York University

Faculty of Science and Engineering

Math 1505

Class Test 2 Version B

NAME (print):______________________________
(Family) (Given)

SIGNATURE:______________________________

STUDENT NUMBER:______________________________

SOLUTION

Instructions:

1. Time allowed: 50 minutes.
2. **NO CALCULATORS OR OTHER AIDS PERMITTED**
3. Show your work. Your work must justify any answers you give. Use page backs for any scrap work.
4. Use pen to fill in cover. If you use pencil for your solutions, you may not submit your paper for regrading.
5. There are 4 questions on 8 pages.

Question	Points	Marks
1	15	
2	5	
3	25	
4	5	
Total	50	

1. (15 points) Let $f(x) = \dfrac{x-4}{x^2}$, $f'(x) = \dfrac{8-x}{x^3}$, $f''(x) = \dfrac{2x-24}{x^4}$.

(a) Find all critical points of f.

$f'(x) = 0$ when $x = 8$

Critical point is $\left(8, \frac{1}{16}\right)$

(b) Find where f is increasing and where f is decreasing.

$f''(x)$	dec	undef	incr	rel max	decr
$f'(x)$	$-$		$+$		$-$
x		0		8	

incr $(0, 8)$

dec $(-\infty, 0)$ and $(8, \infty)$

(c) Determine any relative maxima and minima of f.

~~Relative max~~ rel max at

$\left(8, \frac{1}{16}\right)$

(d) Find where f is concave up and where f is concave down.

$f''(x) = 0$ when $2x - 24 = 0$, $x = 12$

$f(x)$	CD	undef	CD		CU
$f''(x)$	$-$		$-$	0	$+$
x		0		12	

(e) Find any points of inflection.

$\left(12, \frac{8}{144}\right)$ or $\left(12, \frac{1}{18}\right)$

point of inflection

(f) Find any horizontal asymptotes.

