MAT 136
Exam Review Seminar
Course Booklet 1
Solutions

① $\displaystyle\sum_{n=1}^{\infty} 3^{-n} + \frac{1}{n^{3/2}} = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

geometric, $r < 1$, converges

p series, $p > 1$, converges

$\therefore$ the series converges (basic comparison)

② $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{2n} + \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^n + \frac{1}{n^{\frac{1}{2}}}$

geometric, $r < 1$, converges

p- series, $p < 1$, diverges

$\therefore$ the series diverges (basic comparison)

③ $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{\ln(n+1)}} > \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{\ln(n+1)}}$: diverges (Basic Comparison)

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$: p-series, $p < 1$, diverges

④ $$\sum_{n=1}^{\infty} \tan \frac{1}{n}$$ ← behaves like $\frac{1}{n}$ when n is large (and $\sum \frac{1}{n}$ diverges)

$$\lim_{n\to\infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \underbrace{\frac{\sin \frac{1}{n}}{\frac{1}{n}}}_{\to 1} \cdot \underbrace{\frac{1}{\cos \frac{1}{n}}}_{\to 1} = 1$$

∴ series diverges (Limit Comparison Test)

⑤ $$\sum_{n=1}^{\infty} \frac{3 - \cos n}{n^{3/2} - 2} < \sum \frac{4}{n^{3/2} - 2}$$ ← behaves like $\frac{1}{n^{3/2}}$ which converges

$$\lim_{n\to\infty} \frac{\frac{4}{n^{3/2}-2}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{4n^{3/2}}{n^{3/2}-2} \frac{\frac{1}{n^{3/2}}}{\frac{1}{n^{3/2}}} = 4$$

∴ series converges (Limit Comparison Test)

⑥ $\sum_{n=1}^{\infty} \frac{n}{n^3+5}$ ← looks like $\frac{1}{n^2}$ when n is large ← converges

$$\lim_{n\to\infty} \frac{\frac{n}{n^3+5}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3}{n^3+5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = 1$$

∴ series converges (Limit Comparison Test)

⑦ $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ ← easy to integrate

$$\int_2^{\infty} \frac{1}{x \ln x}\,dx = \lim_{M\to\infty} \int_2^M \frac{1}{x \ln x}\,dx$$

$$= \lim_{M\to\infty} \left(\ln \ln x\right)\Big]_2^M$$

$$= \lim_{M\to\infty} \left(\ln \ln M - \ln \ln 2\right) = \infty$$

∴ series diverges (Integral Comparison)

⑧ $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ ← easy to integrate

$$\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx = \lim_{M\to\infty} \int_2^M \frac{1}{x\sqrt{\ln x}}\,dx$$

$$= \lim_{M\to\infty} \left(2\sqrt{\ln x}\right)\Big]_2^M$$

$$= \lim_{M\to\infty} 2\sqrt{\ln M} - 2\sqrt{\ln 2} = \infty$$

∴ series diverges (Integral Comparison)

(9) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ ← use ratio test for factorials

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{n!} \cdot \frac{n^n}{(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} (n+1) \frac{n^n}{(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = L$$

$$\lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right)^n = \ln L$$

$$\lim_{n\to\infty} n \ln\left(\frac{n}{n+1}\right) = \ln L$$

use L'Hopital's Rule →

$$\lim_{n\to\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}} = \ln L$$

$$\lim_{n\to\infty} \frac{\frac{1}{n} - \frac{1}{n+1}}{-\frac{1}{n^2}} = \ln L$$

$$\lim_{n\to\infty} -\frac{n^2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \ln L$$

$$-1 = \ln L$$

$$L = \frac{1}{e} < 1$$

∴ series is absolutely convergent

(10) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ ← use ratio test for factorial

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{((n+1)!)^2}{(2(n+1))!}}{\frac{(n!)^2}{(2n)!}}$$

$$= \lim_{n\to\infty} \left(\frac{(n+1)!}{n!}\right)^2 \left(\frac{(2n)!}{(2n+2)!}\right)$$

$$= \lim_{n\to\infty} (n+1)^2 \left(\frac{1}{(2n+1)(2n+2)}\right)$$

$$= \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2+5n+2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \frac{1}{4} < 1$$

$\therefore$ Series converges absolutely (Ratio Test)

