Name:_______________________________________uwuserid:_________________________________

ECE 486/687: Robot Dynamics and Control
Final Examination

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April 17, 2010
12:30 – 3:00pm (2hrs 30min)

Please read the instructions below before beginning the exam:

1. This is a closed book exam, no aids are permitted.
2. A formula sheet is attached to the back of this exam. You may detach the formula sheet for use during the exam.
3. The examination is an independent activity. Please work alone.
4. You can use pencil or pen to answer the questions. Make sure your answers are legible and dark enough to see.
5. Please provide all answers in the space provided below each question.
6. For all questions, provide justification for your answer. Correct answers with no explanation will not get full marks.
7. No questions will be answered during the exam period. If you believe there is missing or incorrect information in a problem statement, make an assumption, state it clearly at the start of your response, and proceed to provide the solution. If the assumptions do not trivialize the answer, we will grade accordingly.
8. In case pages come apart, write your name and UW userid (QUEST userid) at the top of each page before beginning work. For your own privacy, do not include your student number anywhere on the paper.
9. You have 2.5 hours to complete the exam. You must stay for the first hour. If you are finished early, you may raise your hand to turn in your paper and leave in silence, up to 10 minutes before the end of the period. Afterwards, please remain seated quietly and wait for the the examination period to finish.
10. After reading and understanding these instructions, please fill in and sign your name below. Please use the last name the registrar has on file.

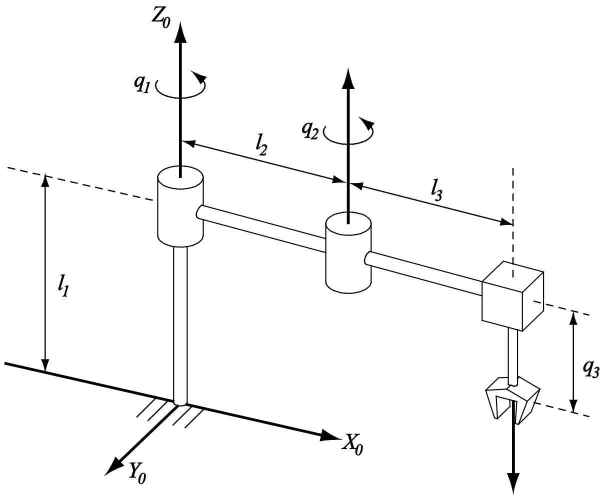
Last Name	**First Name(s)**	**uwuserid**	**Signature**

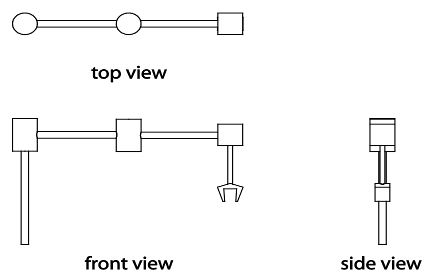
For markers only:

Q1	Q2	Q3	Q4	Q5	Total
/20	/20	/20	/20	/20	

Question 1 [20 marks total]

For this question, we will consider the manipulator shown below:





The forward kinematics of this manipulator is given by:

$$T_{01} = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 & l_2\cos(q_1) \\ \sin(q_1) & \cos(q_1) & 0 & l_2\sin(q_1) \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} \cos(q_2) & \sin(q_2) & 0 & l_3\cos(q_2) \\ \sin(q_2) & -\cos(q_2) & 0 & l_3\sin(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cos(q_1+q_2) & \sin(q_1+q_2) & 0 & l_2\cos(q_1) + l_3\cos(q_1+q_2) \\ \sin(q_1+q_2) & -\cos(q_1+q_2) & 0 & l_2\sin(q_1) + l_3\sin(q_1+q_2) \\ 0 & 0 & -1 & l_1 - q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

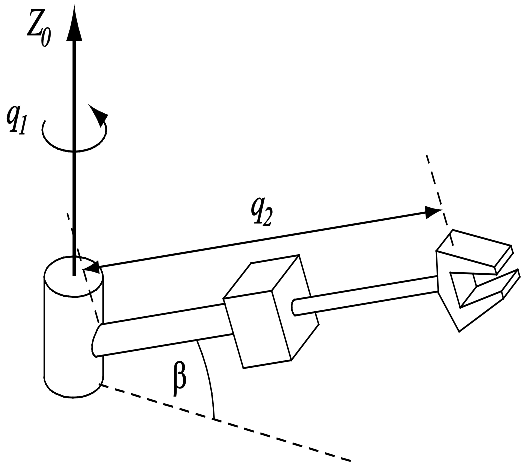
(a) **[10 marks]** Using the method of your choice, find the <u>position</u> inverse kinematics for this manipulator. Indicate which method you are using. Make sure to provide ALL valid solutions.

