Solution to ECO206 2013

Midterm 1

Section: L0201

1 Q1

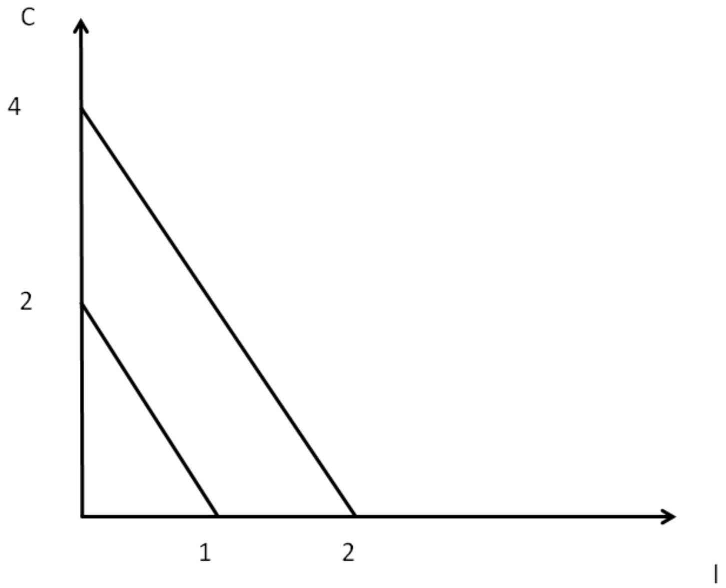
1. Bilbo is choosing between lembas (l) and cram (c) when packing for his long trek. He cares about total calories consumed and doesn't worry about how they taste. A unit of lembas has twice the calories than a unit of cram.

(a) (5) Write down a utility function that captures Bilbo's preferences. Explain your reasoning. Draw his Indifference Curves on a clearly labeled diagram.

Solution: Since a unit of lembas has twice the calories than a unit of cram and all Bilbo cares about is total calories:

$$U(l, c) = 2l + c$$

The MRS is constant at every possible bundle. He is willing to trade 2 units of cram for an extra unit of lembas. They are perfect substitutes.



(b) (8) If the price of Lembas $p_l = 5$ and the price of cram $p_c = 3$. Calculate the optimal quantity of cram and lemba that Bilbo consumes. Show your working clearly.

Solution:

We have a case of perfect substitutes so cannot use the Lagrangian mehod. We can solve the problem by comparing the marginal willingness to pay to the price ratio. From the MRS we know that Bilbo is willing to give up 2 cram for 1 lemba.

$$MRS = \frac{-MU_l}{MU_c} = -2$$

From the price ratio we know that he only has to give up 1.66 cram for an additional lembas.

$$-\frac{p_l}{p_c} = -1.66$$

This gives us a corner solution where he spends all this money on Lembas and none on cram.

An alternate way of doing this is to use the marginal utility per dollar

$$\frac{MU_l}{p_l} = \frac{2}{5}$$

$$\frac{MU_c}{p_c} = \frac{1}{3}$$

Thus we have

$$\frac{MU_l}{p_l} > \frac{MU_c}{p_c}$$

So the optimal allocation is

$$\begin{aligned} l &= \frac{I}{p_l} = \frac{I}{5} \\ c &= 0 \end{aligned}$$

(c) (12)Suppose the price of Lembas rises to $p_l = 7$. Calculate the income and substitution effect of this change for lembas. Draw a clearly labeled graph. Show all the steps of your working clearly.

Solution:

The starting point is the chosen allocation $(l, c) = \left(\frac{I}{5}, 0\right)$ in part (a).

To find the final allocation:

We know that Bilbo is still willing to give up 2 cram for 1 lemba.

$$MRS = \frac{-MU_l}{MU_c} = -2$$

From the price ratio we know that now he has to give up 2.33 cram for an additional lembas.

$$-\frac{p_l}{p_c} = -\frac{7}{3} = -2.33$$

This gives us a corner solution where he spends all this money on cram and none on lembas.