(11) $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(3x-2)^{n+1}}{(n+1)3^{n+1}}}{\frac{(3x-2)^n}{n\,3^n}}$$

$$= \lim_{n\to\infty} \frac{(3x-2)^{n+1}}{(3x-2)^n} \cdot \frac{n}{(n+1)} \cdot \frac{3^n}{3^{n+1}}$$

$$= \frac{3x-2}{3}$$

For convergence
$$-1 < \frac{3x-2}{3} < 1$$
$$-3 < 3x-2 < 3$$
$$-1 < 3x < 5$$
$$-\frac{1}{3} < x < \frac{5}{3}$$

at $x = \frac{5}{3}$ the series is $\sum \frac{1}{n}$ diverges

at $x = -\frac{1}{3}$ the series is $\sum (-1)^n \frac{1}{n}$ converges

$\therefore$ interval of convergence $\left[-\frac{1}{3}, \frac{5}{3}\right)$

(12) $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+1}}\left(\frac{x-1}{2}\right)^{n+1}}{\frac{1}{\sqrt{n}}\left(\frac{x-1}{2}\right)^{n}}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{\frac{1}{2}} \left(\frac{x-1}{2}\right) = \frac{x-1}{2}$$

For convergence

$$-1 < \frac{x-1}{2} < 1$$
$$-2 < x-1 < 2$$
$$-1 < x < 3$$

at $x=3$ the series is $\sum \frac{1}{\sqrt{n}}$ diverges

at $x=-1$ the series is $\sum (-1)^n \frac{1}{\sqrt{n}}$ converges

$\therefore$ the interval of convergence is $[-1, 3)$

(13) $f'(x) = -\frac{\sin x}{\cos x} = -\tan x$ $\quad f''(x) = -\sec^2 x$

$f^{(3)}(x) = -2\sec^2 x \tan x$

$f^{(4)}(x) = -4\sec^2 x \tan^2 x - 2\sec^4 x$ $\quad f^{(4)}(0) = -2$

$\therefore$ coefficient of x^4 is $\frac{-2}{4!} = \frac{-1}{12}$

(14) $f'(x) = e^x \sin x + e^x \cos x$ $\quad f''(x) = 2e^x \cos x$

$f^{(3)}(x) = 2e^x \cos x - 2e^x \sin x$ $\quad f^{(3)}(0) = 2$

$\therefore$ coefficient of x^3 is $\frac{2}{3!} = \frac{1}{3}$

(15) $f'(x) = e^{\sin x} \cos x$ $\qquad f''(x) = e^{\sin x}\cos^2 x - e^{\sin x}\sin x$

$f^{(3)}(x) = e^{\sin x}(\cos^3 x - 3\sin x \cos x - \cos x)$

$f^{(4)}(x) = e^{\sin x}(\cos^4 x - 3\sin x\cos^2 x - \cos^2 x - 3\cos^2 x\sin x - 3\cos^2 x + 3\sin^2 x + \sin x)$

$f^{(4)}(0) = -3$

$\therefore$ coefficient of x^4 is $\frac{-3}{4!} = -\frac{1}{8}$

(16) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ ← easy to integrate

$$\lim_{M\to\infty} \int_2^M \frac{1}{x(\ln x)^2}\,dx = \lim_{M\to\infty} \left(-\frac{1}{\ln x}\right)\Big]_2^M$$

$$= \lim_{M\to\infty}\left(\frac{1}{\ln 2} - \frac{1}{\ln M}\right)$$

$$= \frac{1}{\ln 2}$$

$\therefore$ series converges (Integral Test)

(17) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2} > \sum \frac{1}{n^{\frac{1}{2}}}$ $\quad\therefore$ series diverges

we know: $\ln n < n^{\frac{1}{4}}$ ← any other exponent could be used here

so $\frac{1}{\ln n} > \frac{1}{n^{\frac{1}{4}}}$

$$\left(\frac{1}{\ln n}\right)^2 > \left(\frac{1}{n^{1/4}}\right)^2$$

$$\frac{1}{(\ln n)^2} > \frac{1}{n^{\frac{1}{2}}}$$

(18) Repeat of question 16

(19) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)(\ln \ln n)}$ ← easy to integrate

$$\lim_{M\to\infty} \int_2^M \frac{1}{x(\ln x)(\ln\ln x)}\,dx = \lim_{M\to\infty} \left(\ln\ln\ln x\right)\Big]_2^M$$