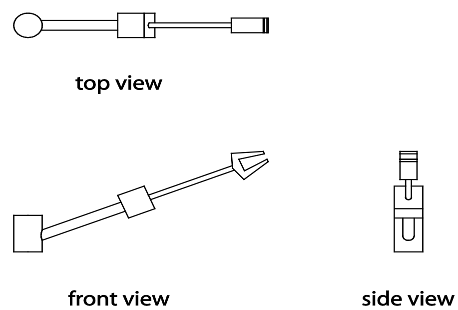
(b) **[5 marks]** Find the linear velocity Jacobian for this manipulator.

(c) **[5 marks]** Is there an alternate way to compute the inverse kinematics of this manipulator, by using the Jacobian computed in part (b)? If yes, outline the alternate solution approach.

Question 2 [20 marks total]

For this question, we will consider the manipulator shown below:





The first link has mass m_1 and the second link has mass m_2. Assume that both links are of length l, have uniform mass density and that the thin bar assumption can be applied (i.e., you can assume that the mass and inertia of the link 2 motor box and the end effector are negligible).

(a) **[15 marks]** Using the method of your choice, find the equations of motion for this manipulator. Indicate which method you are using.

(b) **[5 marks]** Now assume that there is significant friction at the prismatic joint, which can be modeled by $F_{friction} = K_f \dot{q}_2$. How would you modify the equations of motion to account for this friction?

Question 3 [20 marks total]

Given a robot system with the equations of motions described by:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) = U$$

where $\boldsymbol{q}$ is an n-dimensional vector of joint positions

(a) **[5 marks]** What is the control input U required to implement an independent joint PD controller for this system?

(b) **[10 marks]** Let $\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}^d$. Let us consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\dot{\boldsymbol{q}}^T M \dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{q}}^T K \tilde{\boldsymbol{q}}$$

where K is a constant positive definite matrix. Is it possible to show that the PD controller designed in part (a) is stable, using the above Lyapunov function candidate? Why or why not?

(c) **[5 marks]** If you answered yes in (b), what should K be to show stability? If you answered no in (b), what additional terms should U contain to ensure stability under this Lyapunov function candidate?

Question 4 [20 marks total]

(a) **[5 marks]** Outline the probabilistic roadmap planning method. You can use pseudocode to describe the algorithm.

(b) **[5 marks]** Can the probabilistic roadmap planning method be used for on-line path planning in a dynamic environment? Why or why not?

(c) **[10 marks]** Assume that you are given two via points A and B. Your task is to design a linear segments with parabolic blends (LSPB) trajectory between these two points. The desired end point conditions are as follows:

- The position and velocity are zero at via point A.

- The position is $q_B > 0$ and the velocity is $\dot{q}_B > 0$ at via point B.

The robot has a maximum acceleration a_{max}, a maximum deceleration $-a_{max}$ and a maximum velocity v_{max}, where $\dot{q}_B < v_{max}$. Assume that q_B is large enough so that maximum velocity is reached during the trajectory.

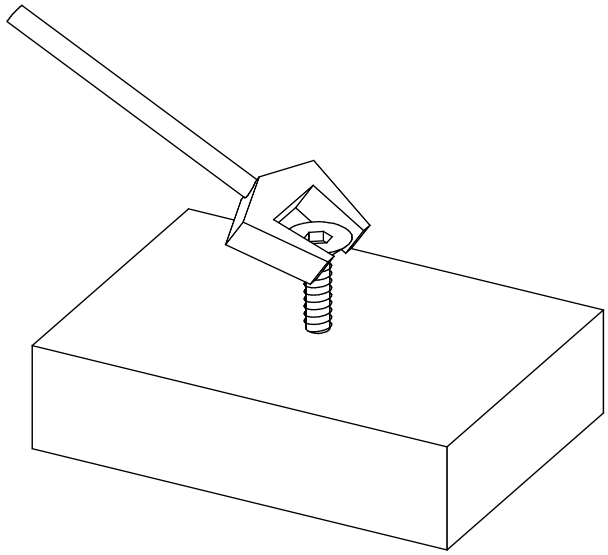
(i) Sketch the LSPB trajectory from via point A to via point B. Show the position, velocity and acceleration profile, and clearly label all trajectory segments.

