York University

Faculty of Science and Engineering

Math 1550

Class Test 1

SOLUTIONS

NAME (print):____
(Family) (Given)

SIGNATURE:____

STUDENT NUMBER:____________________________________

Instructions:

1. Time allowed: 90 minutes.
2. **NO CALCULATORS OR OTHER AIDS PERMITTED**
3. Show your work. Your work must justify any answers you give. Use page backs for any scrap work.
4. Use pen to fill in cover. If you use pencil for your solutions, you may not submit your paper for regrading.
5. There are 8 questions on 8 pages.

Question	Points	Marks
1	10	
2	12	
3	10	
4	14	
5	14	
6	14	
7	16	
8	10	
Total	100	

1. (10 points) Solve the system:

$$\begin{aligned} x \quad + z &= 1 \\ 2x - y + 3z &= 2 \\ 4x + y + 8z &= -1\ . \end{aligned}$$

$x+z=1$

$6x+11z=1$ ~~manum~~ (adding the second & third equations)

Now solving

$6x+6z=6$

$6x+11z=1$

$-5z=5$

$z=-1$

$x+(-1)=1$ $\qquad x=2$

$y=-2+2x+3z=-2+(4)+(-3)$

$=-1$

Solution is

$x=2$

$y=-1$

$z=-1$

2. (12 points) Consider the system

$$\begin{aligned} x + y - z &= 2 \\ 2x - y - 5z &= 1 \\ 3x \quad\; - 6z &= 3\,. \end{aligned}$$

(a) Give all solutions to the system, using parameters in the case of an infinite solution set.

$x + y - z = 2$
$-3y - 3z = -3 \quad E_2 - 2E_1$
$-3y - 3z = -3 \quad E_3 - 3E_1$

$x + y - z = 2$
$y + z = 1$
$0 = 0$

$z = t$
$y = 1 - z = 1 - t$
$x = 2 - y + z = 2 - (1 - t) + t$
$= 1 + 2t$

(b) Is there a solution with $z = -5$? If yes, give it.

Set $\cancel{z}$ $= -5$
$x = -9,\ y = 6,\ z = -5$

(c) Is there a solution with $x = 0$? If yes, give it.

For $x = 0$, set $t = -\frac{1}{2}$

$y = \frac{3}{2}$
$z = -\frac{1}{2}$

3. (10 points) Solve the nonlinear system

$$\begin{aligned} x &= y+6 \\ y &= 3\sqrt{x+4}\,. \end{aligned}$$

From equation 1, $y = 6 - x$

$6 - x = 3\sqrt{x+4}$

$36 - 12x + x^2 = 9(x+4)$

$x^2 - 12x + 36 = 9x + 36$

$x^2 - 21x = 0$

$x(x-21) = 0$

$x = 0$	$x = 21$
$y = -6$	$y = x - 6 = 15$
not a solution as 2nd eqn is not satisfied.	This satisfies both equations.

Solution is $x = 21$, $y = 15$.

4. (14 points) To sell 100 pairs of jeans per week a manufacturer must price them at \$ 18 per pair. To sell 250 pairs of jeans per week a manufacturer must price them at \$ 15 per pair.

   (a) Assuming linear demand, give the demand equation.

$p = 18$ when $q = 100$. $p = 15$ when $q = 250$

Slope is $\dfrac{15-18}{250-100} = \dfrac{-3}{150} = -\dfrac{1}{50}$

$p - 18 = -\frac{1}{50}(q - 100)$ or $p = -\frac{1}{50}q + 20$

Alternatively

$q = 100 + 50(18 - p)$

which can be solved for p to obtain

$p = -\frac{1}{50}q + 20$ as above

   (b) Determine the manufacturer's revenue function and determine the *quantity* he must sell if his revenue is to be a maximum.

