University of Waterloo
Department of Electrical and Computer Engineering

ECE 207 – Signals and Systems (Section 001)

Midterm Exam, Fall 2013

Monday, October 21, 2013, 8:30-9:50 AM

Instructor: *Mohamed-Yahia Dabbagh*

Student's Name: ____

ID Number: ______________________

Instructions:

- This exam has **3** pages, including this one, and **6** questions.
- **Closed book and notes**. **No aids (calculators, cell phones, PDAs, etc) are allowed**. Turn off all electronic devices. Place all bags, books, and notes under the table, at the front, or back of the room.
- Place your **WATCARD** on the table, write your **name** and **ID number** on all your booklets and exam sheets, and fill out the exam attendance sheet when provided by the proctor.
- Question marks are listed by the question. **The maximum total mark is 90**.
- Please, write the exam using a **blue/black pen**, not a pencil. Remarking requests will be denied if you use a pencil.
- Be neat and show your work clearly. Poor presentation will be penalized.
- **No questions will be answered during the exam**. If there is an ambiguity, state your assumptions and proceed.
- **No student can leave the exam room in the first 40 minutes or the last 5 minutes.**
- **If you finish before the end of the exam by more than 5 minutes and wish to leave**, remain seated and raise your hand. A proctor will pick up your exam booklet/s from you, at which point you may leave.
- **When the proctors announce the end of the exam,** put down your pens/pencils, close your exam booklet/s, place the exam sheets and any additional booklets inside the first one, and remain seated in silence **(No Talking).** The proctors will collect the exams, count them, and then announce you may leave. **Violations will be penalized.**

Question	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Total**
Mark	/8	/20	/16	/15	/17	/14	/90

Question 1 [8 marks]: Show whether each of the following signals is periodic or non-periodic. In case of a periodic signal, find its fundamental period.

(a) $x(\mathrm{t}) = 2\sin\left(\frac{\pi}{3}t\right) + \cos(\frac{2\pi}{9}t + \pi)$. [5 marks]

(b) $x[\mathrm{n}] = \sin\left(\frac{\pi}{5}\mathrm{n}\right) + \sin\left(\frac{1}{2}\mathrm{n}\right)$. [3 marks]

Question 2 [20 marks]:

(a) Find the power P_x of the signal $x(t) = \begin{Bmatrix} 1 & for\ t < 0 \\ 2 & for\ t \geq 0 \end{Bmatrix}$. [5 marks]

(b) Find and sketch the odd and even components $x_o(t)$ and $x_e(t)$ of the signal $x(t)$ given in part (a). Note: Pay attention to the values at $t = 0$. [10 marks]

(c) Find the value of the integral $I = \int_{-\infty}^{\infty} \sin(\pi t)\, \delta(2t - 3)\, dt$. [5 marks]

Question 3 [16 marks]: A system multiplies its input $x(t)$ by a ramp function $r(t) = t\, u(t)$ to get its output $y(t) = r(t)\, x(t)$.

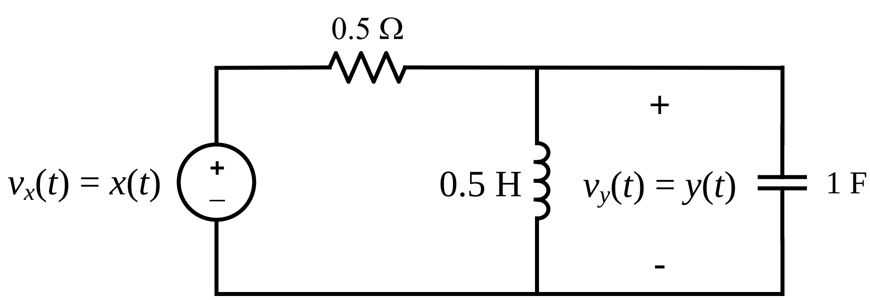
(a) Show with reason whether this system is linear or nonlinear. [6 marks]

(b) Show with reason whether this system is time-invariant. [6 marks]

(c) Show with reason whether this system is causal. [2 marks]

(d) Show with reason whether this system is BIBO stable. [2 marks]

Question 4 [15 marks]: Consider the following circuit.



(a) Derive the system model in the form $Q(D)y(t) = P(D)x(t)$. (**NOTE**: No integrals or $1/D$ terms are allowed in your final answer). [8 marks]

(b) Determine whether this system is asymptotically stable, marginally stable, or unstable. Justify your answer to get the credit. [5 marks]

(c) Determine whether this system is BIBO stable. Justify. [2 mark]

Question 5 [17 marks]: A *linear time-invariant* (LTI) system is described by

$$(D^2 + 3D + 2)y(t) = Dx(t)$$

(a) Find the zero-input response $y_{\mathrm{zi}}(t)$ if $y_{zi}(0) = 2$ and $Dy_{zi}(0) = 3$. [8 marks]

(b) Find the unit impulse response $h(t)$. [9 marks]

<u>Question 6</u> [14 marks]: An LTI system has an impulse response $h(t) = e^{-t}u(t)$.

(a) Find (***without integration***) the system response $y(t)$ due to the input $x(t) = e^{-2t+2}u(t)$. [5 marks]

(b) Repeat part (a) with $x(t) = e^{-t}\, u(t-3)$. [7 marks]

(c) Repeat part (a) with $x(t) = \delta(t-2)$. [2 marks]

Table of Convolutions

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$t\,u(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t), \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^M u(t)$	$t^N u(t)$	$\frac{M!N!}{(M+N+1)!}t^{M+N+1}u(t)$
8	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t), \quad \lambda_1 \neq \lambda_2$
9	$t^M e^{\lambda t}u(t)$	$t^N e^{\lambda t}u(t)$	$\frac{M!N!}{(M+N+1)!}t^{M+N+1}e^{\lambda t}u(t)$
10	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t),$ $\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
11	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1}, \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
12	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t), \quad \lambda_1 \neq \lambda_2$

Useful Formulas: $\cos(a) = \dfrac{e^{ja}+e^{-ja}}{2}$, $\sin(a) = \dfrac{e^{ja}-e^{-ja}}{2j}$

The solution of $ax^2+bx+c=0$ is $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$