MIE100S Dynamics – Spring 2014

Midterm exam solutions

Question #1

Given: $m = 50\ kg$ $v = 17\ m/s$

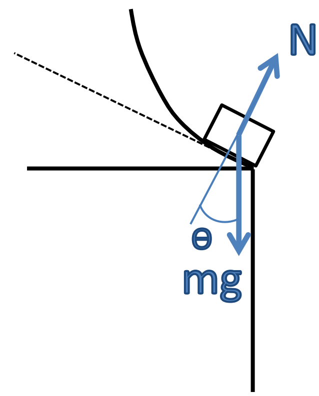
$f_f = 130\ N$ $\rho = 28\ m$ $\theta = 20^o$

a) $a_n = \frac{v^2}{\rho} = \frac{17^2}{28} = 10.32\,\frac{m}{s^2}$

b) $\sum F_n = ma_n$ → $N - mgCos(20) = ma_n$

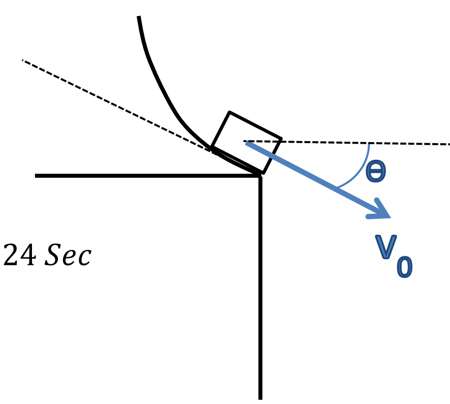
$N = 977\ Newton$ $f_f = \mu_k N = 130\ Newton$

➔ $\mu_k = 0.133$



c) Projectile motion:

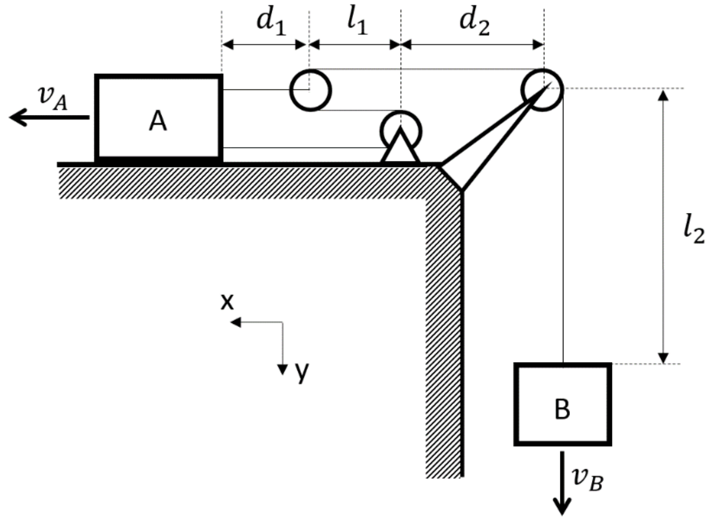
$$\begin{cases} \Delta x = v_0 \,\mathrm{Cos}(\theta)\, \Delta t \\ y = \frac{1}{2}at^2 + V_0 Sin(\theta) + y_0 \end{cases}$$



$-40 = -\frac{1}{2}gt^2 - 17Sin(20)t$ → $t = 2.324\ Sec$

$\Delta x = v_0\,\mathrm{Cos}(\theta)\,\Delta t = 37.123\ m$

Question #2



<u>Given</u>

$$v_A = 0.5 \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \text{ (to the right)}; \mu_k = 0.27;\ m_A = 44\ [\mathrm{kg}]; m_B = 21\ [\mathrm{kg}]$$

Velocities are assumed in direction of increasing length:

- As l_1 increases, A moves left
- As l_2 increases, B moves down

Total length of rope:

$$L_T = 2d_1 + 3l_1 + d_2 + l_2 \tag{1}$$

Taking the time derivative on both sides gives:

$$0 = 3v_A + v_B \tag{2}$$

since $L_T, d_1,$ and d_2 are constant, and $\dot{l}_1 = v_A$ and $\dot{l}_2 = v_B$.

