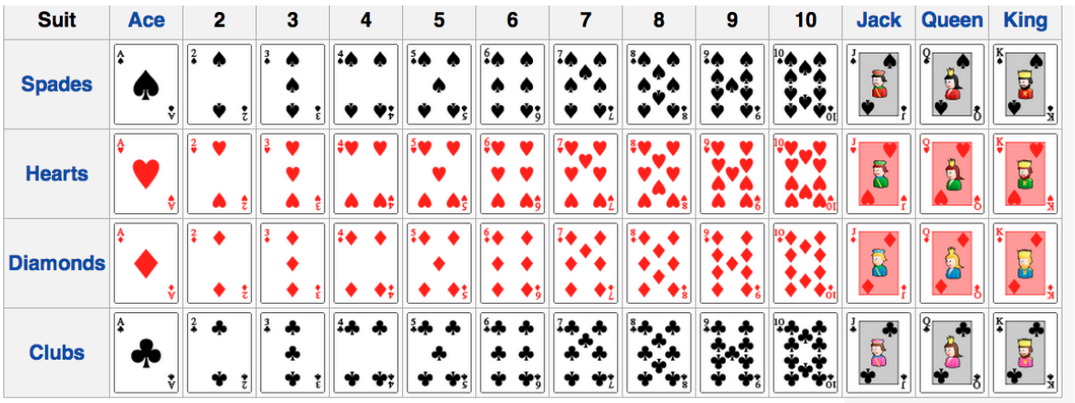
Question 1. (7 points)

Consider the following standard deck of cards and answer the questions. (each part is independent of the other).



a) From a standard deck of cards, one card is drawn. What is the probability that the card is spade and a jack? (1 point)

P(Spade and Jack) = 1/52

b) A standard deck of cards is shuffled and one card is drawn. Find the probability that the card is a queen or an ace. (1 point)

P(Q or A) = P(Q)+P(A) = 1/13 + 1/13 = 2/13

c) A standard deck of cards is shuffled and one card is drawn. Find the probability that the card is a heart or a king. (1 point)

P(H or K) = P(H)+P(K)-P(H and K) = 13/52 + 4/52 -1/52= 16/52

d) WITHOUT REPLACEMENT: If you draw two cards from the deck without replacement, what is the probability that they will both be aces? (1 point)

P(AA) = (4/52)(3/51) = 1/221

e) WITHOUT REPLACEMENT: If you draw two cards from the deck without replacement, what is the probability that the second card will be an ace if the first card is a king? (1.5 point)

P(A|K) = 4/51 since there are four aces in the deck but only 51 cards left after the king has been removed.

f) WITH REPLACEMENT: Find the probability of drawing three queens in a row, with replacement. (We pick a card, write down what it is, then put it back in the deck and draw again.) (1.5 point)

To find the P(QQQ), we find the probability of drawing the first queen which is 4/52. The probability of drawing the second queen is also 4/52 and the third is 4/52. We multiply these three individual probabilities together to get P(QQQ) = P(Q)P(Q)P(Q) = (4/52)(4/52)(4/52) = .00004 which is very small but not impossible.

Question 2. (4 points)

Suppose that David can decide to go to work by one of three modes of transportation, car, bus, or commuter train. Because of high traffic, if he decides to go by car, there is a 50% chance he will be late. If he goes by bus, which has special reserved lanes but is sometimes overcrowded, the probability of being late is only 20%. The commuter train is almost never late, with a probability of only 1%, but is more expensive than the bus.

a) Suppose that David is late one day, and his boss wishes to estimate the probability that he drove to work that day by car. Since he does not know which mode of transportation David usually uses, he gives a prior probability of 1/3 to each of the three possibilities. What is the boss' estimate of the probability that David drove to work? (2 point)

Hint: use the Bays theorem!