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

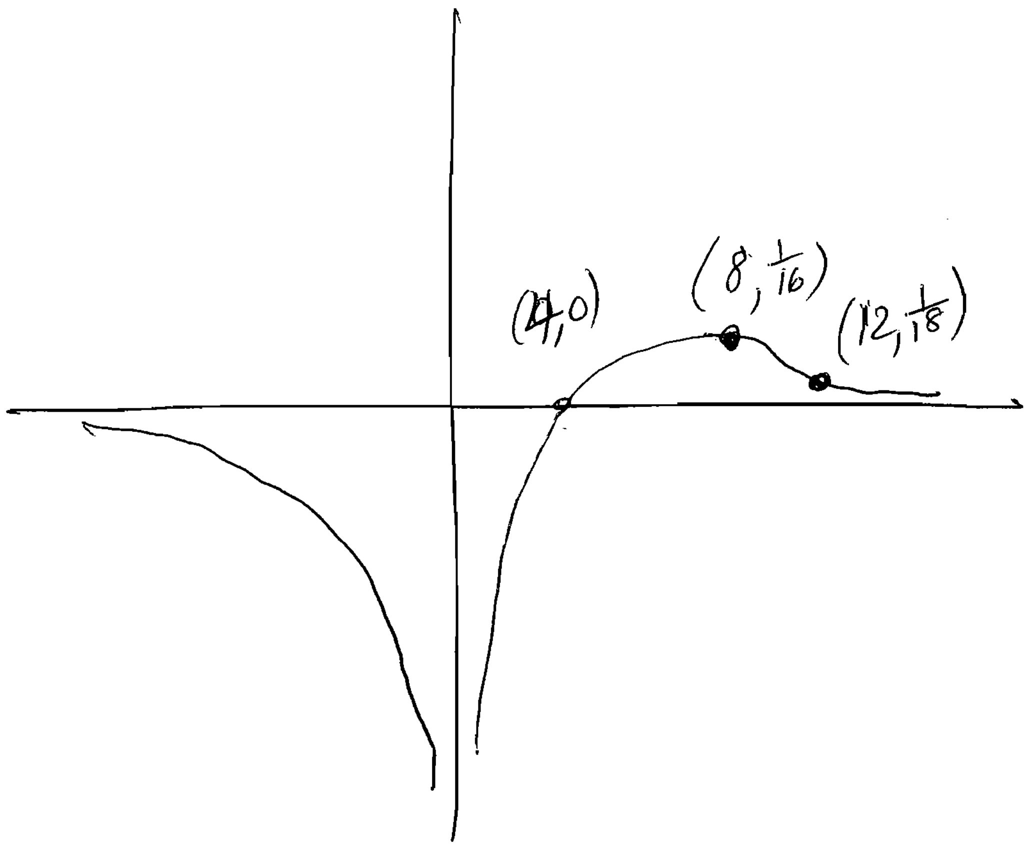
HA at $y = 0$

(g) Find any vertical asymptotes.

VA is $y = 0$

$$\left(\begin{array}{l} \lim_{x \to 0^+} f(x) = -\infty \\ \lim_{x \to 0^-} f(x) = -\infty \end{array} \right)$$

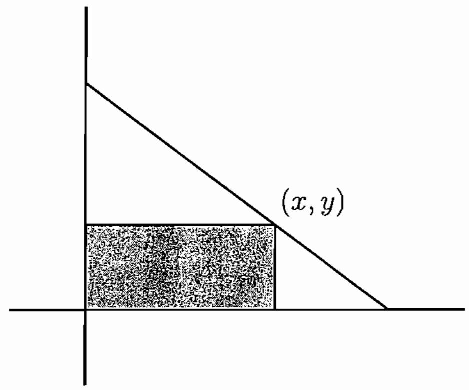
(h) Graph f, labeling the intercepts, relative extrema and points of inflection.



(i) Does f have an absolute maximum or absolute minimum on $\mathbb{R}$? Determine any absolute maximum or absolute minimum of f.

absolute max is $\frac{1}{16}$ at ~~at~~ $x=8$,

2. (5 points)



$4y = 12 - 3x$
$y = 3 - \frac{3}{4}x$

Find the point (x, y) on the line $3x + 4y = 12$ making the area of the shaded rectangle a maximum.

$A(x) = x(3 - \frac{3}{4}x) = 3x - \frac{3}{4}x^2$

$A'(x) = 3 - \frac{3}{2}x$

$A'(x) = 0$ when $\frac{3}{2}x = 3$, $x = 2$

When $x = 2$, $y = 3 - \frac{3}{2} = \frac{3}{2}$

$(x, y) = (2, \frac{3}{2})$

~~2~~ 3. (~~20~~ 25 points) Calculate

(a)

$$\int \frac{2e^{2s}}{e^{2s}+3}\,ds\,.$$

$$u = e^{2s}+3$$
$$du = 2e^{2s}\,ds$$

$$\int \frac{2e^{2s}}{e^{2s}+3}\,ds = \ln|e^{2s}+3| + C$$

(b)

$$\int_{-1}^{0} \frac{t}{(1+t^2)^4}\,dt \quad = \frac{1}{2}\int_{-1}^{0}\frac{2t\,dt}{(1+t^2)^4}$$

$$u = 1+t^2$$
$$du = 2t\,dt$$

x	-1	0
u	2	1

$$= \frac{1}{2}\int_{2}^{1}\frac{2t\,dt}{(1+t^2)^4} = \frac{1}{2}\int_{2}^{1} u^{-4}\,du$$

$$= \frac{1}{2}\,\frac{u^{-3}}{-3}\Bigg|_2^1 = -\frac{1}{6}\left(\frac{1}{u^3}\right)\Bigg|_2^1 = -\frac{1}{6}\left(1-\frac{1}{8}\right) = -\frac{7}{48}$$

$$= \cancel{\frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{}}$$

(c)

$$\int \frac{x+2}{x^2+1}\,dx\,.$$

$$x+2 = \frac{1}{2}(2x)+2$$

$$\frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{2}{x^2+1}\,dx$$

$$= \frac{1}{2}\ln|x^2+1| + 2\arctan(x) + C$$

(d)

