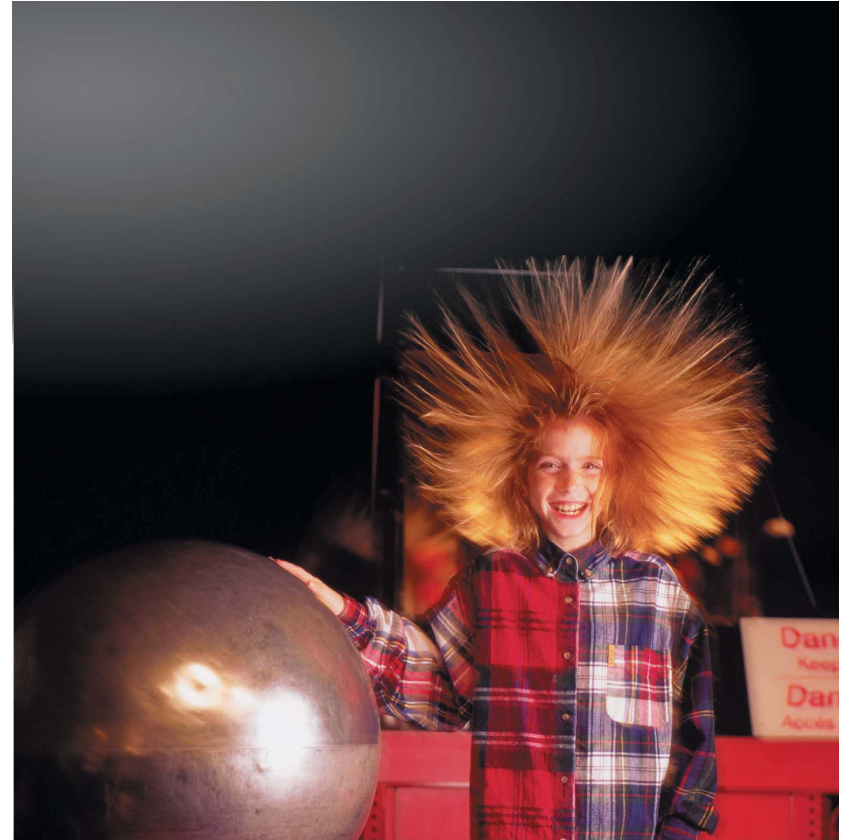


## Chapter 28. Gauss's Law

The nearly spherical shape of the girl's head determines the electric field that causes her hair to stream outward. Using Gauss's law, we can deduce electric fields, particularly those with a high degree of symmetry, simply from the shape of the charge distribution.

**Chapter Goal:** To understand and apply Gauss's law.



# Chapter 28. Gauss's Law

## Topics:

- Symmetry
- The Concept of Flux
- Calculating Electric Flux
- Gauss's Law
- Using Gauss's Law
- Conductors in Electrostatic Equilibrium

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Introduced last lecture

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- Symmetry
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Introduced last lecture

Today

# Chapter 28. Gauss's Law

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- Symmetry
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- Calculating Electric Flux
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- Using Gauss's Law
- Conductors in Electrostatic Equilibrium

Introduced last lecture

Today


Next lecture

## **Chapter 28. Reading Quizzes**

**The amount of electric field  
passing through a surface is  
called**

- A. Electric flux.
- B. Gauss's Law.
- C. Electricity.
- D. Charge surface density.
- E. None of the above.

**The amount of electric field passing through a surface is called**


-  **A. Electric flux.**
- B. Gauss's Law.
- C. Electricity.
- D. Charge surface density.
- E. None of the above.



**Gauss's law is useful for calculating electric fields that are**

- A. symmetric.
- B. uniform.
- C. due to point charges.
- D. due to continuous charges.


**Gauss's law is useful for calculating electric fields that are**

-  **A. symmetric.**
- B. uniform.
- C. due to point charges.
- D. due to continuous charges.

## Gauss's law applies to

- A. lines.
- B. flat surfaces.
- C. spheres only.
- D. closed surfaces.


## Gauss's law applies to

- A. lines.
- B. flat surfaces.
- C. spheres only.
-  **D. closed surfaces.**

**The electric field inside a conductor in electrostatic equilibrium is**

- A. uniform.
- B. zero.
- C. radial.
- D. symmetric.

The electric field inside a conductor in electrostatic equilibrium is

- A. uniform.
-  **B. zero.**
- C. radial.
- D. symmetric.

# Learning Objectives

- To understand the importance of symmetry
- To calculate electric flux
- To use Gauss's law to derive electric fields of interest
- To study the properties of conductors in electrostatic equilibrium

## Why this (Gauss's Law) is difficult

- Requires reasoning with the concept of symmetry
- Presupposes a basic understanding of electric fields
- Uses surface (vector) integrals
- Most students are not familiar with reasoning by symmetry, have just learned about electric fields and have not studied vector integrals



## Aside: Coulomb's Law & Gauss's Law

- Note that Coulomb's Law can be derived from Gauss's law and vice versa
    - Recall Coulomb's Law
    - Only applies to electrostatics
- $$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$
- $$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$
- Gauss's law allows us to find electric field in situations that would be difficult for Coulomb's law
    - Also applies to electrodynamics
      - Electric fields that vary with time

# Symmetry

Some charge distributions have translational, rotational, or reflective symmetry. If this is the case, we can determine something about the field it produces:

**The symmetry of an electric field must match the symmetry of the charge distribution.**

**Symmetry can tell us the shape of the electric field**

For example, the electric field of a cylindrically symmetric charge distribution

- a) cannot have a component parallel to the cylinder axis.
- b) cannot have a component tangent to the circular cross section.

# Cylindrical Symmetry



Infinitely long  
charged cylinder

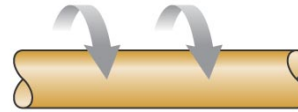
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Original  
cylinder



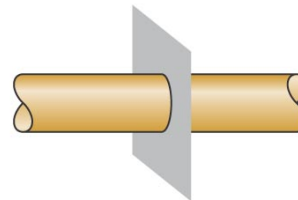
Translation  
parallel to  
the axis



Rotation  
about the  
axis



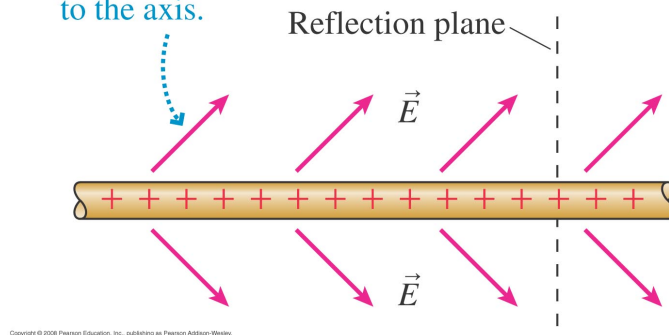
Reflection  
in plane  
containing  
the axis



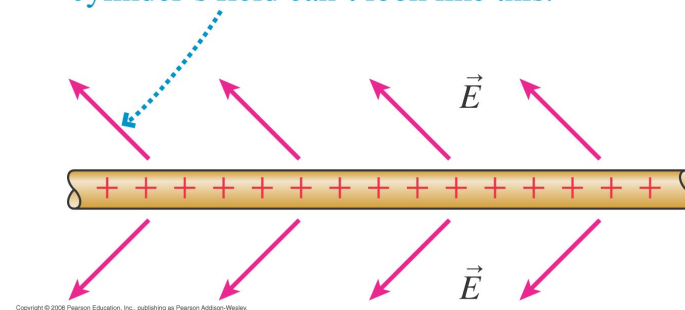
Reflection  
perpendicular  
to the axis

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- (a) Is this a possible electric field of an infinitely long charged cylinder? Suppose the charge and the field are reflected in a plane perpendicular to the axis.



- (b) The charge distribution is not changed by the reflection, but the field is. This field doesn't match the symmetry of the cylinder, so the cylinder's field can't look like this.

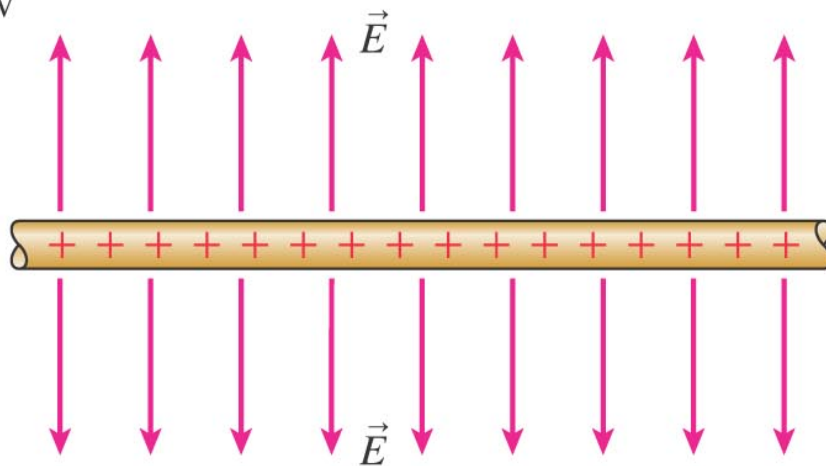


- (a)
- End view of cylinder
- Reflection plane
- The charge distribution is not changed by reflecting it in a plane containing the axis.
- 
- $\vec{E}$
- $\vec{E}$
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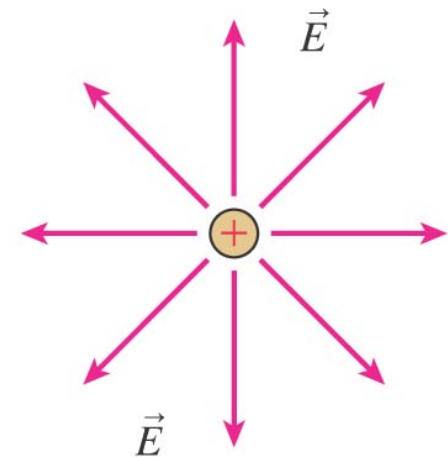
- (b)
- This field *is* changed. It doesn't match the symmetry of the cylinder, so the field can't look like this.
- 
- $\vec{E}$
- $\vec{E}$
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# Shape of Electric field must match symmetry of charge distribution

Side view



End view

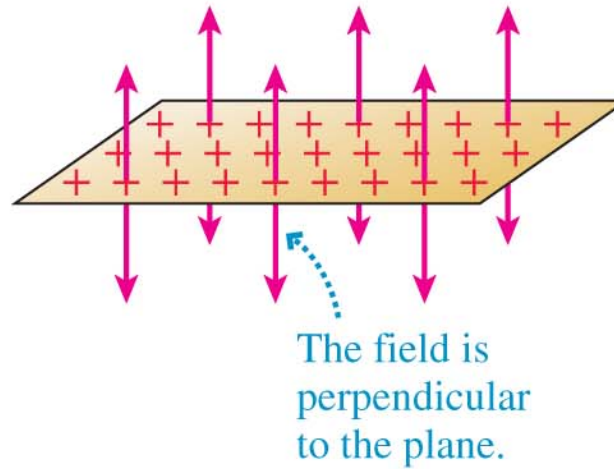


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**FIGURE 28.6** Three fundamental symmetries.