$Pr\{bus\} = Pr\{car\} = Pr\{train\} = 1/3$
$Pr\{late \mid car\} = 0.5$
$Pr\{late \mid train\} = 0.01$
$Pr\{late \mid bus\} = 0.2$

Using bays theorem:

$$Pr\{car \mid late\} = \frac{Pr\{late \mid car\} Pr\{car\}}{Pr\{late \mid car\} Pr\{car\} + Pr\{late \mid bus\} Pr\{bus\} + Pr\{late \mid train\} Pr\{train\}} = 0.7042$$

b) Suppose that a coworker of David's knows that he almost always takes the commuter train to work, never takes the bus, but sometimes, 10% of the time, takes the car. What is the coworker's probability that David drove to work that day, given that he was late? (2 point)

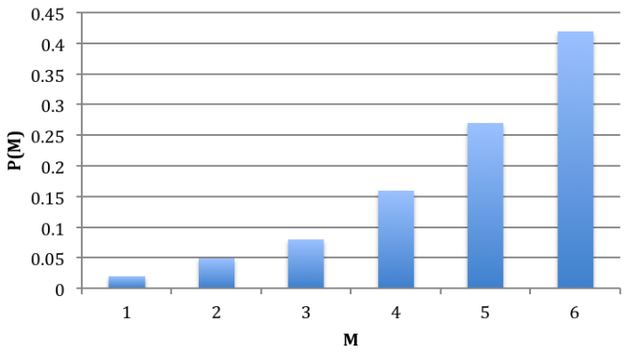
Repeat the identical calculations as the above, but instead of the prior probabilities being1/3, we use Pr{bus}= 0, Pr{car}= 0.1, and Pr {train}=0.9.
Plugging in to the same equation with these three changes, we get Pr{car | late}= 0.8475

Question 3. (6 points)

A small ferry runs every half hour from one side of a large river to the other. The number of cars X on a randomly chosen ferry trip has the probability distribution shown below. You can check that $\mu_x = 3.87$ and $\sigma_x = 1.29$.

Cars	0	1	2	3	4	5
Probability	0.02	0.05	0.08	0.16	0.27	0.42

a) The cost of the trip is 5$. Make a graph of the probability distribution for the random variable M= Money collected on a randomly selected ferry trip. (1 point)



b) Find and interpret μ_M. (1 point)

$\mu_M = 5\mu_x = 5(3.87) = 19.35\$$

On average the ferry makes 19.35 $ per trip

c) Find and interpret σ_M. (1 point)

$\sigma_M = 5\sigma_x = 5(1.29) = 6.45\$$

The individual amounts made on the ferry trips will vary about 6.45$ from the mean an average.

d) The ferry company's expenses are $20 per trip. Define the random variable Y to be the amount of profit (money collected minus expenses) made by the company on a randomly selected trip. That is Y=M-20

I. How does the mean of Y relate to the mean of M ? (1 point)

$\mu_y = \mu_M - 20 = 19.32 - 20 = -0.65$

The ferry loses an average of 1.65$ per trip

II. How does the standard deviation of Y relate to the standard deviation on M? (1 point)

$\sigma_y = \sigma_M = 6.45$

e) Suppose the ferry company decides to increase the cost of a trip to $6. We can calculate the company's profit Y on a randomly selected trip from the number of cars X. Find the mean and standard deviation of Y. (1 point)

$\mu_y = 6\mu_x - 20 = 6(3.87) - 20 = 3.22$

$\sigma_y = 6\sigma_x = 6(1.29) = 7.74$

Question 4. (2 points)

Cars arrive at a toll booth on an exit ramp on Route 408 in Zurich at an average rate of 3.4 cars every five minutes. Assume that the arrival of cars at this toll booth follows the Poisson distribution.

