Math 1505 Quiz 1

7 p.m. to 8:20 p.m. October 26, 2010

Student Name: _____

Student Number: _____

- Closed book, closed notes;
- Non-graphing, non-programmable calculator only;
- No collaboration;
- Please read each problem carefully and show all your work.

Question	Mark	Question	Mark
1	/5	4	/5
2	/8	5	/5
3	/9	6	/8
		Total	/40

1. Solve the inequality $|6 - 5x| < 7$.
   Give your solution using interval notations. [5]

$\Rightarrow -7 < 6-5x < 7$

$\Rightarrow -13 < -5x < 1$

$\Rightarrow \frac{13}{5} > x > -\frac{1}{5}$

Ans. $(-\frac{1}{5}, \frac{13}{5})$

Name: _____ 2

2. Solve $2\cos\theta\sin\theta = \sin\theta$

(a) on $[0, 2\pi)$ [6]

$\sin\theta(2\cos\theta - 1) = 0.$

$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}.$

$\theta = 0, \pi$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}.$

(b) on $(-\infty, \infty)$ [2]

$\theta = 0 + 2\pi k$ or $\pi + 2\pi k$ or $\frac{\pi}{3} + 2\pi k$ or $\frac{5\pi}{3} + 2\pi k$

$\pi k.$

for integer k.

Name: _____ 3

3. (a) Solve [4]

$\log_2(x-3) + \log_2(x+3) = 4.$

$\Rightarrow \log_2(x-3)(x+3) = 4$

$\Rightarrow \log_2(x-3)(x+3) = \log_2 2^4$

$\Rightarrow (x-3)(x+3) = 2^4 = 16$

$\Rightarrow x^2 - 9 = 16$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5$ or -5

↑

but $\log_2(-5-3)$ is undefined

Ans: $x = 5$

(b) Solve [5]

$$e^x - 12e^{-x} - 1 = 0.$$

Let $u = e^x$ $\Rightarrow u - 12u^{-1} - 1 = 0$

$\Rightarrow u^2 - 12 - u = 0$

$\Rightarrow (u-4)(u+3) = 0$

$\Rightarrow u = 4$ or $u = -3$

$e^x = 4$ $\quad e^x = -3$ no solution

$x = \ln 4$

Answer: $x = \ln 4$

4. Solve [5]

$$\begin{aligned} 2x - z &= 1 \\ -x + y + 3z &= -1 \\ x - y + z &= -3. \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ -1 & 1 & 3 & -1 \\ 1 & -1 & 1 & -3 \end{array}\right]$$

$\Rightarrow$ row3+row2

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ -1 & 1 & 3 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right]$$

$\Rightarrow$ 2×row2+row1

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 0 & 2 & 5 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right]$$

$\Rightarrow$ $2x - z = 1$

$2y + 5z = -1$

$4z = -4$

$\Rightarrow$ $z = -1$

$y = \frac{1}{2}(-1 - 5z) = 2$

$x = \frac{1}{2}(1 + z) = 0$

Answer $x = 0$, $y = 2$, $z = -1$

Name: _____ 6

5. Consider

$$\begin{aligned} 2x - y &= 1 \\ 3x + ay &= 0. \end{aligned}$$

For which values of a are there no solutions, exactly one solution, and infinitely many solutions? [5]

$$\left[\begin{array}{cc|c} 2 & -1 & 1 \\ 3 & a & 0 \end{array}\right]$$

$$\Rightarrow \quad \left[\begin{array}{cc|c} 2 & -1 & 1 \\ 0 & 2a+3 & -3 \end{array}\right] \quad 2\times\text{row}2 - 3\times\text{row}1$$

$$\Rightarrow \quad \begin{aligned} 2x - y &= 1 \\ (2a+3)y &= -3 \end{aligned}$$

if $a = -\frac{3}{2}$ then $(2a+3)y = -3$ becomes $0 = -3$

and the system of equations has no solution.

if $a \neq -\frac{3}{2}$ then $2a+3 \neq 0$ and $y = -\frac{3}{2a+3}$

$$x = \frac{1}{2}(1+y) = \frac{a}{2a+3}$$

the system has exactly one solution

Name: _____ 7

6. (a) Evaluate [4]

$$\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x^2}.$$

$$= \lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x^2} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$$

$$= \lim_{x\to 0} \frac{1-(1-x^2)}{x^2(1+\sqrt{1-x^2})}$$

$$= \lim_{x\to 0} \frac{x^2}{x^2(1+\sqrt{1-x^2})}$$

$$= \lim_{x\to 0} \frac{1}{1+\sqrt{1-x^2}}$$

$$= \frac{1}{1+\sqrt{1-0^2}}$$

$$= \frac{1}{2} \quad //$$

(b) Evaluate [4]

$$\lim_{x\to 1} \sec\left(\frac{x^2 - x + \pi}{x^2 - 4}\right)$$

Let $f(t) = \sec t$ and $g(x) = \frac{x^2 - x - \pi}{x^2 - 4}$

note $\lim_{x\to 1} x^2 - 4 = 1^2 - 4 = -3 \neq 0$.

$\Rightarrow \lim_{x\to 1} g(x) = \frac{1^2 - 1 - \pi}{1^2 - 4} = -\frac{\pi}{3}$

$\lim_{x\to 1} \sec\left(\frac{x^2 - x + \pi}{x^2 - 4}\right) = \lim_{t\to -\frac{\pi}{3}} \sec t = \sec\left(-\frac{\pi}{3}\right) = \frac{1}{\cos(-\frac{\pi}{3})} = \frac{1}{1/2} = 2$

Special angles:

Angle θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0