**Planar symmetry**

Basic  
symmetry:

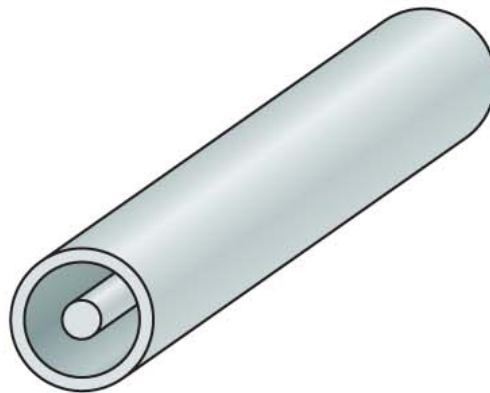
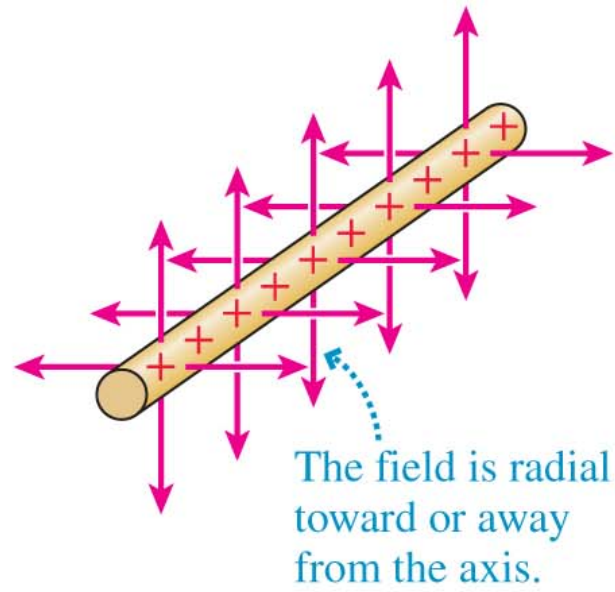


More  
complex  
example:



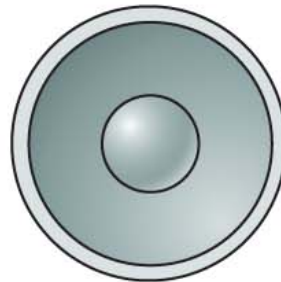
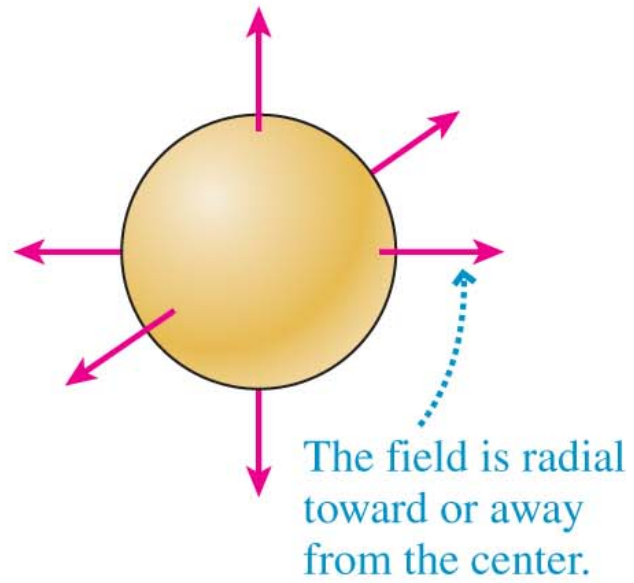
Infinite parallel-plate capacitor

## Cylindrical symmetry



Coaxial cylinders

## Spherical symmetry

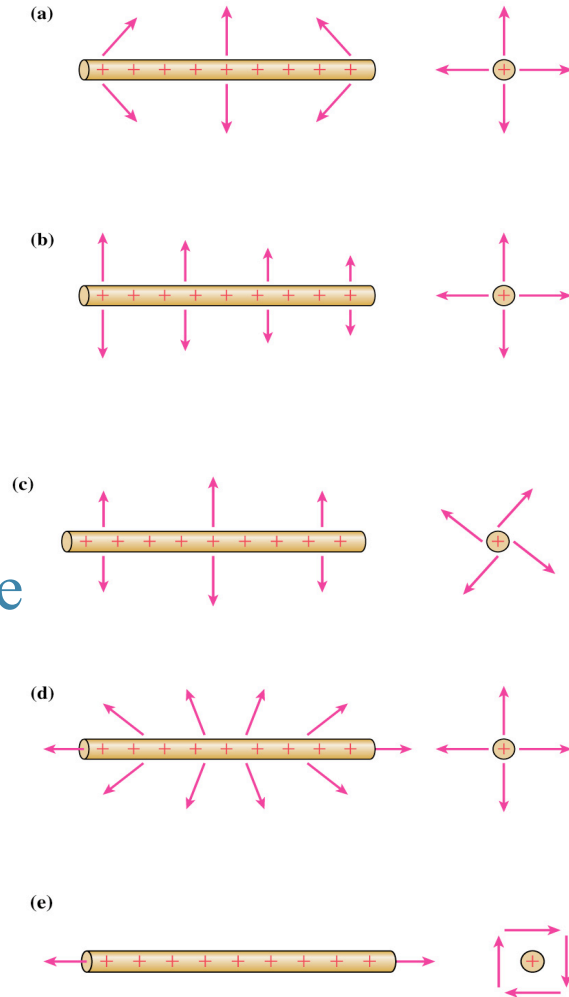


Concentric spheres



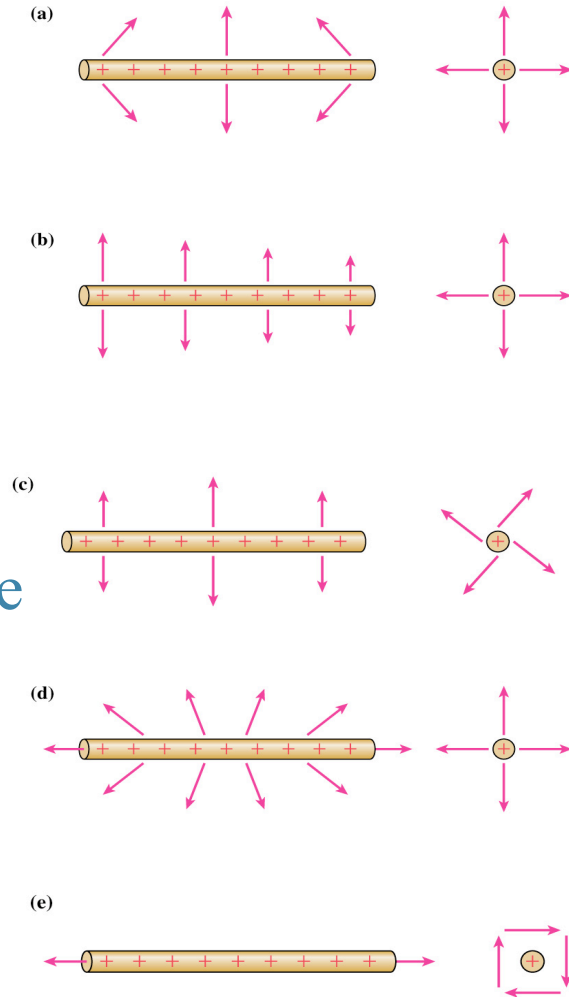
A uniformly charged rod has a *finite* length  $L$ . The rod is symmetric under rotations about the axis and under reflection in any plane containing the axis. It is *not* symmetric under translations or under reflections in a plane perpendicular to the axis other than the plane that bisects the rod. Which field shape or shapes match the symmetry of the rod?

- A. c and e
- B. a and d
- C. e only
- D. b only
- E. none of the above



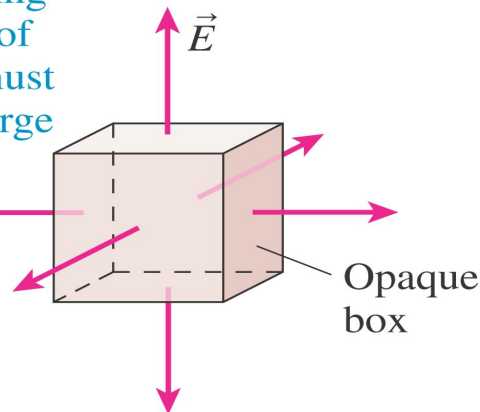
A uniformly charged rod has a *finite* length  $L$ . The rod is symmetric under rotations about the axis and under reflection in any plane containing the axis. It is *not* symmetric under translations or under reflections in a plane perpendicular to the axis other than the plane that bisects the rod. Which field shape or shapes match the symmetry of the rod?

- ✓ A. c and e
- B. a and d**
- C. e only
- D. b only
- E. none of the above



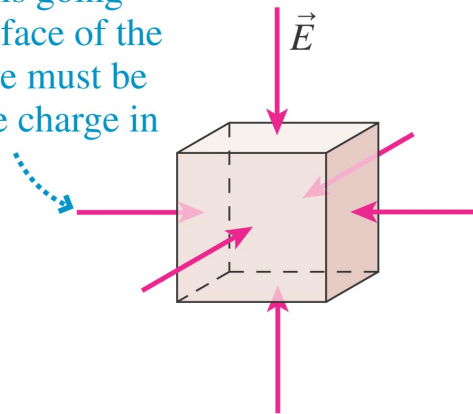
# The Concept of Flux

- (a) The field is coming out of each face of the box. There must be a positive charge in the box.



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- (b) The field is going into each face of the box. There must be a negative charge in the box.



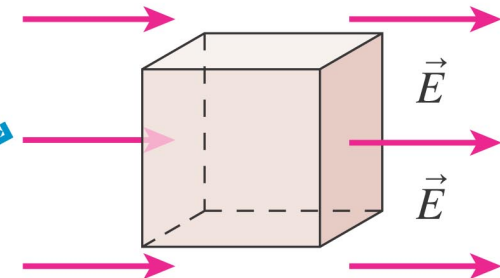
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The electric field from each surface “tells” us what charge is in the box.

There is a connection between how much electric field passes in/out/through the box and how much charge is inside.

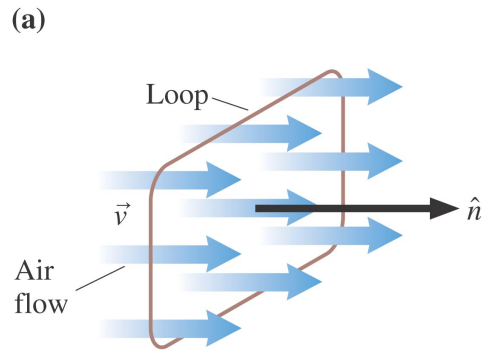
The amount of electric field that passes through a surface is called the electric flux.

- (c) A field passing through the box implies there's no net charge in the box.