a) What is the probability that two or more cars will arrive at this toll booth during the next five minutes? (1 point)

$$\lambda = 3.4$$

$$P(0) = \frac{\left(3.4^{0}\right)\left(2.71828^{-3.4}\right)}{0!} = \frac{(1)(0.033373)}{1} = 0.0334$$

$$P(1) = \frac{\left(3.4^{1}\right)\left(2.71828^{-3.4}\right)}{1!} = \frac{(3.4)(0.033373)}{1} = 0.1135$$

$$P(x \geq 2) = 1.0 - P(x < 2) = 1.0 - 0.0334 - 0.1135 = 0.8531$$

b) What is the probability that exactly four cars will arrive at this toll booth during the next 10 minutes? (1 point)

$$\lambda = 6.8$$

$$P(4) = \frac{\lambda^{x} e^{-\lambda}}{x!} = \frac{\left(6.8^{4}\right)\left(2.71828^{-6.8}\right)}{4}$$

$$P(4) = \frac{(2{,}138.138)(0.001114)}{24} = 0.0992$$

Question 5 (3 point)

A survey was conducted on January 2013 in Ottawa area to determine the proportion of residents who are satisfied with the service they received while admitted to the emergency department of the Ottawa Hospital. For a random sample of 804 adults, aged 18 or older, who were admitted to the emergency department over the year 2012, let X be the number in the sample who are satisfied with the service they received. Suppose the actual percent of Ottawa residents who are satisfied with the service is 45%. Please answer the following questions.

a) The variable X is binomial (yes I am telling you this ☺) with what parameters? (0.5 point)

Binomial with p = 804 and P= 0.45

b) How likely is it (or "what is the probability") that a random sample of 804 adults will have at least 386 adults who are satisfied with the service? Please explain your answer in a simple language. (2 points)

Hint: you can use normal approximation to the binomial!

P(X>=386) = 1- P(X<386) = 1 – P(z< $\frac{386 - 804 * 0.45}{\sqrt{804 * 0.45 * 0.55}}$)=0.4

c) On average, about how many people in the sample will be satisfied? (0.5 point)

$$\mu_x = np = 804(0.45) = 361.8$$

Questions 6. (3 Points)

Over the past few years, concerns have grown over increasing traffic in the Berlin. One way the Berlin Department of Transportation measures traffic congestion is by looking at travel time. To help people plan their travel, Berlin Department of Transportation estimates that average trip from central train station to Berlin wall at 5:40 pm (at peak) takes 17 minutes and, 95% the time, the trip will not exceed 30 minutes. They also believe this travel time approximates a normal distribution

a) What is the standard deviation of peak travel time for the trip from central train station to the Berlin wall? (1 point)

From the question, we know that 95 times in 100, the trip will take 30 minutes or less. So, the entire area to the left 30 is 95%. To use the normal table to determine z, subtract .5 from the probability, and you are left with 45%, corresponding to a z-score of 1.645.

$$z = \frac{x-\mu}{\sigma} \rightarrow \sigma = \frac{30-17}{1.645} = 7.903$$

So, the standard deviation is about 8 (7.9) minutes.

b) Using your answer in part a, what proportion of drive times fall between 20 and 30 minutes? (1 point)

We are looking for P(20<x<30).
P(20<x<30)=P(x<30)-P(x<20)=.95-.6480=.302 or 30% of drive times fall between 20 and 30 minutes.

c) Write a paragraph explaining your results to a non-technical policy audience. (1 point)

The average trip at peak travel time, 5:40 pm, takes about 17 minutes and most of the time (19/20 times) it takes less than 30 minutes. However, 68% of the time, it will be between about 9 (9.1) and 25 (24.9) minutes. In 3 out of 10 trips, the trip will take between 20 and 30 minutes.

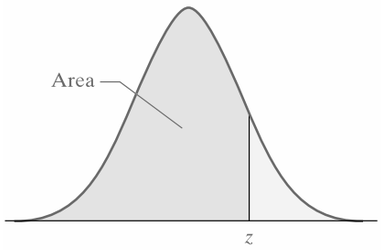


Table IV *(continued)*

Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

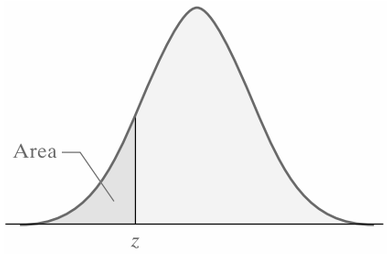


Table IV

Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641