Revenue is $pq = \left(-\frac{1}{50}q + 20\right)q$

$= -\frac{1}{50}q^2 + 20q$

It is maximum when

$q = -\dfrac{20}{2\left(-\frac{1}{50}\right)} = 500$ pairs per week.

5. (14 points) Let the supply and demand equations for a particular commodity be, respectively, $20p - 3q = 30$ and $4p + 2q = 45$, with p in dollars and q in ~~thousands of~~ units per week.

(a) Determine the values of p and q at market equilibrium.

$$20p - 3q = 30$$
$$4p + 2q = 45$$

$$20p - 3q = 30$$
$$20p + 10q = 225$$

$$-13q = -195$$
$$q = 15$$
$$20p - 3(15) = 30, \quad 20p = 75$$
$$p = \frac{75}{20} = 3.75$$

(b) Find the new equilibrium *quantity* if the producer receives a subsidy of \$ 1 per unit.

Supply equation becomes
$$20(p+1) - 3q = 30$$

Solve

$$20p - 3q = 10$$
$$4p + 2q = 45$$

$$20p - 3q = 10$$
$$20p + 10q = 225$$

$$-13q = -215$$
$$q = \frac{215}{13}$$ ~~× 1000~~ units per week

6. (14 points) A manufacturer sells his product at \$ 8.35 per unit, selling all he produces. His fixed cost is \$ 2300 and his variable cost is \$ 7.20 per unit.

Total cost is $2300 + 7.20q$

Total revenue is $8.35q$

(a) At what level of production will he break even?

$$8.35q = 2300 + 7.20q$$
$$1.15q = 2300$$
$$q = 2000$$

(b) At what level of production will he have a profit of \$ 4600 ?

$$8.35q - (2300 + 7.20q) = 4600$$
$$1.15q = 6900$$
$$q = 6000$$

(c) At what level of production will he have a loss of \$ 1150 ?

$$8.35q - (2300 + 7.20q) = -1150$$
$$1.15q = 1150$$
$$q = 1000$$

7. (16 points)

(a) Let $\log 3 = x$ and $\log 4 = y$.

i. Express $\log(16\sqrt{3})$ in terms of x and y.

$$\log(16\sqrt{3}) = \log(4^2 3^{1/2}) = 2\log 4 + \frac{1}{2}\log 3$$

$$= 2y + \frac{1}{2}x$$

ii. Express $\log_3 10 + \log_4 100$ in terms of x and y.

$$\log_3 10 + \log_4 100 = \log_3 10 + 2\log_4 10$$

$$= \frac{1}{\log_{10} 3} + 2\left(\frac{1}{\log_{10} 4}\right)$$

$$= \frac{1}{x} + \frac{2}{y}$$

(b) Simplify $e^{\ln x} + \ln e^x + \ln 1$.

$$e^{\ln x} + \ln e^x + \ln 1$$

$$= x + x + 0$$

$$= 2x$$

(c) Solve $\log(x+2) = \log(3x+15) - 1$.

$$\log(x+2) = \log(3x+15) - \log 10$$

$$\log(x+2) = \log\left(\frac{3x+15}{10}\right)$$

$$x + 2 = \frac{3x+15}{10}$$

$$10x + 20 = 3x + 15$$

$$7x = -5$$

$$x = -\frac{5}{7}$$

8. (10 points)

(a) If money triples in 15 years at rate of interest r compounded continuously, find an expression for r.

$$3P = Pe^{r(15)}$$
$$3 = e^{r(15)}$$
$$\ln 3 = 15r$$
$$r = \frac{\ln 3}{15}$$

(b) At a nominal rate of 7% per year compounded quarterly, write an expression for the number of years it takes \$ 1000 to double?

$$2P = P\left(1 + \frac{.07}{4}\right)^{4t}$$
$$2 = (1.0175)^{4t}$$
$$\log 2 = 4t \log(1.0175)$$
$$t = \frac{\log 2}{4 \log(1.0175)} \text{ years}$$

The end