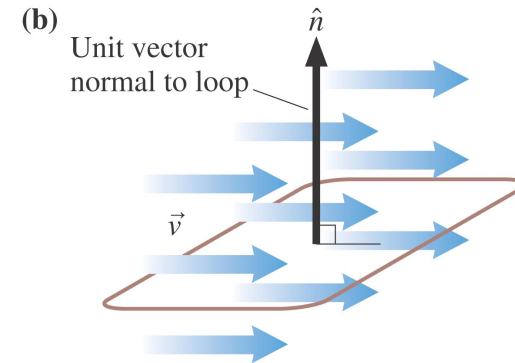
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# Amount of air flowing through loop depends on the angle between velocity ( $\vec{v}$ ) and normal ( $\hat{n}$ )



The air flowing through the loop is maximum when  $\theta = 0^\circ$ .

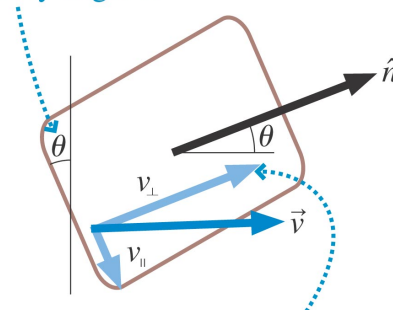
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No air flows through the loop when  $\theta = 90^\circ$ .

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(c) The loop is tilted by angle  $\theta$ .



$v_{\perp} = v \cos \theta$  is the component of the air velocity perpendicular to the loop.

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## The Electric Flux

The electric flux measures the amount of electric field passing through a surface of area  $A$  whose normal to the surface is tilted at angle  $\theta$  from the field.

$$\Phi_e = E_{\perp} A = EA \cos \theta$$

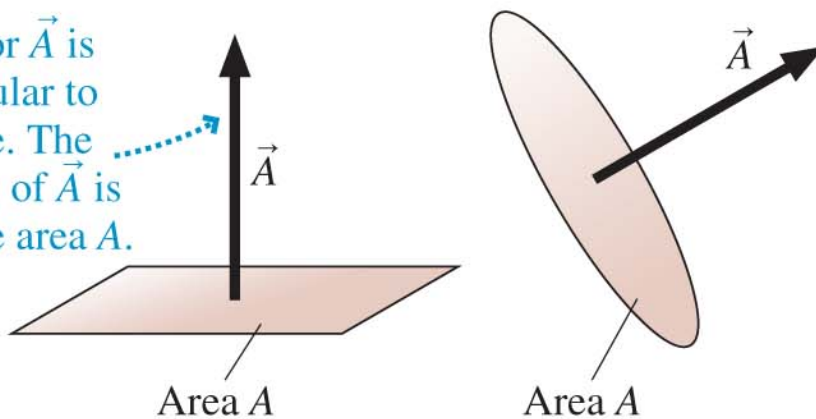
We can define the electric flux more concisely using the dot-product:

$$\Phi_e = \vec{E} \cdot \vec{A} \quad (\text{electric flux of a constant electric field})$$

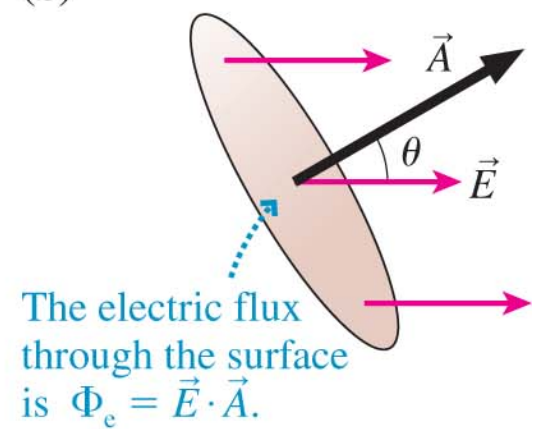
**FIGURE 28.13** The electric flux can be defined in terms of the area vector  $\vec{A}$ .

(a)

Area vector  $\vec{A}$  is perpendicular to the surface. The magnitude of  $\vec{A}$  is the surface area  $A$ .

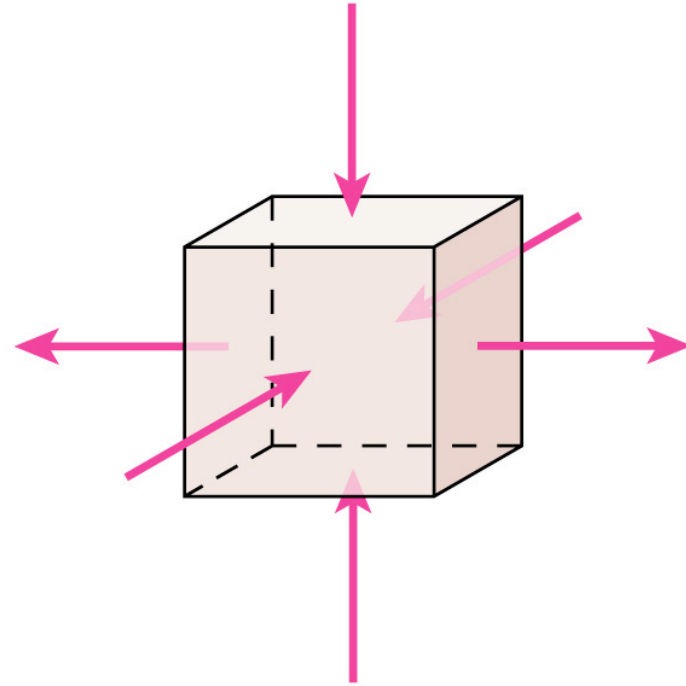


(b)



**This box contains**

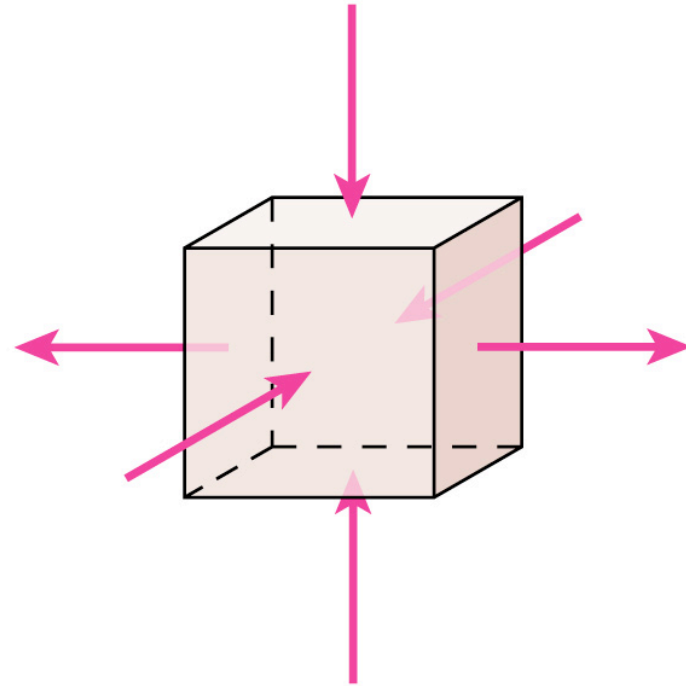
- A. a net positive charge.
- B. a net negative charge.
- C. a negative charge.
- D. a positive charge.
- E. no net charge.



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**This box contains**

- A. a net positive charge.
- ✓ **B. a net negative charge.**
- C. a negative charge.
- D. a positive charge.
- E. no net charge.



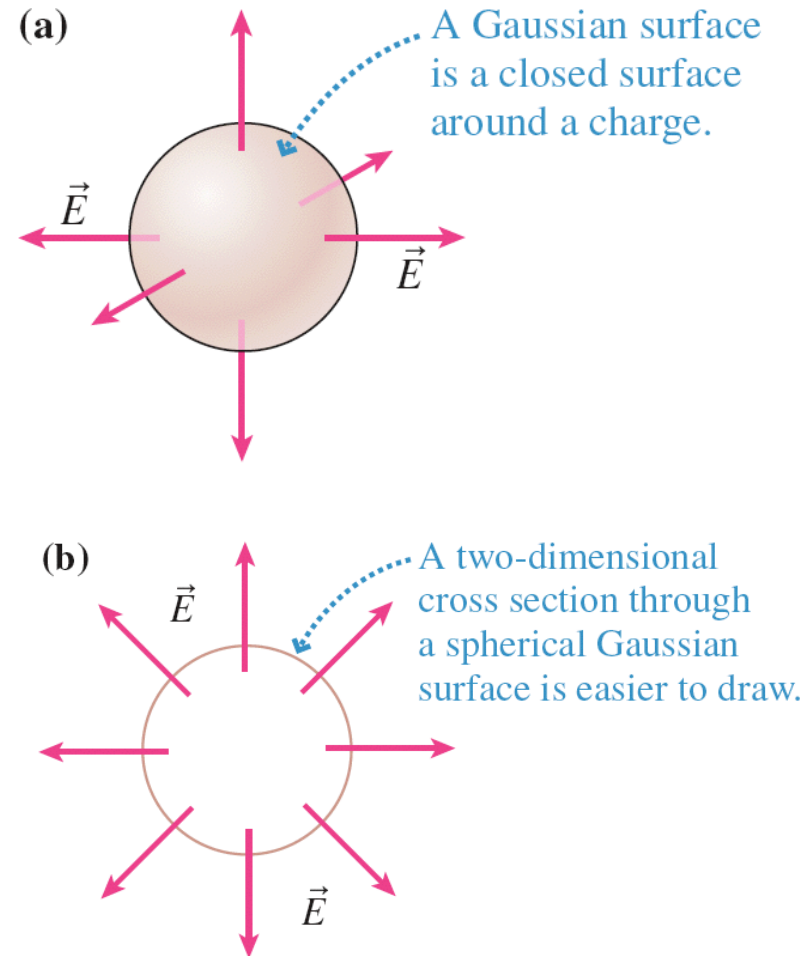
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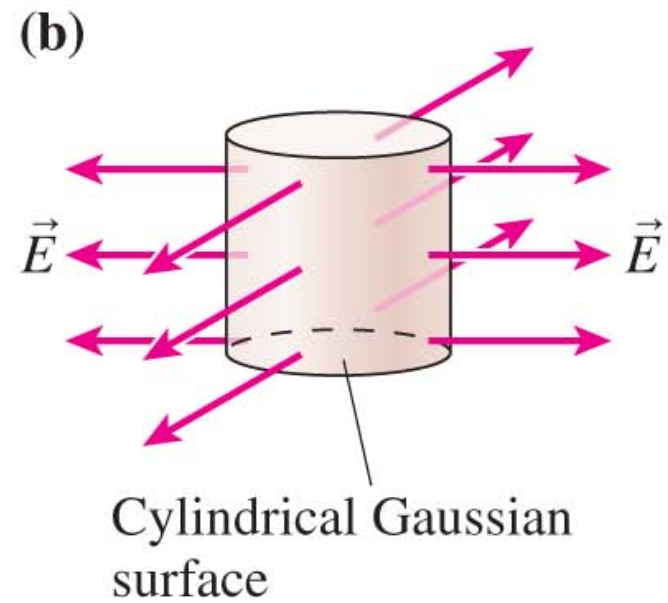
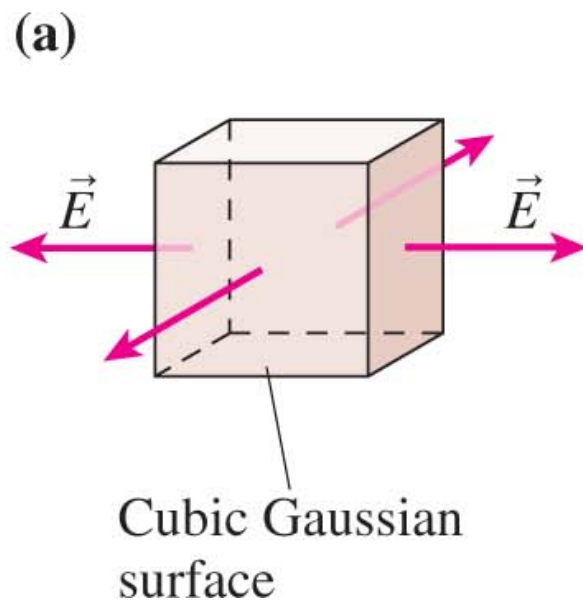
# Gaussian Surfaces