Alternately, If $P_l' = 7$

$$\frac{MU_l}{P_l'} = \frac{2}{7}$$

So

$$\frac{MU_l}{P_l'} < \frac{MU_c}{P_c}$$

So all money will be spent on Cram.

$$\begin{aligned} l &= 0 \\ c &= \frac{I}{P_c} = \frac{I}{3} \end{aligned}$$

To get the substitution effects, we keep the utility same as the starting bundle we need to target

$$u\left(\frac{I}{5}, 0\right) = 2 * \frac{I}{5} + 0$$

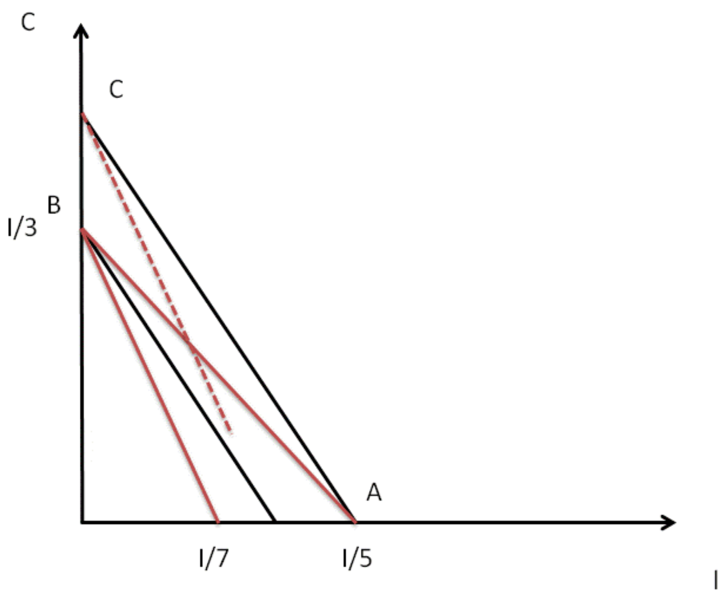
With the new prices we know that Bilbo will consume at a corner where $l = 0$. We need to find the minimum income needed to reach this utility with the new prices. The number of cram we need to consume to his this level of utility is

$$c = \frac{2I}{5}$$

as 1 unit of cram results in 1 calorie. The cost of this is

$$\frac{2I}{5} * 3$$

The substitution effect on lembas is going from the point $\left(\frac{I}{5}, 0\right)$ to $\left(0, \frac{2I}{5}\right)$ i.e. $-\frac{I}{5}$ and the income effect is going from point $\left(0, \frac{2I}{5}\right)$ to $\left(0, \frac{I}{3}\right)$ i.e. 0.



For cram, the substitution ubstitution effect is $c^C - c^A = \frac{2I}{5}$, income effect is $c^B - c^C = \frac{I}{15}$

2 Q2

Ziggy's preferences over lettuce (l) and chocolate (c) are given by $u(l, c) = 100l^{0.5} + c$ The prices of l and c are $p_l = 10$ and $p_c = 1$ respectively. His income is 100

(a) (12) The government wants to encourage people to get healthier (i.e. eat more lettuce) so it gives everyone an lump sum increase in income of 50. That is, the government gives him additional income of 50 . Calculate the increase in Ziggy's lettuce consumption due to this.

Solution:

$$u(l, c) = 100l^{0.5} + c$$

$$\max_{l,c} u(l,c)$$
$$s.t \quad : \quad p_l l + p_c c = I$$

$$L = 100l^{0.5} + c - \lambda \left[p_l l + p_c c - I \right]$$

F.O.C

$$\begin{aligned} \frac{\partial L}{\partial j} &= 100 * 0.5 * l^{-0.5} - \lambda p_l = 0 \\ \frac{\partial L}{\partial b} &= l - \lambda p_c = 0 \\ \frac{\partial L}{\partial \lambda} &= p_l l + p_c c - I = 0 \end{aligned}$$

Solve and get

$$\begin{aligned} \lambda &= 1 \\ l &= \frac{2500}{p_l^2} = 25 \\ c &= \frac{I - p_l l}{p_c} \end{aligned}$$

When $I = 100, l = 25$, we have $c < 0$. Thus it is a corner solution.

$$\begin{aligned} l &= 10 \\ c &= 0 \end{aligned}$$

When $I = 150, l = 25$, we have $c < 0$. Thus it is still a corner solution.

$$\begin{aligned} l &= 15 \\ c &= 0 \end{aligned}$$

So Ziggy's lettuce consumption increases by 5 units.