$$\int \sqrt{z}\,\ln(z)\,dz\,.$$

$$u = \ln(z) \qquad dv = \sqrt{z}\,dz$$

$$du = \frac{1}{z}\,dz \qquad v = \frac{2}{3}z^{\frac{3}{2}}$$

$$\frac{2}{3}z^{\frac{3}{2}}\ln(z) - \frac{2}{3}\int z^{1/2}\,dz$$

$$= \frac{2}{3}z^{\frac{3}{2}}\ln(z) - \frac{2}{3}\,\frac{z^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$$

(e)

$$\int \frac{4x-5}{(x-2)(x-1)}\,dx\,.$$

$$\frac{4x-5}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{\cancel{B}(x-1)}$$

$$\cancel{A}$$

$$A(x-1) + B(x-2) = 4x-5$$

$$A + B = 4$$

$$-A - 2B = -5$$

$$B = 1$$

$$A = 3$$

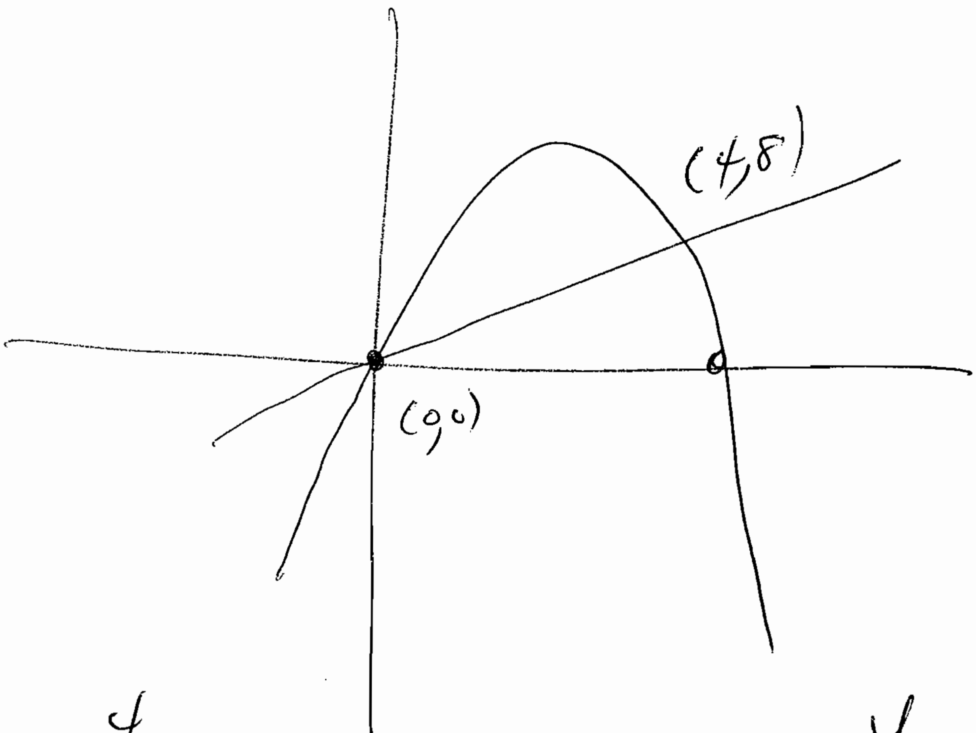
$$= \int \frac{3}{x-2} + \frac{1}{x-1}\,dx$$

$$= 3\ln|x-2| + \ln|x-1| + C$$

4. (5 points) Find the area of the region bounded by the graphs of $f(x) = 6x - x^2$ and $g(x) = 2x$.

$2x = 6x - x^2$

$x^2 - 4x = 0$



$$\int_0^4 \left((6x - x^2) - 2x\right) dx = \int_0^4 (4x - x^2)\, dx$$

$$= 2x^2 - \frac{x^3}{3}\Big|_0^4 = 32 - \frac{64}{3}$$

$$= \frac{32}{3}$$