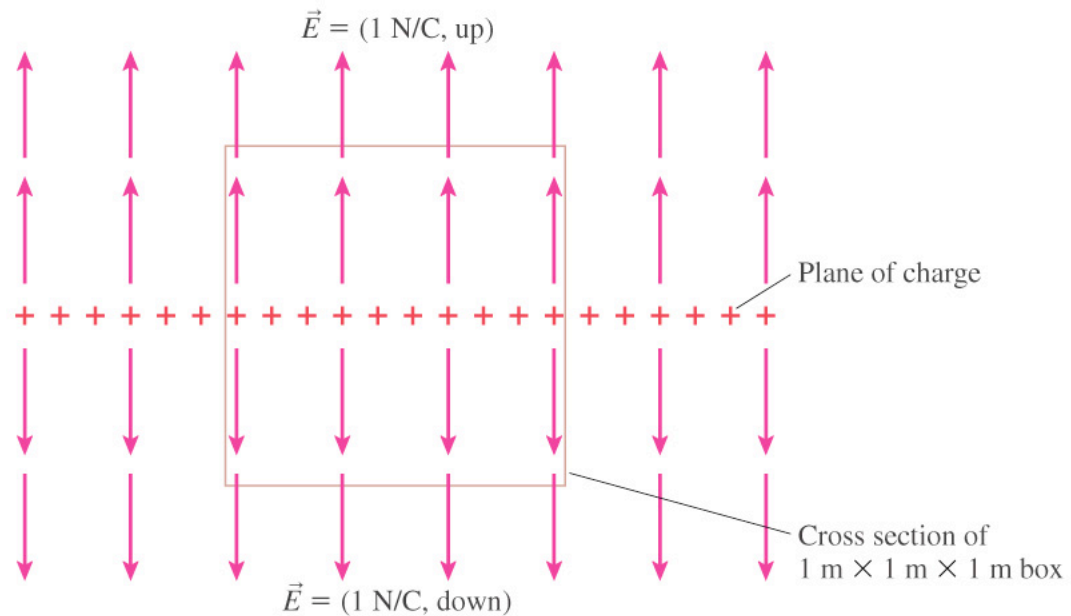
- Suppose we surround a region of space with a closed surface.
  - So there is inside & outside
- In Electrostatics we call this surface a **Gaussian Surface**
  - Does not need to correspond to a physical surface
  - Gaussian surfaces are most useful when they match the shape of the electric field

**FIGURE 28.8** Gaussian surface surrounding a charge. A two-dimensional cross section is usually easier to draw.



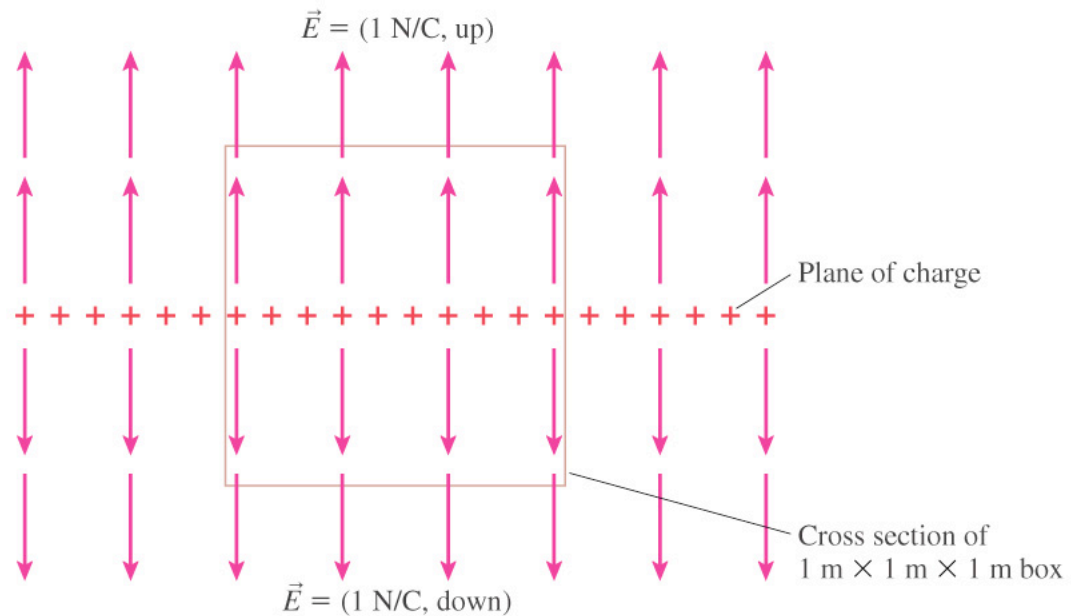
**FIGURE 28.9** Gaussian surface is most useful when it matches the shape of the field.





**The total electric flux through this box is**

- A.  $6 \text{ Nm}^2/\text{C}$ .
- B.  $4 \text{ Nm}^2/\text{C}$ .
- C.  $2 \text{ Nm}^2/\text{C}$ .
- D.  $1 \text{ Nm}^2/\text{C}$ .
- E.  $0 \text{ Nm}^2/\text{C}$ .



**The total electric flux through this box is**

- A.  $6 \text{ Nm}^2/\text{C}$ .
- B.  $4 \text{ Nm}^2/\text{C}$ .
- ✓ C.  $2 \text{ Nm}^2/\text{C}$ .
- D.  $1 \text{ Nm}^2/\text{C}$ .
- E.  $0 \text{ Nm}^2/\text{C}$ .

## EXAMPLE 28.1 The electric flux inside a parallel-plate capacitor

### QUESTION:

#### EXAMPLE 28.1 The electric flux inside a parallel-plate capacitor

Two  $100 \text{ cm}^2$  parallel electrodes are spaced  $2.0 \text{ cm}$  apart. One is charged to  $+5.0 \text{ nC}$ , the other to  $-5.0 \text{ nC}$ . A  $1.0 \text{ cm} \times 1.0 \text{ cm}$  surface between the electrodes is tilted to where its normal makes a  $45^\circ$  angle with the electric field. What is the electric flux through this surface?

## Electric Flux of a non-Uniform Field

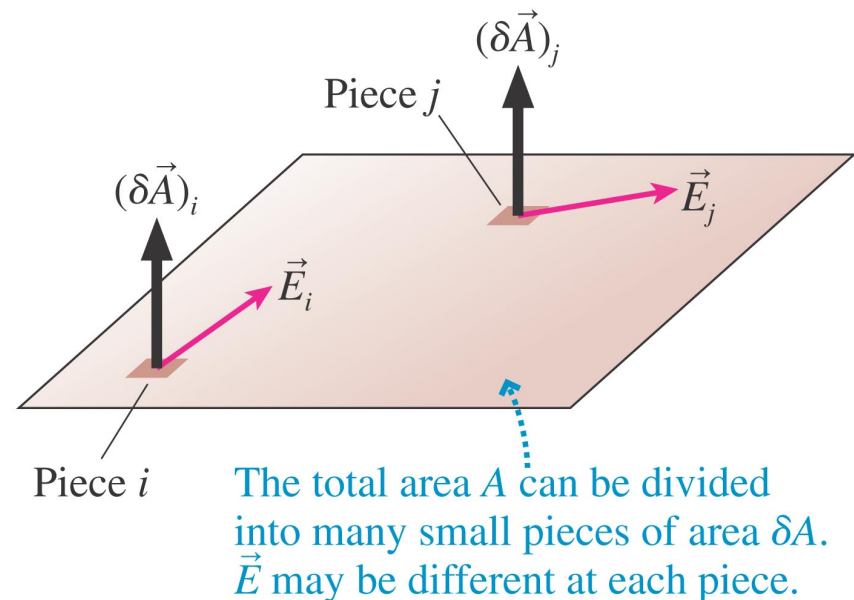
- We just did an example of the electric flux through a surface of a uniform field

$$\Phi_e = \vec{E} \cdot \vec{A}$$

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- What if the electric field is not uniform
  - Evaluate integral over the surface

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



# Flux Through a Curved Surface

- Electric Flux

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

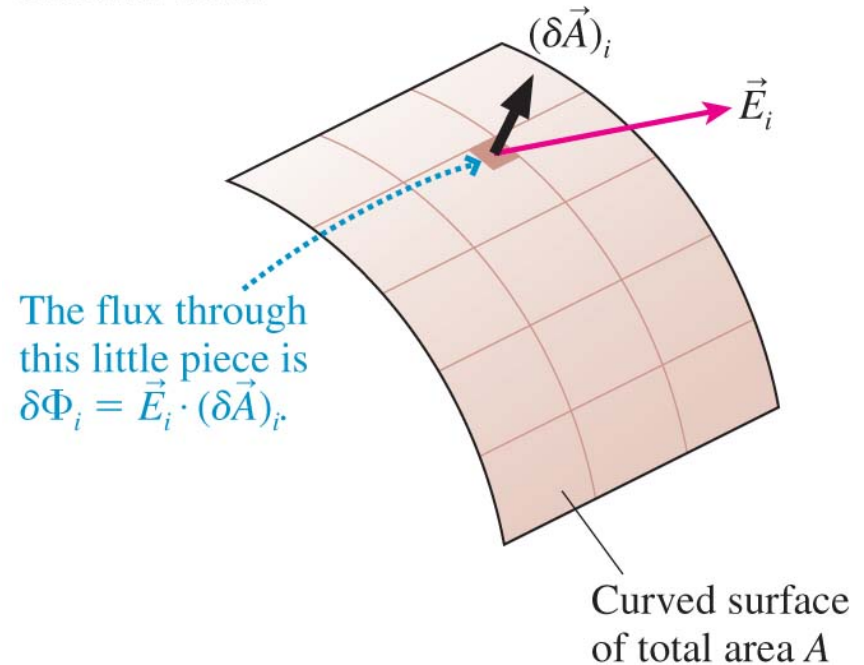
- If electric field is constant then

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = E \cos \vartheta \int_{\text{surface}} dA$$

- The last integral is just the sum of all the little areas which equals the total surface area  $A$

$$A = \int_{\text{surface}} dA$$

**FIGURE 28.15** A curved surface in an electric field.



## The Electric Flux through a Closed Surface

The electric flux through a closed surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

The electric flux is still the summation of the fluxes through a vast number of tiny pieces, pieces that now cover a closed surface.

**NOTE:** A closed surface has a distinct inside and outside. The area vector  $d\vec{A}$  is defined to always point *toward the outside*. This removes an ambiguity that was present for a single surface, where  $d\vec{A}$  could point to either side.