$$= \lim_{M\to\infty} \ln\ln\ln M - \ln\ln\ln 2$$

$$= \infty$$

(20) $\sum_{n=1}^{\infty} \frac{1}{n \ln(1+n^3)}$ ← looks like $\frac{1}{n \ln n^3} = \frac{1}{3n \ln n}$

when n is large

$$\lim_{n\to\infty} \frac{\frac{1}{n\ln(1+n^3)}}{\frac{1}{n\ln n^3}} = \lim_{n\to\infty} \frac{\cancel{n} \ln n^3}{\cancel{n} \ln(n^3+1)} \quad \text{use L'Hopitals Rule}$$

$$= \lim_{n\to\infty} \frac{3n^2}{n^3} \cdot \frac{n^3+1}{3n^2} = 1$$

Since $\frac{1}{n\ln n}$ diverges (see question 7) this series diverges (~~Basic Compari~~ Limit Comparison Test)

(21) $\sum_{n=2}^{\infty} \frac{1}{n\ln(n^4)} = \frac{1}{4}\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

Since $\frac{1}{n\ln n}$ diverges (see question 7) this series diverges

(22) $\displaystyle\sum_{n=10}^{\infty} \frac{n^2}{2n^3-1999}$ ← Behaves like $\frac{1}{n}$ when n is large

$$\lim_{n\to\infty} \frac{\frac{n^2}{2n^3-1999}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3}{2n^3-1999} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \frac{1}{2}$$

$\therefore$ series diverges (Limit Comparison)

(23) $\displaystyle\sum_{n=2}^{\infty} \frac{n+1}{n^{3/2}-2n}$ ← Behaves like $\frac{1}{n^{\frac{1}{2}}}$ when n is large

$$\lim_{n\to\infty} \frac{\frac{n+1}{n^{3/2}-2n}}{\frac{1}{n^{\frac{1}{2}}}} = \lim_{n\to\infty} \frac{n^{3/2}+n^{\frac{1}{2}}}{n^{3/2}+2n} \cdot \frac{\frac{1}{n^{3/2}}}{\frac{1}{n^{3/2}}} = 1$$

$\therefore$ series diverges (Limit comparison)

(24) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan\left(\frac{1}{n}\right)}{n}$ ← $\tan\left(\frac{1}{n}\right)$ behaves like $\frac{1}{n}$ so series behaves like $\frac{1}{n^2}$ when n is large

$$\lim_{n\to\infty} \frac{\frac{\tan\left(\frac{1}{n}\right)}{n}}{\frac{1}{n^2}} = \lim_{n\to\infty} n\tan\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} \cdot \frac{1}{\cos\frac{1}{n}} = 1$$

$\therefore$ series converges.

(25) $\sum_{n=1}^{\infty} n \sin\frac{1}{n} = \sum_{n=1}^{\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}}$ ← goes to 1 when n is large

$\therefore$ series diverges (Terms don't approach 0)

(26) $\sum_{n=1}^{\infty} \frac{3^n}{2^n+4^n} < \sum_{n=1}^{\infty} \frac{3^n}{4^n} = \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ ← geometric $r<1$

$\therefore$ series converges (Basic Comparison)

(27) $\sum_{n=1}^{\infty} \arcsin\left(\frac{1}{n}\right) > \sum_{n=1}^{\infty} \frac{1}{n}$

$\therefore$ series diverges (Basic Comparison)

(28) $\sum_{n=1}^{\infty} \frac{1}{n+n\cos n} > \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n}$

$\therefore$ series diverges (Basic Comparison)

(29) $\sum_{n=1}^{\infty} \arctan\left(\frac{n^2-1}{n^2+1}\right)$

$\lim_{n\to\infty} \arctan\left(\frac{n^2-1}{n^2+1}\right) = \arctan(1) = \frac{\pi}{4}$

Terms in the series don't go to 0

$\therefore$ series diverges.

(30) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n - n}{4^n + n^2}$ ← Behaves like $\left(\frac{5}{4}\right)^n$ when n is large

↖ geometric $r > 1$

$$\lim_{n\to\infty} \frac{\frac{5^n - n}{4^n + n^2}}{\frac{5^n}{4^n}} = \lim_{n\to\infty} \frac{(5^n - n)\frac{1}{4^n}}{(4^n + n^2)\frac{1}{5^n}} = 1$$

∴ series diverges.