(ii) Derive the time law equation for each segment you identified in part (c)(i).

(iii) Compute the coefficients for each segment as a function of the endpoint conditions and robot constraints.

Question 5 [20 marks total]

For this question, we will consider the task of turning a screw into a fixed workpiece, as shown below:



(a) **[5 marks]** For the task illustrated above:

(i) On the figure above, define and sketch an appropriate constraint frame.

(ii) Find the natural and artificial constraints.

(b) **[15 marks]** Now assume that the interaction components of the task above will be controlled with an impedance controller.

(i) What will be the effective force seen by the end effector under the impedance controller? Clearly define all terms in your equation.

(ii) Assume that the task space equations of motion for the robot can be written as:

$$\Lambda(\boldsymbol{q})\ddot{\boldsymbol{X}} + \Upsilon(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{X}} + \Pi(\boldsymbol{q}) = U_f$$

What should the control input U_f be to implement the desired end effector force you specified in part (b)(i)?

(iii) Sketch the impedance controller block diagram. Clearly label all blocks and signals.

Robot Dynamics & Control: Useful Formulae

$$\cos^2\theta + \sin^2\theta = 1 \qquad c^2 = a^2 + b^2 - 2ab\cos\theta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\pi/2+\theta) = -\sin\theta \qquad \sin(\pi/2+\theta) = \cos\theta$$
$$\cos(\pi+\theta) = -\cos\theta \qquad \sin(\pi+\theta) = -\sin\theta$$
$$\cos(\pi/2-\theta) = \sin\theta \qquad \sin(\pi/2-\theta) = \cos\theta$$
$$\cos(\pi-\theta) = -\cos\theta \qquad \sin(\pi-\theta) = \sin\theta$$
$$\cos(-\theta) = \cos\theta \qquad \sin(-\theta) = -\sin\theta$$
$$\frac{d}{d\theta}\sin\theta = \cos\theta \qquad \tfrac{d}{d\theta}\cos\theta = -\sin\theta$$

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_{i-1,i} = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(DH1) $x_i \perp z_{i-1}$
(DH2) $x_i \cap z_{i-1} \neq \oslash$

$$\dot{R}(t) = S(\boldsymbol{\omega})R(t)$$

$$S(\boldsymbol{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{v}_e^0 \\ \boldsymbol{\omega}_e^0 \end{bmatrix} = \begin{bmatrix} \dots & \boldsymbol{z}_i^0 \times \boldsymbol{d}_{ie}^0 & \dots & \boldsymbol{z}_j^0 & \dots \\ \dots & \boldsymbol{z}_i^0 & \dots & \boldsymbol{0} & \dots \end{bmatrix} \begin{bmatrix} \vdots \\ \dot{q}_{i+1} \\ \vdots \\ \dot{q}_{j+1} \\ \vdots \end{bmatrix} = J_0\dot{\boldsymbol{q}}$$

$$\mathcal{L} = \mathcal{T} - \mathcal{V}$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\lambda}_i} - \frac{\partial\mathcal{L}}{\partial\lambda_i} = \xi_i$$

$$D = \sum_{i=1}^{n}[m_i(J_{VC}^{(i)0})^T J_{VC}^{(i)0} + (J_\omega^{(i)0})^T R_{0i} I_i^i R_{0i}^T J_\omega^{(i)0}]$$

$$\sum_j d_{kj}\ddot{q}_j + \sum_{i,j} c_{ijk}\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k$$

$$c_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}$$

$$\begin{aligned}
I_{xx} &= \int\int\int (y^2+z^2)\rho(x,y,z)dxdydz \\
I_{yy} &= \int\int\int (x^2+z^2)\rho(x,y,z)dxdydz \\
I_{zz} &= \int\int\int (x^2+y^2)\rho(x,y,z)dxdydz \\
I_{xy} = I_{yx} &= -\int\int\int xy\rho(x,y,z)dxdydz \\
I_{xz} = I_{zx} &= -\int\int\int xz\rho(x,y,z)dxdydz \\
I_{yz} = I_{zy} &= -\int\int\int yz\rho(x,y,z)dxdydz
\end{aligned}$$