# Evaluating surface integrals

- To use Gauss's Law you need to evaluate surface integrals
- There are **ONLY** two cases for constant Electric field

## TACTICS BOX 28.1 Evaluating surface integrals



- ① If the electric field is everywhere tangent to a surface, the electric flux through the surface is  $\Phi_e = 0$ .
- ② If the electric field is everywhere perpendicular to a surface *and* has the same magnitude  $E$  at every point, the electric flux through the surface is  $\Phi_e = EA$ .

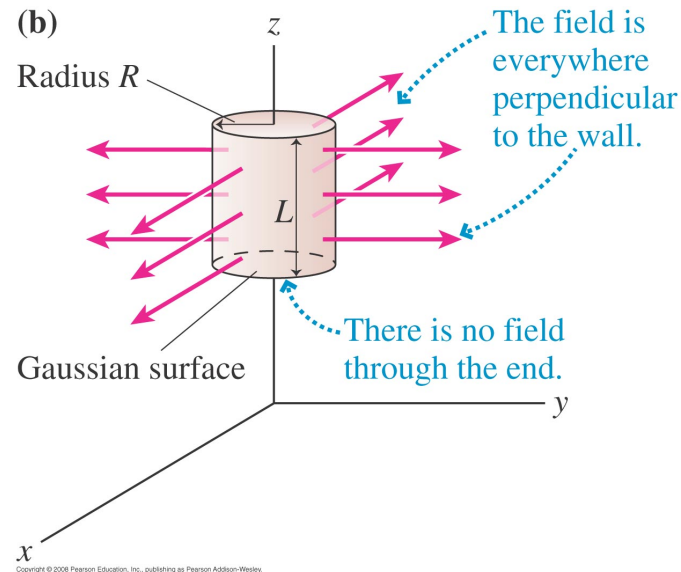
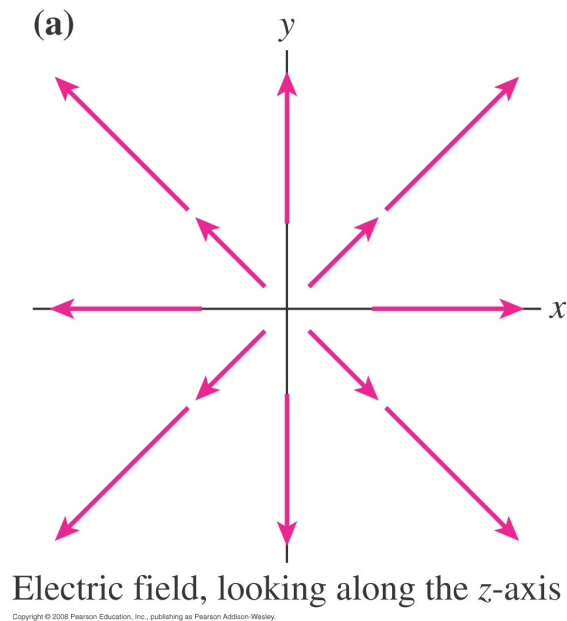
## TACTICS BOX 28.2 Finding the flux through a closed surface



- ① Divide the closed surface into pieces that are everywhere tangent to the electric field and everywhere perpendicular to the electric field.
- ② Use Tactics Box 28.1 to evaluate the surface integrals over these surfaces, then add the results.



- Calculate the Electric Flux through a closed cylinder
  - A cylindrical charge distribution has created the electric field  $\vec{E} = E_0 \left( \frac{r^2}{r_0^2} \right) \hat{r}$  where  $E_0$  and  $r_0$  are constants and the unit vector  $\hat{r}$  lies in the x-y plane. Calculate the electric flux through a closed cylinder of length  $L$  and radius  $R$ .



## Gauss's Law

For any *closed* surface enclosing total charge  $Q_{\text{in}}$ , the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

This result for the electric flux is known as **Gauss's Law**.

## Using Gauss's Law

1. Gauss's law applies only to a *closed* surface, called a Gaussian surface.
2. A Gaussian surface is not a physical surface. It need not coincide with the boundary of any physical object (although it could if we wished). It is an imaginary, mathematical surface in the space surrounding one or more charges.
3. We can't find the electric field from Gauss's law alone. We need to apply Gauss's law in situations where, from symmetry and superposition, we already can guess the *shape* of the field.

Show relationship between  
Coulomb's Law and Gauss's Law

**PROBLEM-SOLVING  
STRATEGY 28.1**

## Gauss's law



**MODEL** Model the charge distribution as a distribution with symmetry.

**VISUALIZE** Draw a picture of the charge distribution.

- Determine the symmetry of its electric field.
- Choose and draw a Gaussian surface with the *same symmetry*.
- You need not enclose all the charge within the Gaussian surface.
- Be sure every part of the Gaussian surface is either tangent to or perpendicular to the electric field.

**SOLVE** The mathematical representation is based on Gauss's law

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

- Use Tactics Boxes 28.1 and 28.2 to evaluate the surface integral.

**ASSESS** Check that your result has the correct units, is reasonable, and answers the question.

## Electric Field Outside a Sphere of Charge

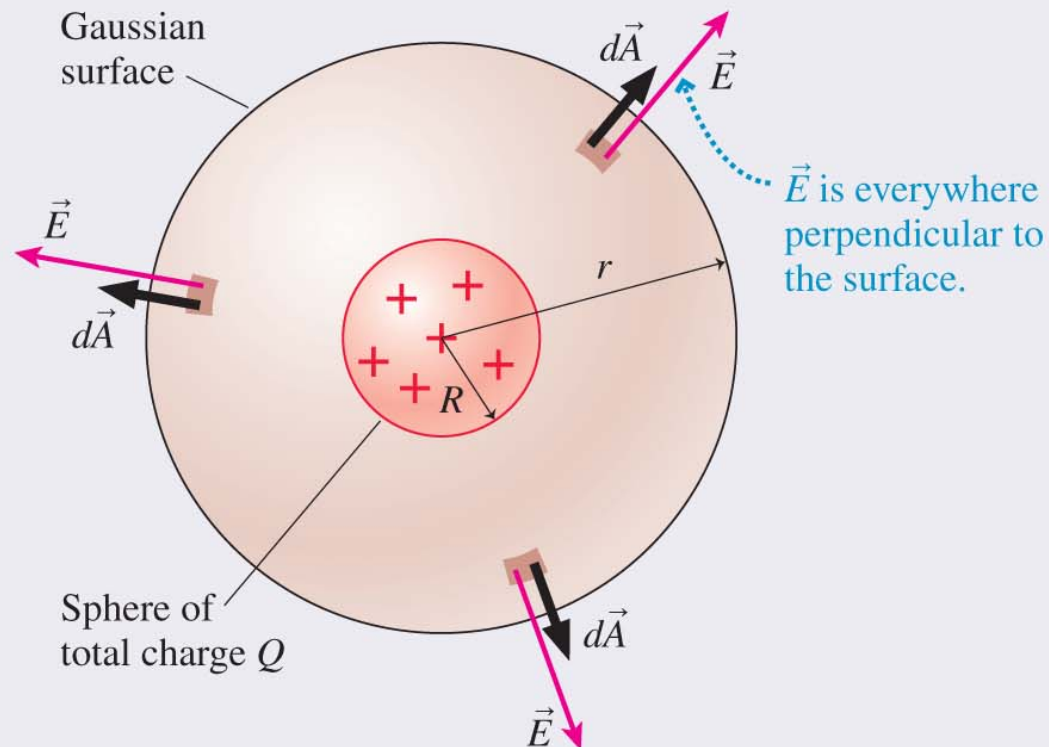
- Last week (chapter 27) we asserted that the electric field outside a sphere with total charge  $Q$  is the same as the field of a point charge  $Q$  at the center of the sphere.

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \quad \text{for } r \geq R$$

– Now we will prove using Gauss's Law

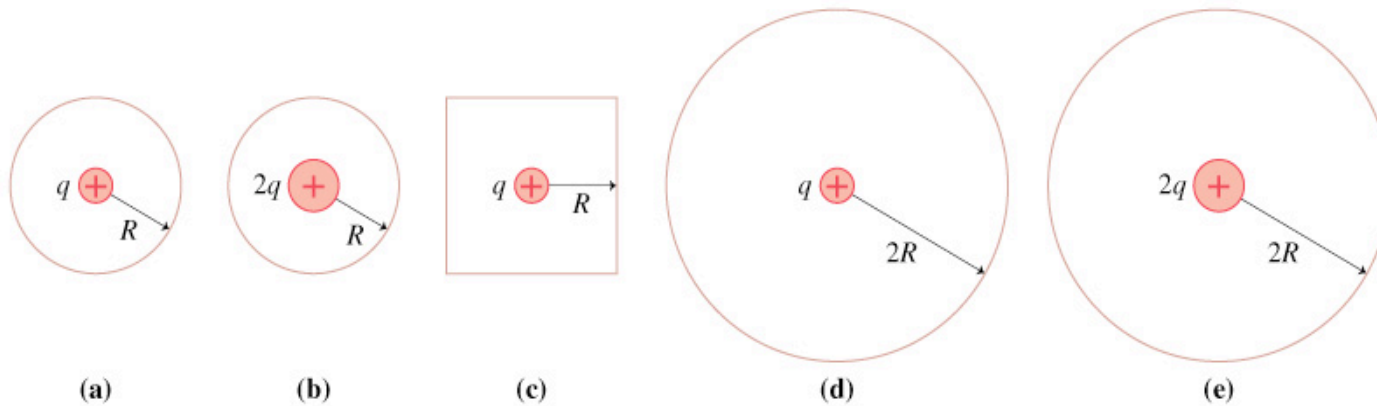
## EXAMPLE 28.3 Outside a sphere of charge

**FIGURE 28.23** A spherical Gaussian surface surrounding a sphere of charge.



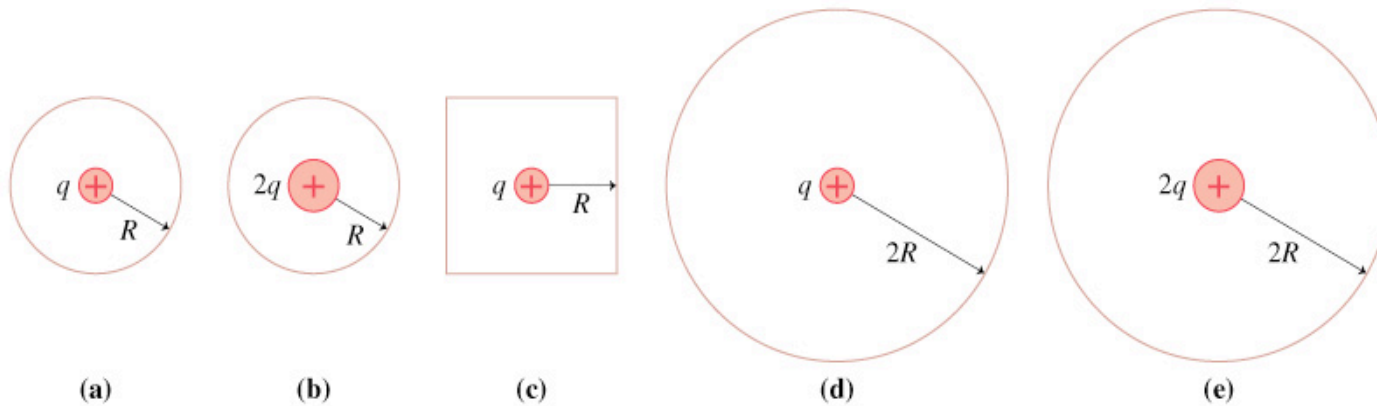


These are two-dimensional cross sections through three-dimensional closed spheres and a cube. Rank order, from largest to smallest, the electric fluxes  $\Phi_a$  to  $\Phi_e$  through surfaces a to e.



