Carleton University
Department of Electronics

Solutions for the Sample Practice Questions (Set 1)

Note:
To be consistent in the solutions, we will "generally" use the following notations:
***(a)** Lowercase letters to represent the time-domain signals (e.g.):* $v_i = v_i(t)$*,*
***(b)** Bold lowercase letters for the vectors of the time-dependent signals (e.g.):* $\mathbf{b} = \mathbf{b}(t)$ *and* $\mathbf{x} = \mathbf{x}(t)$*,*
***(c)** Uppercase letters to represent the signals in frequency-domain (e.g.):* V_i*,*
***(d)** Bold uppercase letters for the vectors of the frequency-domain signals (e.g.):* $\mathbf{X}$ *and* $\mathbf{b}$*.*

MNA Matrix Equations

Question 5: For the following circuits,

(a) Write the MNA frequency-domain matrix equations in the form $\overbrace{(\mathbf{G} + s\mathbf{C})}^{\mathbf{A}}\mathbf{X} = \mathbf{b}$.

(b) Write the MNA time-domain matrix equations.

Note: The entries that are complex numbers should be shown in "$a + jb$" format.

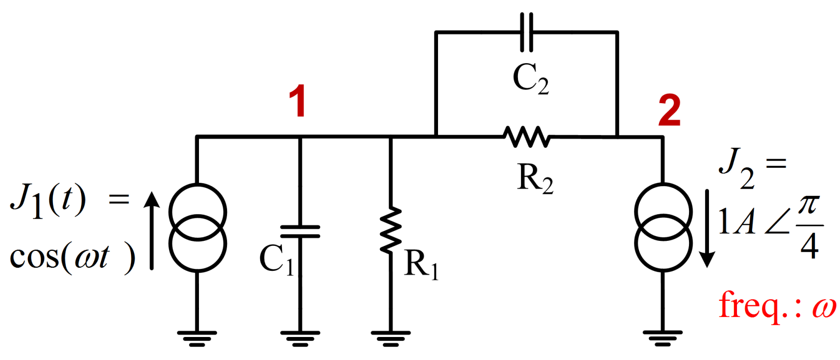

Figure 1: Circuit 5.1

Solution:

$$\mathbf{G} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} C_1 + C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix}$$

frequency-domain:

$$\mathbf{G}\,\mathbf{X} + s\mathbf{C}\,\mathbf{X} = \mathbf{b} \qquad \textbf{(FD-equation)}$$

$$J_1 = cos\,(\omega t + 0^o) \xrightarrow{\mathbf{FD}} \mathbf{J}_1 = 1$$

$$J_2 = 1\angle\frac{\pi}{4} = 1e^{\frac{\pi}{4}j} = cos(\frac{\pi}{4}) + jsin\frac{\pi}{4} = 0.7071 + 0.7071j$$

$$\mathbf{b} = \begin{bmatrix} +J_1 \\ -J_2 \end{bmatrix} = \begin{bmatrix} +1 \\ -0.7071 - 0.7071j \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(FD-equation)** to have the complete form for the frequency-domain MNA equations, as

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + s \begin{bmatrix} C_1 + C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} +1 \\ -0.7071 - 0.7071j \end{bmatrix}$$

time-domain:

$$\mathbf{G}\,\mathbf{x}(t) + \mathbf{C}\,\frac{d}{dt}\mathbf{x}(t) = \mathbf{b}(t) \qquad \textbf{(TD-equation)}$$

$$J_2 = 1\angle\frac{\pi}{4} \xrightarrow{\mathbf{TD}} J_2(t) = cos\left(\omega\, t + \frac{\pi}{4}\right)$$

$$\mathbf{b}(t) = \begin{bmatrix} +J_1(t) \\ -J_2(t) \end{bmatrix} = \begin{bmatrix} +cos\,(\omega\, t) \\ -cos\left(\omega\, t + \frac{\pi}{4}\right) \end{bmatrix},$$

$$\mathbf{x}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}, \text{ and } \qquad \frac{d}{dt}\mathbf{x}(t) = \begin{bmatrix} \frac{dv_1(t}{dt}) \\ \frac{dv_2(t)}{dt} \end{bmatrix}$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(TD-equation)** to have the compelete form for the frequency-domain MNA equations, as

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix} \begin{bmatrix} \frac{d\,v_1(t)}{dt} \\ \frac{d\,v_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} +cos\,(\omega\, t) \\ -cos\left(\omega\, t + \frac{\pi}{4}\right) \end{bmatrix} \qquad \blacksquare$$