Thin rod assumption, bar concentrated along the y-axis

$$I_c = \begin{bmatrix} \frac{ml^2}{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{ml^2}{12} \end{bmatrix}$$

Recursive Newton-Euler Equations

$$\begin{aligned}
\boldsymbol{\omega}_i^i &= \begin{cases} R_{i-1,i}^T\boldsymbol{\omega}_{i-1}^{i-1} + R_{i-1,i}^T\boldsymbol{k}\dot{q}_i & \textit{revolute} \\ R_{i-1,i}^T\boldsymbol{\omega}_{i-1}^{i-1} & \textit{prismatic} \end{cases} \\
\boldsymbol{\alpha}_i^i &= \begin{cases} R_{i-1,i}^T\boldsymbol{\alpha}_{i-1}^{i-1} + R_{i-1,i}^T\boldsymbol{k}\ddot{q}_i + \boldsymbol{\omega}_i^i \times (R_{i-1,i}^T\boldsymbol{k})\dot{q}_i & \textit{revolute} \\ R_{i-1,i}^T\boldsymbol{\alpha}_{i-1}^{i-1} & \textit{prismatic} \end{cases} \\
\boldsymbol{a}_{c,i}^i &= \begin{cases} R_{i-1,i}^T\boldsymbol{a}_{e,i-1}^{i-1} + \boldsymbol{\alpha}_i^i \times \boldsymbol{r}_{i,c_i}^i + \boldsymbol{\omega}_i^i \times (\boldsymbol{\omega}_i^i \times \boldsymbol{r}_{i,c_i}^i) & \textit{revolute} \\ R_{i-1,i}^T\boldsymbol{a}_{e,i-1}^{i-1} + \boldsymbol{\alpha}_i^i \times \boldsymbol{r}_{i,c_i}^i + \boldsymbol{\omega}_i^i \times (\boldsymbol{\omega}_i^i \times \boldsymbol{r}_{i,c_i}^i) + R_{i-1,i}^T\boldsymbol{k}\ddot{q}_i + 2\dot{q}_i\boldsymbol{\omega}_i^i \times R_{i-1,i}^T\boldsymbol{k} & \textit{prismatic} \end{cases} \\
\boldsymbol{a}_{e,i}^i &= \begin{cases} R_{i-1,i}^T\boldsymbol{a}_{e,i-1}^{i-1} + \boldsymbol{\alpha}_i^i \times \boldsymbol{r}_{i,i+i}^i + \boldsymbol{\omega}_i^i \times (\boldsymbol{\omega}_i^i \times \boldsymbol{r}_{i,i+1}^i) & \textit{revolute} \\ R_{i-1,i}^T\boldsymbol{a}_{e,i-1}^{i-1} + \boldsymbol{\alpha}_i^i \times \boldsymbol{r}_{i,i+i}^i + \boldsymbol{\omega}_i^i \times (\boldsymbol{\omega}_i^i \times \boldsymbol{r}_{i,i+1}^i) + R_{i-1,i}^T\boldsymbol{k}\ddot{q}_i + 2\dot{q}_i\boldsymbol{\omega}_i^i \times R_{i-1,i}^T\boldsymbol{k} & \textit{prismatic} \end{cases}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{f}_i &= R_{i,i+1}\boldsymbol{f}_{i+1} - m_i\boldsymbol{g}_i + m_i\boldsymbol{a}_{c,i} \\
\boldsymbol{h}_i &= R_{i,i+1}\boldsymbol{h}_{i+1} - \boldsymbol{f}_i \times \boldsymbol{r}_{i,c_i} + (R_{i,i+1}\boldsymbol{f}_{i+1}) \times \boldsymbol{r}_{i+1,c_i} + I\boldsymbol{\alpha}_i + \boldsymbol{\omega}_i \times I\boldsymbol{\omega}_i
\end{aligned}$$

$$\boldsymbol{\tau}_i = \begin{cases} \left[R_{i-1,i}\boldsymbol{h}_i\right]^T\boldsymbol{k} & \textit{revolute} \\ \left[R_{i-1,i}\boldsymbol{f}_i\right]^T\boldsymbol{k} & \textit{prismatic} \end{cases}$$