(31) $\displaystyle\sum_{n=1}^{\infty} \sqrt[3]{\frac{3n^2+1}{n^4 - n^3}}$ ← Behaves like $\sqrt[3]{\frac{3n^2}{n^4}} = \sqrt[3]{3}\left(\frac{1}{n^{2/3}}\right)$

↑ diverges (p-series)

$$\lim_{n\to\infty} \left(\frac{\frac{3n^2+1}{n^4 - n^3}}{\frac{1}{n^2}}\right)^{\frac{1}{3}} = \lim_{n\to\infty} \left(\frac{3n^4 + n^2}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}\right)^{\frac{1}{3}} = \sqrt[3]{3}$$

∴ series diverges

(32) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{4/3} \ln n} > \sum_{n=2}^{\infty} \frac{1}{n^{9/10}}$ ∴ series diverges

We know:

$\ln n < n^{1/10}$ when n is large

$$\frac{1}{\ln n} > \frac{1}{n^{1/10}}$$

$$\frac{1}{n^{4/3} \ln n} > \frac{1}{n^{4/3}\, n^{1/10}} = \frac{1}{n^{9/10}}$$

↑ we wanted a series whose behaviour we know.

(33) $$\sum_{n=1}^{\infty} \frac{\ln n}{n^2} < \sum_{n=1}^{\infty} \frac{n^{\frac{1}{2}}}{n^2} = \sum \frac{1}{n^{3/2}}$$

$\therefore$ series converges (Basic Comparison)

(34) $$\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2} < \sum_{n=1}^{\infty} \frac{e}{n^2}$$

$\therefore$ series converges (Basic Comparison)

(35) $$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{3^n}$$ ← use ratio test for factorials

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{\frac{(n+1)!}{3^{n+1}}}{\frac{n!}{3^n}}$$

$$= \lim_{n\to\infty} \frac{3^n}{3^{n+1}} \cdot \frac{(n+1)!}{n!}$$

$$= \lim_{n\to\infty} \frac{1}{3} (n+1) = \infty$$

$\therefore$ series diverges.

(36) $$\sum_{n=1}^{\infty} (-1)^n \frac{2}{n^{\frac{1}{3}}}$$ ← alternating series where terms go to 0 (converges)

Taking absolute value:

$$\sum_{n=1}^{\infty} \frac{2}{n^{\frac{1}{3}}}$$ ← p series $(p < 1)$ diverges

$\therefore$ series is conditionally convergent

(37) $\sum (-1)^n \frac{5^n}{n!}$ ← use ratio test for factorials

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)!}}{\frac{5^n}{n!}} = \lim_{n\to\infty} \frac{5^{n+1}}{5^n} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty} 5 \cdot \frac{1}{n+1} = 0$$

∴ series converges absolutely

(38) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n-4}$ ← alternating series where the terms go to zero (converges)

Taking absolute values:

$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n-4}$ ← behaves like $\frac{1}{\sqrt{n}}$ which diverges

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n-4}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{n}{n-4} = 1$$

∴ series is conditionally convergent

(39) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n^2)}$ ← alternating series where the terms go to zero (converges)

Taking absolute values:

$$\sum_{n=2}^{\infty} \frac{1}{\ln(n^2)} = \frac{1}{2}\sum_{n=2}^{\infty} \frac{1}{\ln n} > \sum_{n=2}^{\infty} \frac{1}{n}$$ ← diverges

∴ the series is conditionally convergent.

(40) $\sum_{n=1}^{\infty} (-1)^n \arcsin\left(\frac{-1}{n}\right) = \sum (-1)^{n+1} \arcsin\left(\frac{1}{n}\right)$

since arcsin is an ODD function

This is an alternating series whose terms go to zero (converges)

Taking absolute values:

$$\sum_{n=1}^{\infty} \arcsin\left(\frac{1}{n}\right) > \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{diverges}$$

∴ series converges conditionally

(41) $\sum_{n=1}^{\infty} (-1)^n \frac{n-1}{2n^2+n+1}$ ← alternating series whose terms go to zero (converges)

Taking absolute values:

$$\sum_{n=1}^{\infty} \frac{n-1}{2n^2+n+1}$$ ← behaves like $\frac{1}{n}$ (diverges)

$$\lim_{n\to\infty} \frac{\frac{n-1}{2n^2+n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2-n}{2n^2+n+1} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \frac{1}{2}$$

∴ series converges conditionally

(42) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n-2^n}$ ← alternating series whose terms go to zero (converges)