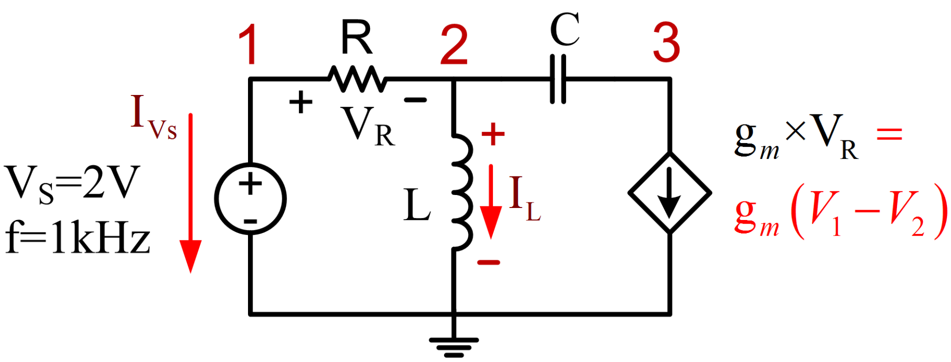

Figure 2: Circuit 5.6

Solution:

$$\mathbf{G} = \begin{bmatrix} \frac{1}{R} & -\frac{1}{R} & 0 & +1 & 0 \\ -\frac{1}{R} & \frac{1}{R} & +1 & 0 & 0 \\ +g_m & -g_m & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & C & -C & 0 & 0 \\ 0 & -C & C & 0 & 0 \\ 0 & 0 & 0 & -L & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

frequency-domain:

$$\mathbf{G}\,\mathbf{X} + s\mathbf{C}\,\mathbf{X} = \mathbf{b} \qquad \textbf{(FD-equation)}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_L \\ I_{Vs} \end{bmatrix}$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(FD-equation)** to have the complete form for the frequency-domain MNA equations. **time-domain:**

$$\mathbf{G}\,\mathbf{x}(t) + \mathbf{C}\,\frac{d}{dt}\mathbf{x}(t) = \mathbf{b}(t) \qquad \textbf{(TD-equation)}$$

$$V_s = 2\angle 0, \quad f = 1kHz \rightarrow \omega = 2\pi f = 2k\pi \quad \xrightarrow{\textbf{TD}} \quad v_s(t) = 2cos\,(\omega\,t)$$

$$\mathbf{b}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_s(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2cos(\omega t) \end{bmatrix},$$

$$\mathbf{x}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ i_L(t) \\ i_{vs}(t) \end{bmatrix}, \qquad \frac{d}{dt}\mathbf{x}(t) = \begin{bmatrix} \frac{d\,v_1(t)}{dt} \\ \frac{d\,v_2(t)}{dt} \\ \frac{d\,v_3(t)}{dt} \\ \frac{d\,i_L(t)}{dt} \\ \frac{d\,i_{vs}(t)}{dt} \end{bmatrix}.$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(TD-equation)** to have the compelete form for the frequency-domain MNA equations. ■

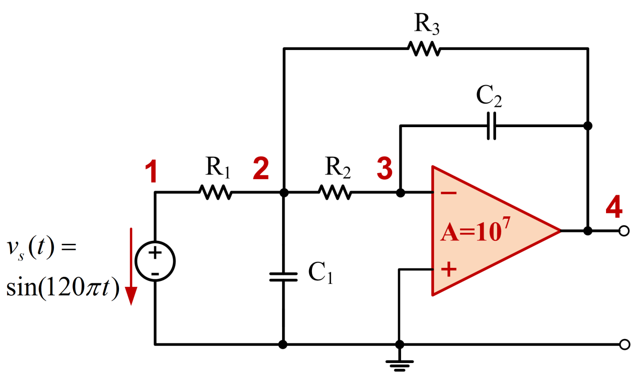

Figure 3: Circuit 5.7

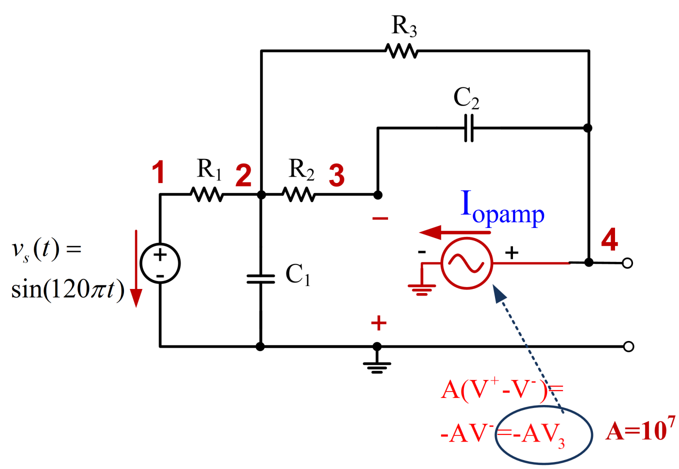

Figure 4: Circuit 5.7

<u>Solution:</u>

In this example, Opamp is modeled as a *Voltage-Control-Voltage-Source (VCVS)*. The *Opamp equation* in the above circuit is as follows:

$V_{out} = A \times (V^+ - V^-)$

$V_4 = A \times (0 - V_3) \quad \Rightarrow \quad \underline{V_4 + AV_3 = 0}$

$$\mathbf{G} = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 0 & 0 & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} & 0 & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & \frac{1}{R_3} & 0 & 1 & 0 \\ 0 & 0 & +10^7 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_2 & -C_2 & 0 & 0 \\ 0 & 0 & -C_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