- A.  $\Phi_a > \Phi_c > \Phi_b > \Phi_d > \Phi_e$
- B.  $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$
- C.  $\Phi_e > \Phi_d > \Phi_b > \Phi_c > \Phi_a$
- D.  $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$
- E.  $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$

These are two-dimensional cross sections through three-dimensional closed spheres and a cube. Rank order, from largest to smallest, the electric fluxes  $\Phi_a$  to  $\Phi_e$  through surfaces a to e.



- A.  $\Phi_a > \Phi_c > \Phi_b > \Phi_d > \Phi_e$   
 ✓ B.  $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$   
 C.  $\Phi_e > \Phi_d > \Phi_b > \Phi_c > \Phi_a$   
 D.  $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$   
 E.  $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$

## **Chapter 28. Summary Slides**

# General Principles

## Gauss's Law

For any *closed* surface enclosing net charge  $Q_{\text{in}}$ , the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

The electric flux  $\Phi_e$  is the same for *any* closed surface enclosing charge  $Q_{\text{in}}$ .

# General Principles

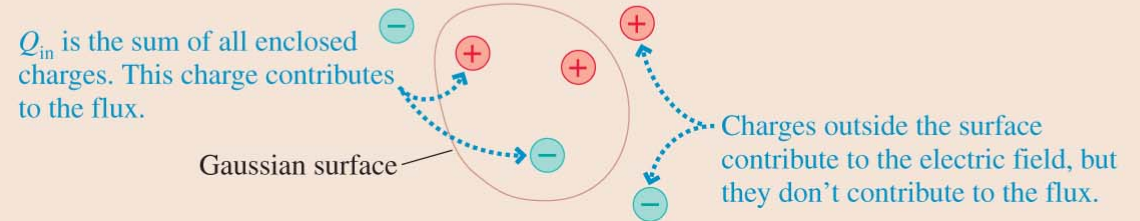
## Symmetry

The symmetry of the electric field must match the symmetry of the charge distribution.

In practice,  $\Phi_e$  is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.

# Important Concepts

**Charge** creates the electric field that is responsible for the electric flux.

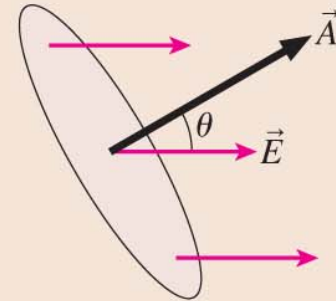


# Important Concepts

**Flux** is the amount of electric field passing through a surface of area  $A$ :

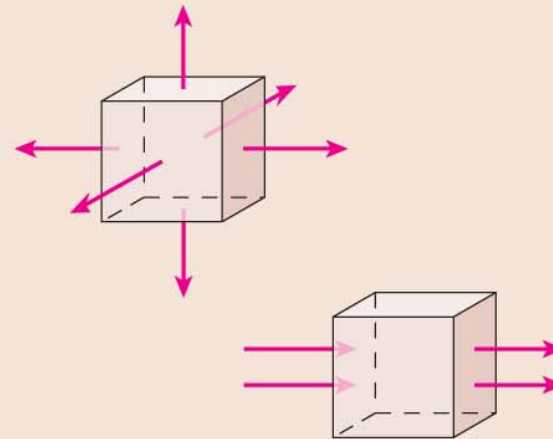
$$\Phi_e = \vec{E} \cdot \vec{A}$$

where  $\vec{A}$  is the **area vector**.



## For closed surfaces:

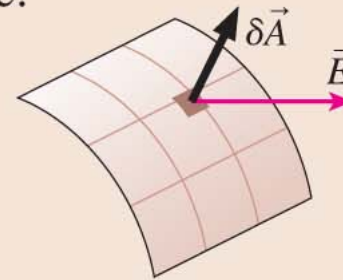
A net flux in or out indicates that the surface encloses a net charge. Field lines through but with no *net* flux mean that the surface encloses no *net* charge.



# Important Concepts

**Surface integrals** calculate the flux by summing the fluxes through many small pieces of the surface:

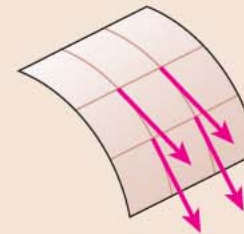
$$\Phi_e = \sum \vec{E} \cdot \delta \vec{A}$$
$$\rightarrow \int \vec{E} \cdot d\vec{A}$$



## Two important situations:

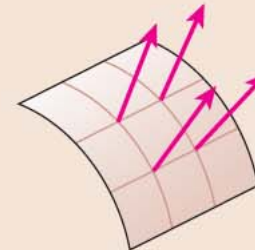
If the electric field is everywhere tangent to the surface, then

$$\Phi_e = 0$$



If the electric field is everywhere perpendicular to the surface *and* has the same strength  $E$  at all points, then

$$\Phi_e = EA$$





**Which Gaussian surface would allow you to use Gauss's law to determine the electric field outside a uniformly charged cube?**

- A. A cube whose center coincides with the center of the charged cube and which has parallel faces.
- B. A sphere whose center coincides with the center of the charged cube.
- C. Neither A nor B.
- D. Either A or B.

**Which Gaussian surface would allow you to use Gauss's law to determine the electric field outside a uniformly charged cube?**

- A. A cube whose center coincides with the center of the charged cube and which has parallel faces.
- B. A sphere whose center coincides with the center of the charged cube.
- ✓ **C. Neither A nor B.**
- D. Either A or B.

# Conductors in Electrostatic Equilibrium

**The electric field is zero at all points within a conductor in electrostatic equilibrium.**

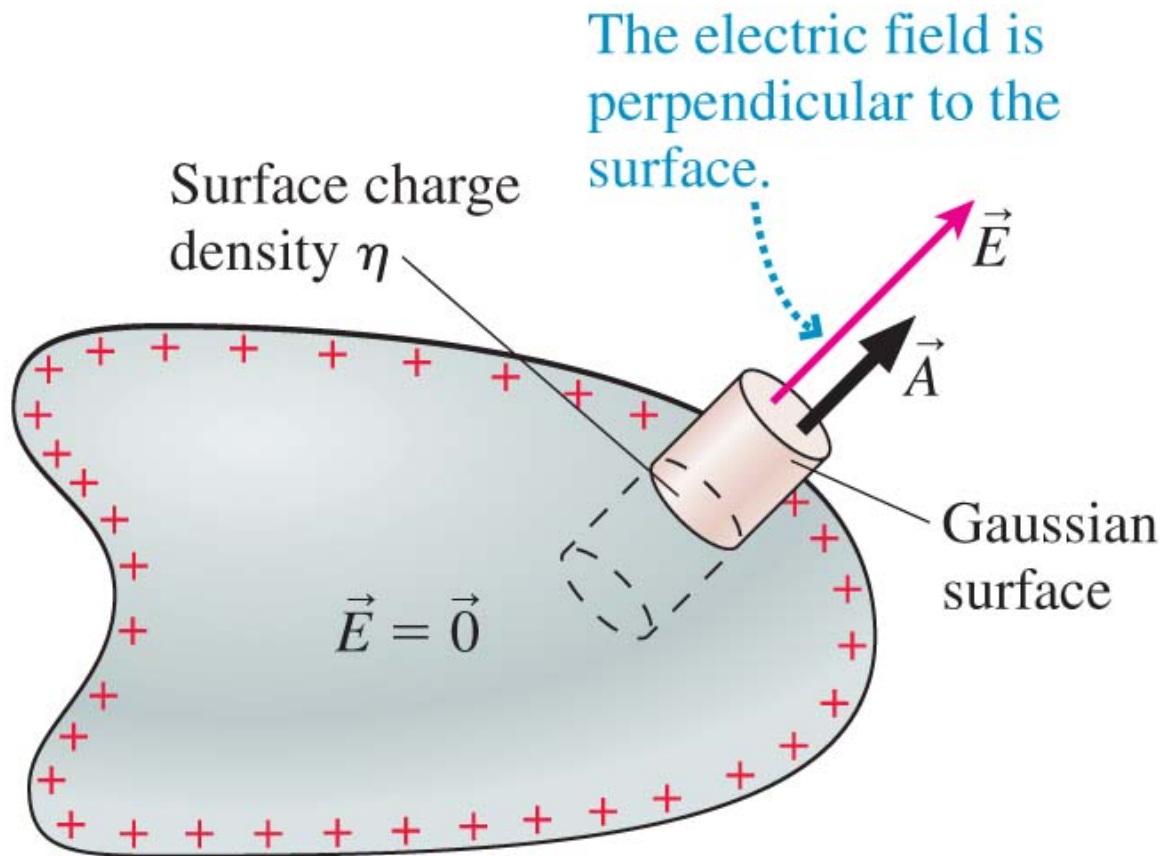
If this weren't true, the electric field would cause the charge carriers to move and thus violate the assumption that all the charges are at rest.

The electric field at the surface of a charge carrier is

$$\vec{E}_{\text{surface}} = \left( \frac{\eta}{\epsilon_0}, \text{perpendicular to surface} \right)$$

where  $\eta$  is the surface charge density of the conductor.

**FIGURE 28.30** A Gaussian surface extending through the surface of the conductor has a flux only through the outer face.



# Tactics: Finding the electric field of a conductor in electrostatic equilibrium

## **TACTICS** Finding the electric field of a conductor in **BOX 28.3** electrostatic equilibrium



- ❶ The electric field is zero at all points within the volume of the conductor.
- ❷ Any excess charge resides entirely on the *exterior* surface.
- ❸ The external electric field at the surface of a charged conductor is perpendicular to the surface and of magnitude  $\eta/\epsilon_0$ , where  $\eta$  is the surface charge density at that point.
- ❹ The electric field is zero inside any hole within a conductor unless there is a charge in the hole.

Exercises 20–24



## EXAMPLE 28.7 The electric field at the surface of a charged metal sphere

### QUESTION:

#### EXAMPLE 28.7 The electric field at the surface of a charged metal sphere

A 2.0-cm-diameter brass sphere has been given a charge of 2.0 nC. What is the electric field strength at the surface?

## EXAMPLE 28.7 The electric field at the surface of a charged metal sphere

**MODEL** Brass is a conductor. The excess charge resides on the surface.

## EXAMPLE 28.7 The electric field at the surface of a charged metal sphere

**VISUALIZE** The charge distribution has spherical symmetry. The electric field points radially outward from the surface.