Hence

$$v_B = -3v_A \tag{3}$$

(a) Kinetic energy of crate B at instant shown:

Velocity of B:

$$v_A = -0.5\left[\frac{\text{m}}{\text{s}}\right]$$

Since $v_B = -3v_A$:

$$v_B = 1.5\left[\frac{\text{m}}{\text{s}}\right]$$

Kinetic energy:

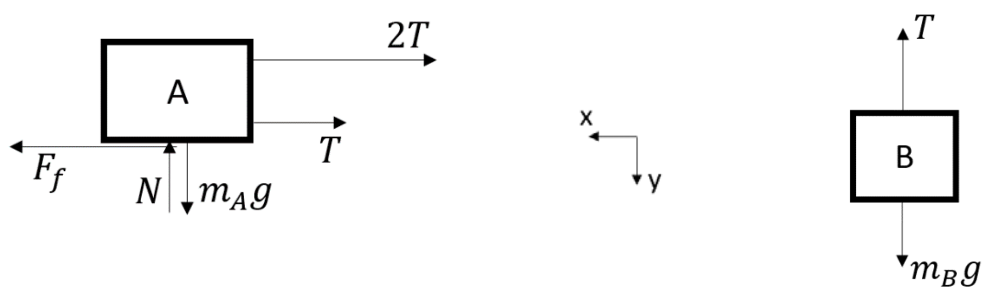
$$T_{k,B} = \frac{1}{2}m_B v_B^2$$

$$T_{k,B} = \frac{1}{2}21 \cdot 1.5^2$$
$$= 23.625\ [\text{J}]$$

$$\boldsymbol{T_{k,B} = 23.625\ [\text{J}]}$$

(b) Acceleration of crate B at the instant shown

Free body diagram for A and B:



$$\sum \boldsymbol{F} = m\boldsymbol{a}$$

A, x-direction: $\sum F_x = -3T + F_f = m_A a_A$ where $F_f = N\mu_k$, and $N = m_A g$ (4)

B, y-direction: $\sum F_y = -T + m_B g = m_B a_B$ (5)

Taking the time derivative on both sides of equation (3) gives

$$a_B = -3a_A \tag{6}$$

Combine equations (4), (5), and (6)

$$g - \frac{T}{m_B} = 3\left(\frac{3T}{m_A} - g\mu_k\right)$$

Rearrange for T

$$T = \frac{g(1 + 3\mu_k)}{\frac{1}{m_B} + \frac{9}{m_A}}$$

Hence

$$T = \frac{9.81(1 + 3 \cdot 0.27)}{\frac{1}{21} + \frac{9}{44}} = 70.415 \text{ [N]} \tag{7}$$

Substitute result of equation (7) into equation (5)

$$a_B = \frac{m_B g - T}{m_B}$$

$$a_B = \frac{21 \cdot 9.81 - 70.451}{21} = 6.455 \left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$$

$$\boldsymbol{a_B = 6.455} \left[\frac{\mathbf{m}}{\mathbf{s}^2}\right] \text{ (downward)}$$

Question #3

(a) Total elastic potential energy

$$2\frac{1}{2}k(\Delta x)^2 = 2\frac{1}{2}(200)(1.5-0.3)^2 = 288 \text{ J}$$

(b) Velocity before the collision

$$v_B = -2\, i \text{ m/s} \qquad v_A = ?$$

$$2\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv_C^2 + mgh_C \quad => \quad v_C = \sqrt{2\frac{k(\Delta x)^2 - m_A g h_C}{m_A}} = 9.541 \text{ m/s}$$

$$v_A = 9.541 \text{ m/s}$$

$$\sum_{i=1}^{2} m_i v_i = \text{constant} => m_A v_A + m_B v_B = (m_A + m_B)v$$

$$v = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{4(9.541) - 5(2)}{9} = 3.129 \text{ m/s}$$

$$v_{after\ impact} = 3.129 \text{ m/s}$$

(c) Heat generated

$$\frac{1}{2}(m_A + m_B)v^2 + Heat = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$Heat = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 - \frac{1}{2}(m_A + m_B)v^2$$

$$Heat = \frac{1}{2}(4)(9.541)^2 + \frac{1}{2}(5)(2)^2 - \frac{1}{2}(4+5)3.129^2 = 148.003 \text{ J}$$