(b) (5) What explain the size and sign of the results you found in part (a). Explain your reasoning. Would you recommend that the government implement the policy? Why or why not?

Solution: Ziggy's preferences are quasilinear in lettuce which is why the Lagrangian gives us 25 as the optimal l in both cases. Quasilinear preferences should have no change in l due to an increase in income, keeping opportunity costs the same. In general, I would not recommend the policy because it would just result in more chocolate being bought. However, for Ziggy specifically, since we start and end at a corner, any increases in income translate into increases in lettuce consumption. If for some reason Ziggy's income rose such that he could afford $l = 25$, I would not recommend the policy then.

(c) (12) Instead of the policy in part (a) the government decides to subsidize the price of lettuce instead.That is, the price of lettuce becomes $10 - s$ where s is the subsidy. How much of a subsidy should the govt give him to get him to consume 5 more units of lettuce than he does with the lump sum gift from the government?

Solution:

We know that the optimal choice of l is 25 with the old prices. As the price of l falls, from the substitution effect, at an interior optimum $\left(MRS = -\frac{p_1}{p_2}\right)$ Ziggy would want to consume more than 25 units (cheaper l so buy more). Quasilinear preferences so income effect $= 0$. If we are looking to raise his consumption from 15 to 20 units of l we know that the only reason he would want to do that is if he is still at a corner solution where he spends all of this income on l. The subsidy that would get him to this solves

$$
\begin{aligned}
l &= \frac{100}{10 - s} = 20 \\
&\Rightarrow s = 5
\end{aligned}
$$

3 Q3

3. Jill has preferences over x_1 and x_2 given by $u(x_1, x_2) = 0.5 \ln(x_1) + 0.5 \ln(x_2)$ where p_1 and p_2 are their respective prices. Jill enters the market with an endowment of e_1 units of x_1 and e_2 units of x_2

(a) (10) What is her own price demand curve for x_1? Calculate it using Lagrangians and draw it on a clearly labeled diagram. Show all your working.

Solution:

$$u(x_1, x_2) = 0.5 \ln(x_1) + 0.5 \ln(x_2)$$

$$\max_{x_1, x_2} u(x_1, x_2)$$
$$s.t \quad : \quad p_1 x_1 + p_2 x_2 = p_1 e_1 + p_2 e_2$$

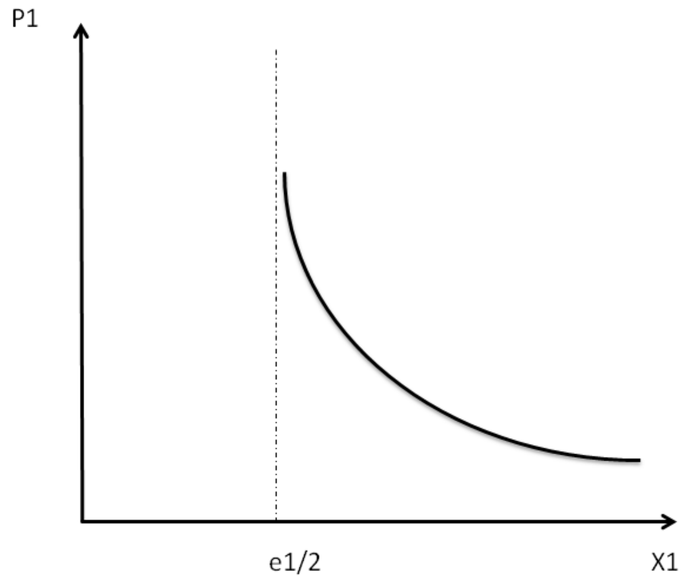
$$L = 0.5 \ln(x_1) + 0.5 \ln(x_2) - \lambda \left[p_1 x_1 + p_2 x_2 - p_1 e_1 - p_2 e_2\right]$$