## EXAMPLE 28.7 The electric field at the surface of a charged metal sphere

**SOLVE** We can solve this problem in two ways. One uses the fact that a sphere is the one shape for which any excess charge will spread out to a *uniform* surface charge density. Thus

$$\eta = \frac{q}{A_{\text{sphere}}} = \frac{q}{4\pi R^2} = \frac{2.0 \times 10^{-9} \text{ C}}{4\pi (0.010 \text{ m})^2} = 1.59 \times 10^{-6} \text{ C/m}^2$$

From Equation 28.20, we know the electric field at the surface has strength

$$E_{\text{surface}} = \frac{\eta}{\epsilon_0} = \frac{1.59 \times 10^{-6} \text{ C/m}^2}{8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = 1.8 \times 10^5 \text{ N/C}$$

## EXAMPLE 28.7 The electric field at the surface of a charged metal sphere

Alternatively, we could have used the result, obtained earlier in the chapter, that the electric field strength outside a sphere of charge  $Q$  is  $E_{\text{outside}} = Q_{\text{in}}/(4\pi\epsilon_0 r^2)$ . But  $Q_{\text{in}} = q$  and, at the surface,  $r = R$ . Thus

$$\begin{aligned} E_{\text{surface}} &= \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = (9.0 \times 10^9 \text{ N m}^2/\text{C}^2) \frac{2.0 \times 10^{-9} \text{ C}}{(0.010 \text{ m})^2} \\ &= 1.8 \times 10^5 \text{ N/C} \end{aligned}$$

As we can see, both methods lead to the same result.

## **Chapter 28. Summary Slides**

# General Principles

## Gauss's Law

For any *closed* surface enclosing net charge  $Q_{\text{in}}$ , the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

The electric flux  $\Phi_e$  is the same for *any* closed surface enclosing charge  $Q_{\text{in}}$ .

# General Principles

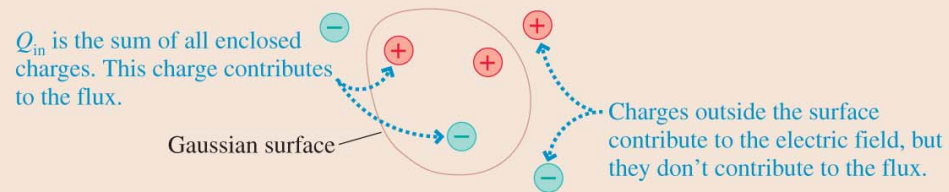
## Symmetry

The symmetry of the electric field must match the symmetry of the charge distribution.

In practice,  $\Phi_e$  is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.

# Important Concepts

**Charge** creates the electric field that is responsible for the electric flux.

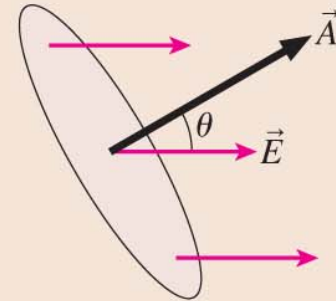


# Important Concepts

**Flux** is the amount of electric field passing through a surface of area  $A$ :

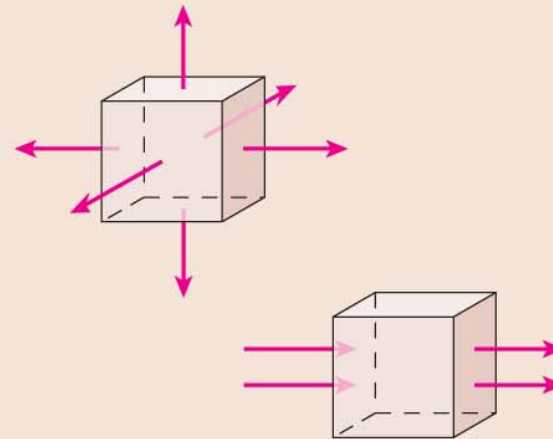
$$\Phi_e = \vec{E} \cdot \vec{A}$$

where  $\vec{A}$  is the **area vector**.



## For closed surfaces:

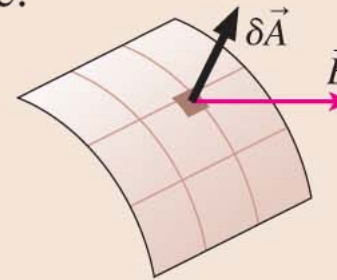
A net flux in or out indicates that the surface encloses a net charge. Field lines through but with no *net* flux mean that the surface encloses no *net* charge.



# Important Concepts

**Surface integrals** calculate the flux by summing the fluxes through many small pieces of the surface:

$$\Phi_e = \sum \vec{E} \cdot \delta \vec{A}$$
$$\rightarrow \int \vec{E} \cdot d\vec{A}$$



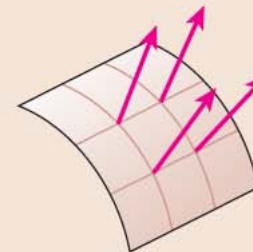
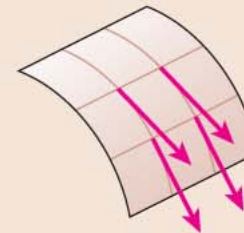
## Two important situations:

If the electric field is everywhere tangent to the surface, then

$$\Phi_e = 0$$

If the electric field is everywhere perpendicular to the surface *and* has the same strength  $E$  at all points, then

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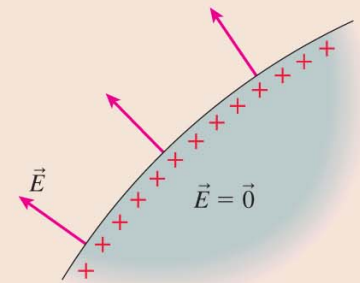




# Applications

## Conductors in electrostatic equilibrium

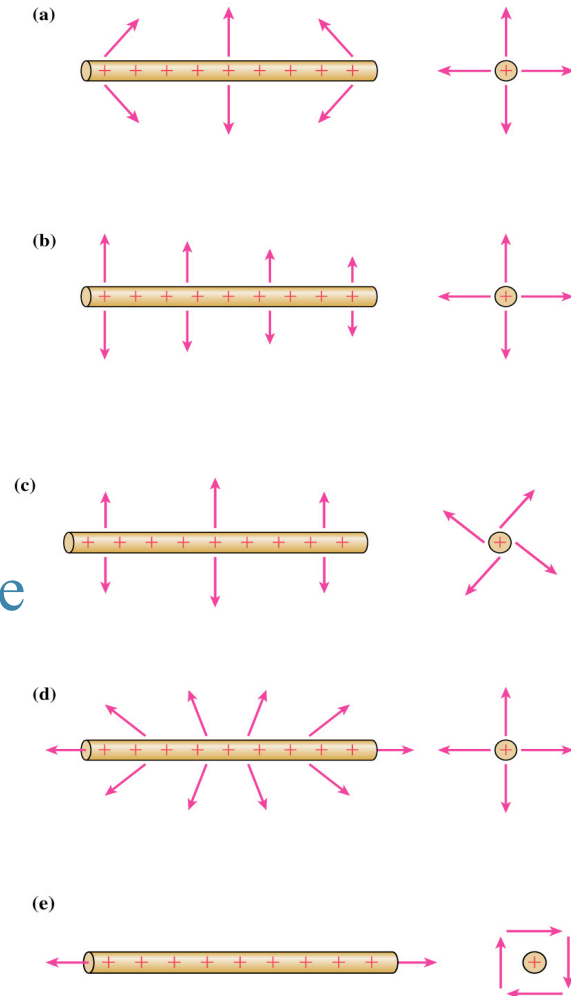
- The electric field is zero at all points within the conductor.
- Any excess charge resides entirely on the exterior surface.
- The external electric field is perpendicular to the surface and of magnitude  $\eta/\epsilon_0$ , where  $\eta$  is the surface charge density.
- The electric field is zero inside any hole within a conductor unless there is a charge in the hole.



## **Chapter 28. Clicker Questions**

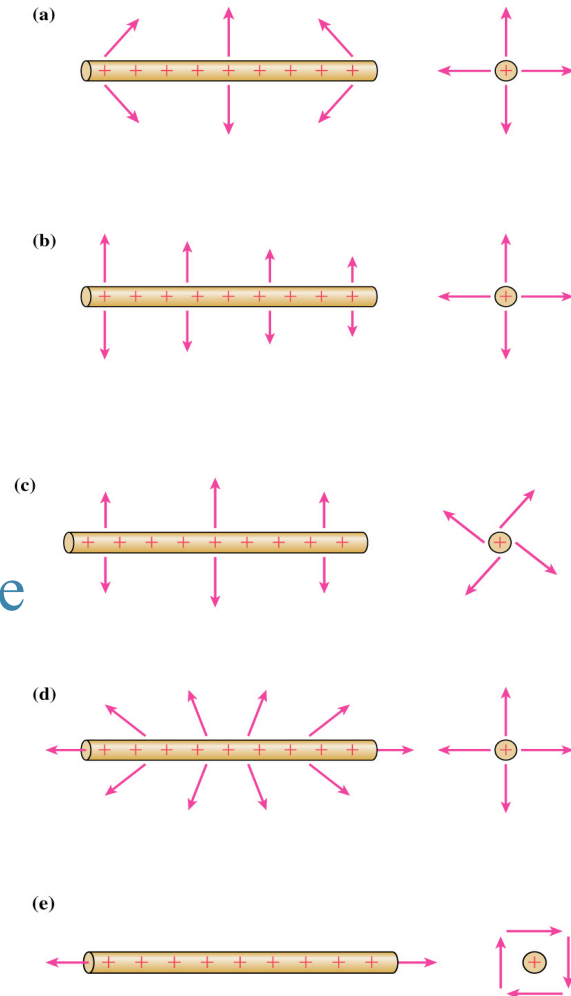
A uniformly charged rod has a *finite* length  $L$ . The rod is symmetric under rotations about the axis and under reflection in any plane containing the axis. It is *not* symmetric under translations or under reflections in a plane perpendicular to the axis other than the plane that bisects the rod. Which field shape or shapes match the symmetry of the rod?

