Phys 1410 Quiz 1

Name: _____ Student Number: _____

NO CALCULATORS ALLOWED!

1. (5 marks) Evaluate the following.

a) cos (-75°) $= \cos 75^\circ$

$= \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

b) tan (-5π/4) $= -\tan \frac{\pi}{4}$

$= -1$

c) Arcsin (-1) $= \frac{3\pi}{2}$ or 270°

d) Arcsec ($-2^{1/2}$) $= \text{Arccos}\left(\frac{-1}{\sqrt{2}}\right)$

$= \frac{3\pi}{4}, \frac{5\pi}{4}$

e) What is the period of y = tan 3x?

Period of $\tan x$ is π.

" $\tan 3x$ is $\frac{\pi}{3}$.

2. (2 marks) Evaluate $(1020)^{1/3}$ to 0.1% accuracy using the binomial theorem?

$$(1020)^{1/3} = (1000+20)^{1/3}$$

$$= 10\left[1 + \frac{20}{1000}\right]^{1/3}$$

$$\simeq 10\left[1 + \frac{1}{3}(0.02)\right]$$

$$= 10.067$$

3. (4 marks) Find the first four nonzero terms of the Taylor expansion of y = ln (1+x) expanded about x = 0.

$y = \ln(1+x)$ $\qquad y(0) = 0$

$\frac{dy}{dx} = \frac{1}{1+x}$ $\qquad \frac{dy}{dx}(x=0) = 1$

$\frac{d^2y}{dx^2} = -(1+x)^{-2}$ $\qquad \frac{d^2y}{dx^2}(x=0) = -1$

$\frac{d^3y}{dx^3} = 2(1+x)^{-3}$ $\qquad \frac{d^3y}{dx^3}(x=0) = 2$

$\frac{d^4y}{dx^4} = -3.2(1+x)^{-4}$ $\qquad \frac{d^4y}{dx^4}(x=0) = -3!$

$$\therefore \ln(1+x) = 0 + 1\cdot x - \frac{1}{2}x^2 + \frac{2}{3!}x^3 - \frac{3!}{4!}x^4 + \cdots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

4. Consider the point A(1,2,1), B(3,4,0) and C(-1,2,1).

a) (2 marks) Find the vectors pointing

i. From A to B

ii. From A to C

$\vec{AB} = (3,4,0) - (1,2,1) = (2,2,-1)$

$\vec{AC} = (-1,2,1) - (1,2,1) = (-2,0,0)$

b) (2 marks) Find the lengths of the lines connecting

i. A and B

ii. A and C

$|\vec{AB}| = \sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{9} = 3$

$|\vec{AC}| = \sqrt{(-2)^2 + 0^2 + 0^2} = 2$

c) (2 marks) Evaluate $\vec{AB} \times \vec{AC}$.

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ -2 & 0 & 0 \end{vmatrix}$$

$= (0,2,4)$

d) (1 mark) Find the area of the triangle ABC.

triangle area $= \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$= \frac{1}{2} \sqrt{0^2 + 2^2 + 4^2}$

$= \sqrt{5}$ square units

e) (2 marks) Prove the angle at A in the triangle is greater than 90°.

$\cos \angle A = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}|\,|\vec{AC}|}$

$= \frac{-4}{3 \cdot 2}$

$= -\frac{2}{3}$

< 0

$\therefore \angle A > 90°.$

Total = 20