Taking absolute values:

$$\sum_{n=1}^{\infty} \frac{1}{3^n-2^n}$$ ← looks like $\frac{1}{3^n}$ (converges)

$$\lim_{n\to\infty} \frac{\frac{1}{3^n-2^n}}{\frac{1}{3^n}} = \lim_{n\to\infty} \frac{3^n}{3^n-2^n} = \lim_{n\to\infty} \frac{1}{1-\left(\frac{2}{3}\right)^n} = 1$$

∴ series converges absolutely

43) $$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{(x-3)^{n+1}}{\sqrt{n+1+1}}}{\frac{(x-3)^n}{\sqrt{n+1}}}$$

$$= \lim_{n\to\infty}\frac{(x-3)^{n+1}}{(x-3)^n}\cdot\frac{\sqrt{n+1}}{\sqrt{n+2}}$$

$$= x-3$$

For convergence:

$$-1 < x-3 < 1$$

$$2 < x < 4$$

When $x = 2$ The series is:

$\sum(-1)^n\frac{1}{\sqrt{n+1}}$ (converges)

When $x = 4$ The series is:

$\sum\frac{1}{\sqrt{n+1}}$ (diverges)

$\therefore$ Interval of convergence is $[2, 4)$

44) $$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{3^{n+1}x^{n+1}}{3^n x^n} = 3x$$

For convergence:

$$-1 < 3x < 1$$

$$-\tfrac{1}{3} < x < \tfrac{1}{3}$$

when $x = \frac{1}{3}$ the series is:

$\sum(-1)^n$ (doesn't converge)

When $x = -\frac{1}{3}$ the series is

$\sum 1$ (diverges)

$\therefore$ Interval of convergence is $(-\frac{1}{3}, \frac{1}{3})$

(45) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{2^{n+1}}{\sqrt{n+1}}(x+3)^{n+1}}{\frac{2^n}{\sqrt{n}}(x+3)^n}$

$$= \lim_{n\to\infty}\frac{2^{n+1}}{2^n}\cdot\frac{\sqrt{n}}{\sqrt{n+1}}\cdot(x+3)$$

$$= 2(x+3) = 2x+6$$

For convergence:

$-1 < 2x+6 < 1$

$-7 < 2x < -5$

$-\frac{7}{2} < x < -\frac{5}{2}$

When $x = -\frac{5}{2}$ the series is: $\sum(-1)^n\frac{1}{\sqrt{n}}$ (converges)

When $x = -\frac{7}{2}$ the series is: $\sum\frac{1}{\sqrt{n}}$ (diverges)

$\therefore$ Interval of convergence is $\left(-\frac{7}{2}, -\frac{5}{2}\right]$

(46) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{(2x+3)^{n+1}}{(n+1)\ln(n+1)}}{\frac{(2x+3)^n}{n\ln n}}$

$$= \lim_{n\to\infty}\frac{n}{n+1}\cdot\frac{\ln n}{\ln(n+1)}\cdot\frac{(2x+3)^{n+1}}{(2x+3)^n}$$

$$= 2x+3$$

For convergence:

$-1 < 2x+3 < 1$

$-4 < 2x < -2$

$-2 < x < -1$

When $x = -1$ the series is $\sum(-1)^n\frac{1}{n\ln n}$ (converges)

When $x = -2$ the series is $\sum\frac{1}{n\ln n}$ (diverges)

$\therefore$ Interval of convergence is $(-2, -1]$

(47)

$f'(x) = 3e^{3x}\cos 2x - 2e^{3x}\sin 2x$

$f''(x) = 9e^{3x}\cos 2x - 12e^{3x}\sin 2x - 4e^{3x}\cos 2x$

$f^{(3)}(x) = -9e^{3x}\cos 2x - 62\ \sin 2x$

$f^{(3)}(0) = -9$

$\therefore$ coefficient of x^3 is $\frac{-9}{3!} = -\frac{3}{2}$

(48)

$f'(x) = -2\cos x\ \sin x$

$f''(x) = 2\sin^2 x - 2\cos^2 x$

$f^{(3)}(x) = 8\sin x\ \cos x$

$f^{(4)}(x) = 8\cos^2 x - 8\sin^2 x$

$f^{(4)}(0) = 8$

$\therefore$ coefficient of x^4 is $\frac{8}{4!} = \frac{1}{3}$