- A. c and e
- B. a and d
- C. e only
- D. b only
- E. none of the above



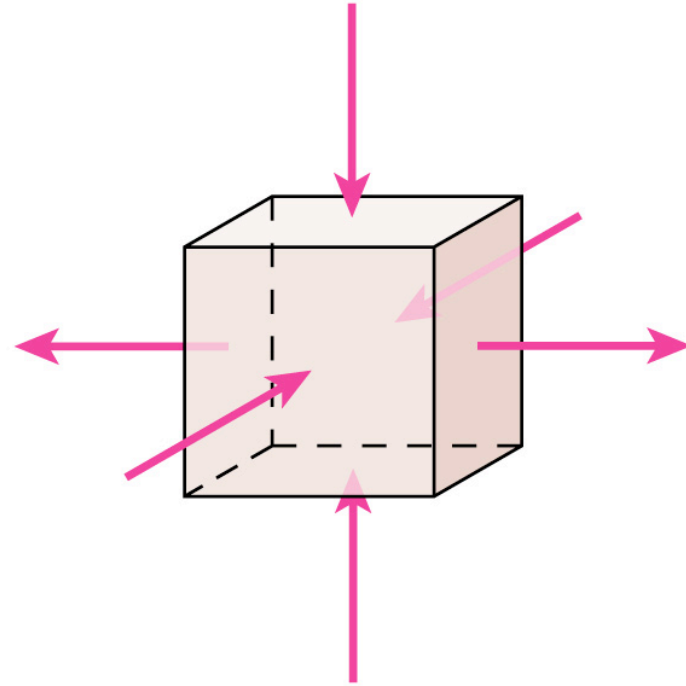
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- ✓ A. c and e
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- D. b only
- E. none of the above



**This box contains**

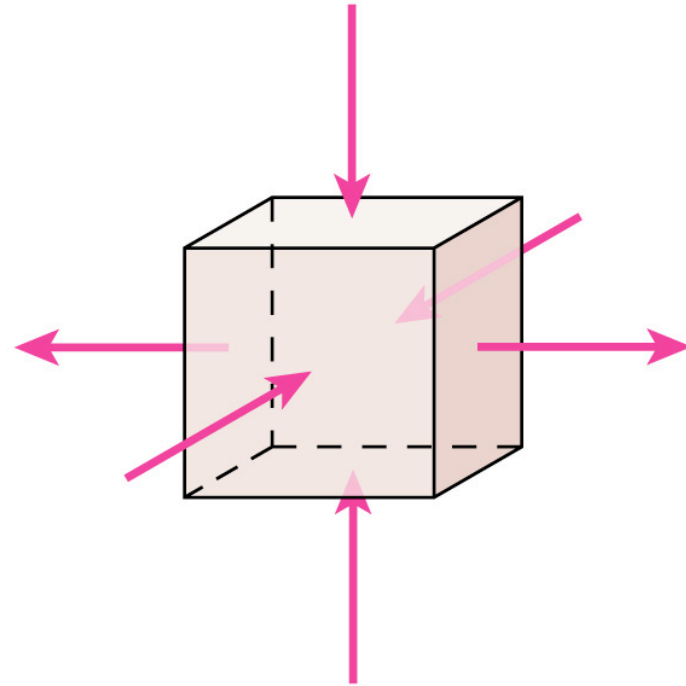
- A. a net positive charge.
- B. a net negative charge.
- C. a negative charge.
- D. a positive charge.
- E. no net charge.



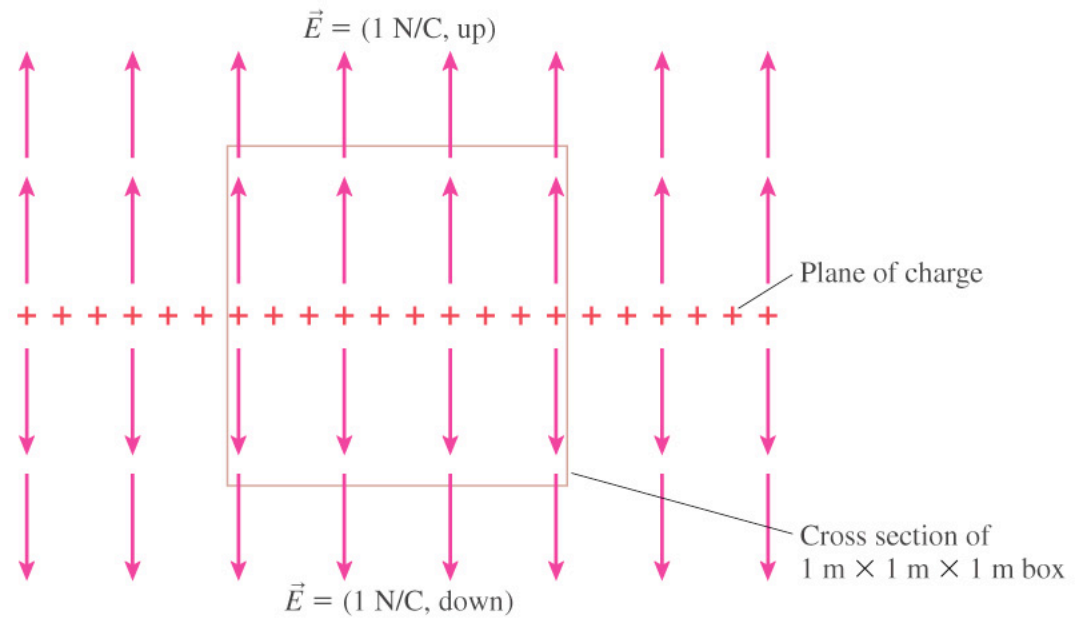
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**This box contains**

- A. a net positive charge.
- ✓ **B. a net negative charge.**
- C. a negative charge.
- D. a positive charge.
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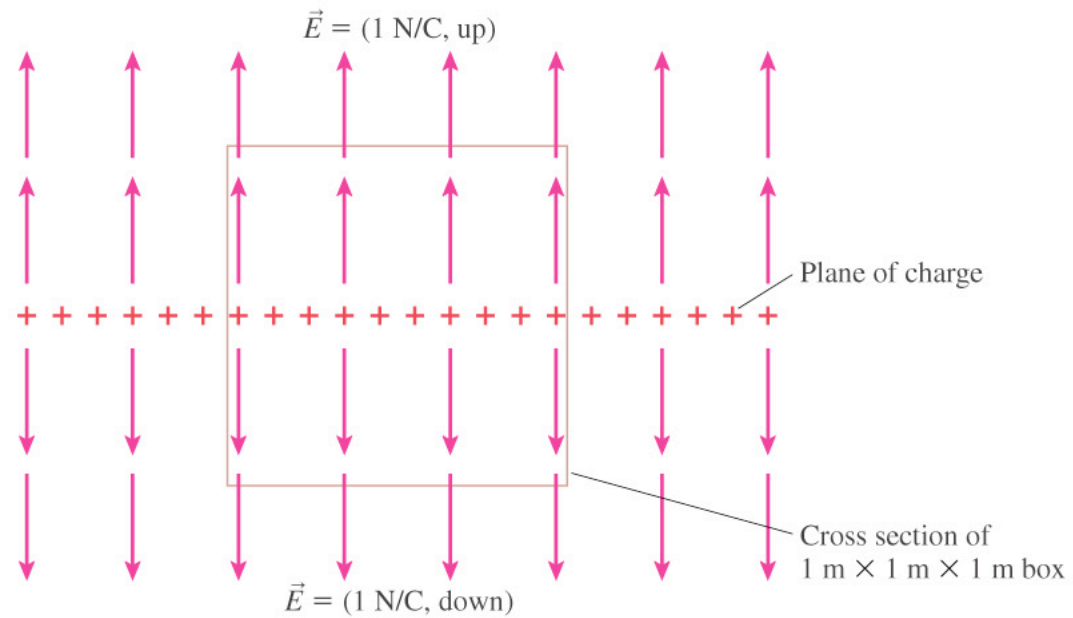


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**The total electric flux through this box is**

- A.  $6 \text{ Nm}^2/\text{C}$ .
- B.  $4 \text{ Nm}^2/\text{C}$ .
- C.  $2 \text{ Nm}^2/\text{C}$ .
- D.  $1 \text{ Nm}^2/\text{C}$ .
- E.  $0 \text{ Nm}^2/\text{C}$ .

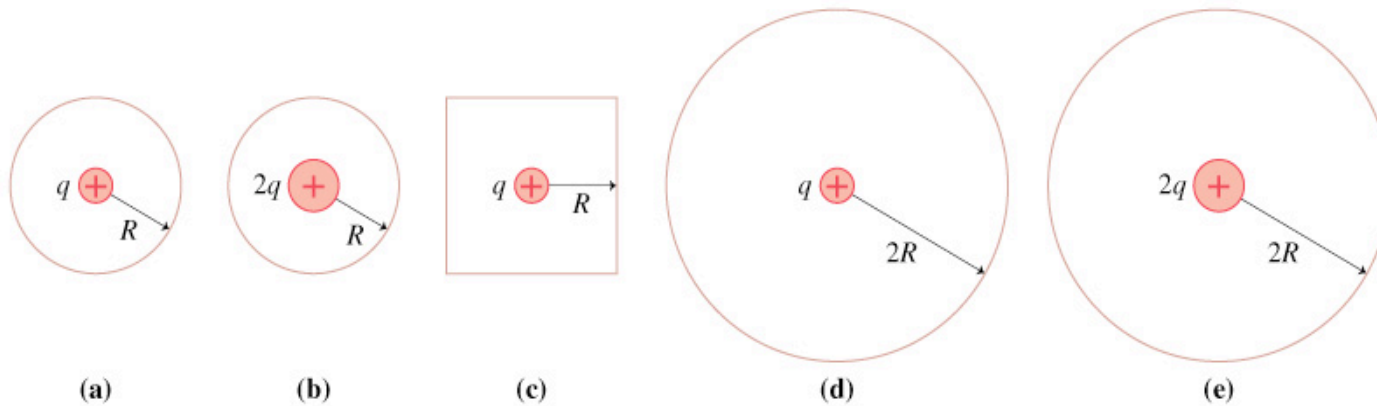


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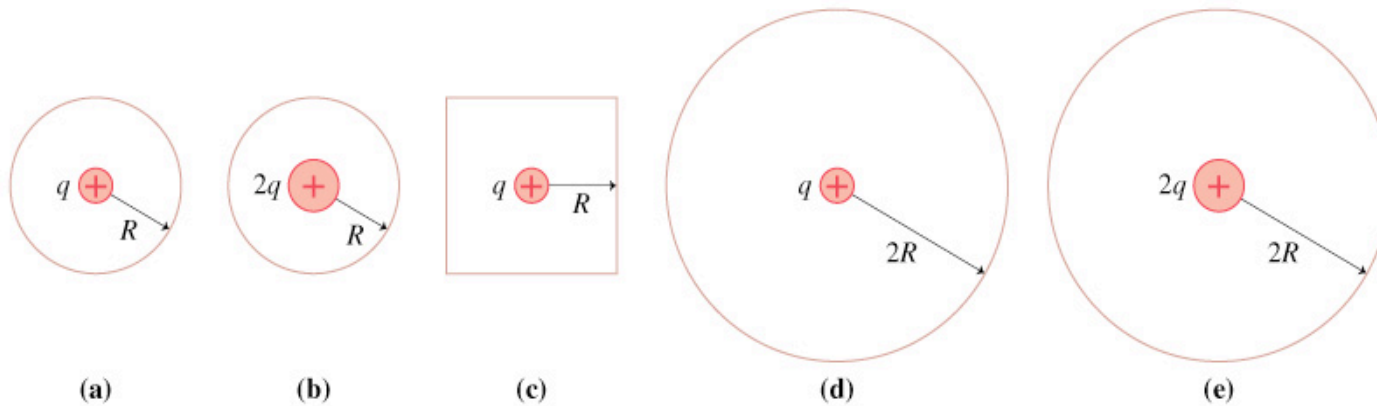


These are two-dimensional cross sections through three-dimensional closed spheres and a cube. Rank order, from largest to smallest, the electric fluxes  $\Phi_a$  to  $\Phi_e$  through surfaces a to e.



- A.  $\Phi_a > \Phi_c > \Phi_b > \Phi_d > \Phi_e$
- B.  $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$
- C.  $\Phi_e > \Phi_d > \Phi_b > \Phi_c > \Phi_a$
- D.  $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$
- E.  $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$

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 ✓ B.  $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$   
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 D.  $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$   
 E.  $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$

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