F.O.C

$$\begin{aligned} \frac{\partial L}{\partial j} &= \frac{0.5}{x_1} - \lambda p_1 = 0 \\ \frac{\partial L}{\partial b} &= \frac{0.5}{x_2} - \lambda p_2 = 0 \\ \frac{\partial L}{\partial \lambda} &= p_1 x_1 + p_2 x_2 - p_1 e_1 - p_2 e_2 = 0 \end{aligned}$$

Solve and get

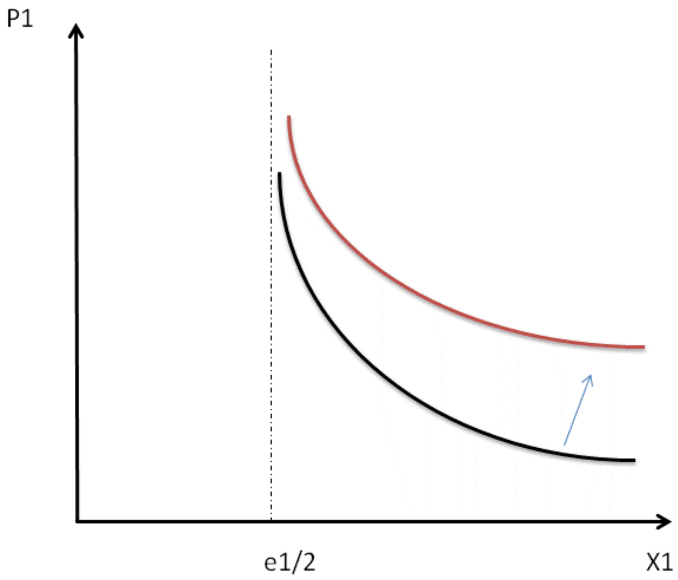
$$\begin{aligned} x_1 &= \frac{p_1 e_1 + p_2 e_2}{2p_1} = \frac{e_1}{2} + \frac{p_2 e_2}{2p_1} \\ x_2 &= \frac{p_1 e_1 + p_2 e_2}{2p_2} = \frac{p_1 e_2}{2p_2} + \frac{e_2}{2} \end{aligned}$$



(b) (4) If the price of x_2 increased, how would it affect Jill's own price demand curve for x_1 ? Explain your reasoning and show the change on a clearly labeled graph.

Solution:

When p_2 increases, Jill's own price demand curve for x_1 will move outwards. From the substitution effect as $p_2 \uparrow$ x_1 becomes relatively more expensive so $x_1 \uparrow$. From the income effect, we can see that x_1 is a normal good and when $p_2 \uparrow$ the value of Jill's endowment increases. Both the income and substitution effect move in the same direction so for the same price, p_1, Jill demands a higher qyt of x_1.



(c) (7) If the market price of x_1 increases. Would Jill spend more or less on x_1? Explain your reasoning and show your calculations clearly.

Solution: When p_1 increases, Jill buy less x_1.

$$x_1 = \frac{e_1}{2} + \frac{p_2 e_2}{2p_1}$$

when p_1 increase, x_1 will decrease. The substitution effect is bigger than the income effect. However the change in her spending on x_1 depends on the elasticity of her demand

$$\begin{aligned} e_d &= \frac{dx_1}{dp_1}\frac{p_1}{x_1} \\ &= -\frac{p_2 e_2}{2p_1^2}\frac{p_1}{\left(\frac{e_1}{2} + \frac{p_2 e_2}{2p_1}\right)} \\ &= -\frac{p_2 e_2}{p_1 e_1 + p_2 e_2} \end{aligned}$$

Thus we can see that $e_d \geq -1$ which means that her total spending will not decrease when p_1 increases.