<u>frequency-domain:</u>

$$\mathbf{G}\,\mathbf{X} \,+\, s\mathbf{C}\,\mathbf{X} \;=\; \mathbf{b} \qquad \textbf{(FD-equation)}$$

$$v_s(t) \;=\; sin\,(120\pi\,t) \;=\; cos\left(120\pi\,t - \frac{\pi}{2}\right) \;\xrightarrow{\textbf{FD}}\; V_s = 1\angle\frac{\pi}{2} \;=\; cos\left(\frac{\pi}{2}\right) + j\,sin\left(\frac{\pi}{2}\right) \;=\; 1j$$

$$\mathbf{b} \;=\; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_s \end{bmatrix} \;=\; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1j \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_{opamp} \\ I_{Vs} \end{bmatrix}$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(FD-equation)** to have the complete form for the frequency-domain MNA equations.

<u>time-domain:</u>

$$\mathbf{G}\,\mathbf{x}(t) \,+\, \mathbf{C}\,\frac{d}{dt}\mathbf{x}(t) \;=\; \mathbf{b}(t) \qquad \textbf{(TD-equation)}$$

$$\mathbf{b}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ v_s(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ sin(120\pi\,t) \end{bmatrix}, \qquad \mathbf{x}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \\ i_{opamp}(t) \\ i_{vs}(t) \end{bmatrix}, \text{ and } \qquad \frac{d}{dt}\mathbf{x}(t) = \begin{bmatrix} \frac{d\,v_1(t)}{dt} \\ \frac{d\,v_2(t)}{dt} \\ \frac{d\,v_3(t)}{dt} \\ \frac{d\,v_4(t)}{dt} \\ \frac{d\,i_{opamp}(t)}{dt} \\ \frac{d\,i_{vs}(t)}{dt} \end{bmatrix}$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(TD-equation)** to have the complete form for the frequency-domain MNA equations. ■

Question 4: For the following Op-amp circuits,

(a) Write the MNA <u>frequency-domain</u> matrix equations in the form $\overbrace{(\mathbf{G} + s\mathbf{C})}^{\mathbf{A}}\,\mathbf{X} = \mathbf{b}$.

(b) Write the MNA <u>time-domain</u> matrix equations.

<u>Solution:</u>

In this example, Ideal-Opamp is modeled as a *Voltage-Control-Voltage-Source (VCVS)* with infinity gain ($A = \infty$).

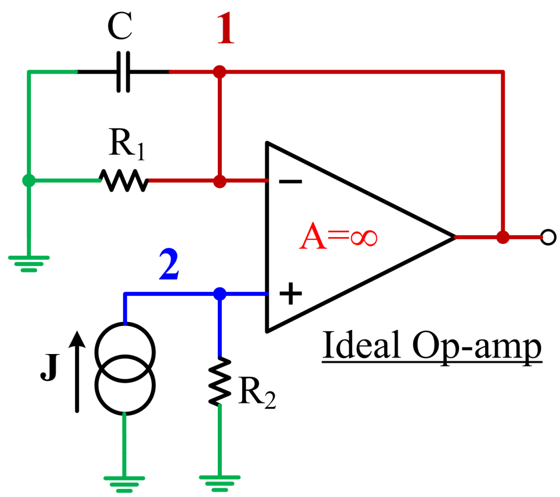

Figure 5: Circuit 4.2

$$\mathbf{G} = \begin{bmatrix} \frac{1}{R_1} & 0 & +1 \\ 0 & \frac{1}{R_2} & 0 \\ +1 & -1 & 0 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

frequency-domain:

$$\mathbf{G}\,\mathbf{X} + s\mathbf{C}\,\mathbf{X} = \mathbf{b} \qquad \textbf{(FD-equation)}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ J \\ 0 \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} V_1 \\ V_2 \\ I_{opamp} \end{bmatrix}$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(FD-equation)** to have the complete form for the frequency-domain MNA equations.

time-domain:

$$\mathbf{G}\,\mathbf{x}(t) + \mathbf{C}\,\frac{d}{dt}\mathbf{x}(t) = \mathbf{b}(t) \qquad \textbf{(TD-equation)}$$

$$\mathbf{b}(t) = \begin{bmatrix} 0 \\ j(t) \\ 0 \end{bmatrix}, \qquad \mathbf{x}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ i_{opamp}(t) \end{bmatrix}, \text{ and } \qquad \frac{d}{dt}\mathbf{x}(t) = \begin{bmatrix} \frac{d\,v_1(t)}{dt} \\ \frac{d\,v_2(t)}{dt} \\ \frac{d\,i_{opamp}(t)}{dt} \end{bmatrix}$$

Now, the above matrices $\mathbf{G}$ and $\mathbf{C}$ and the vectors $\mathbf{X}$ and $\mathbf{b}$ are substituting in the **(TD-equation)** to have the complete form for the frequency-domain MNA equations. ■