

MAT 2377 3X (Spring 2011)

Assignment 2

Deadline : Thursday, June 2 in class.

Note : You must give complete details in your solutions. To receive points for the question, you must clearly justify your final answer.

Please answer the following 6 questions.

1. Let X be a discrete random variable with the following probability mass function :

$$f(x) = (8/15)(1/2)^x \quad x = 0, 1, 2, 3.$$

- (a) Does f satisfy the conditions to be a probability mass function ?
 - (b) Compute $P(X \leq 1)$.
 - (c) Compute $P(2 \leq X < 4)$.
 - (d) Compute $P(X \leq 1 \text{ or } X > 2)$.
 - (e) Compute the mean of X and the standard deviation of X .
 - (f) Give the cumulative distribution function for X .
2. Sample of 15 parts from a metal punching process are selected regularly. Typically, 1% of the parts require rework. Let X be the number of parts in the sample of 15 that need rework. A process problem is suspected if X exceeds its mean by more than three standard deviations.
- (a) What is the probability that X exceeds its mean by more than three standard deviations ?
 - (b) How many samples of size 15 should we expect to select in order to observe a sample for which X exceeds its mean by more than three standard deviations ?
3. Printed circuit cards are placed in a functional test after being populated with semiconductor chips. A lot contains 50 cards, and 10 are selected without replacement for functional testing. If 5 cards among the 50 cards are defective, what is the probability that at least 1 defective card is in the sample ?
4. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. The receiver of the message uses "majority" decoding. By majority decoding we mean that the message is decoded as "0" if there are at least 3 zeroes in the message and as "1" otherwise.

- (a) Among the five transmitted bits, what is the expected number of correctly transmitted bits?
 - (b) What is the probability that the message is decoded properly?
5. The surface flaws in plastic panels used in the interior of automobiles can be modeled as a Poisson process with a rate of 0.045 flaws per square foot of plastic panel. Assume an automobile interior contains 10 square feet of plastic panel.
- (a) What is the probability that there are no surface flaws in an automobile's interior?
 - (b) If 10 cars are sold to a rental company, what is the probability that at most one of the 10 cars has any surface flaws?
6. An automated egg carton loader has a 1% probability of cracking an egg, and a customer will complain if more than one egg per dozen is cracked. Assume each egg load is an independent event.
- (a) What is the distribution of cracked eggs per dozen? Include parameter values.
 - (b) What is the probability that a carton of a dozen eggs results in more than one cracked egg?
 - (c) Approximate the probability that among 100 dozen of eggs (i.e. 1200 eggs), there will be at most 1 cracked egg in total.
 - (d) What is the probability that we need more than 100 loaded eggs to observe a cracked egg?